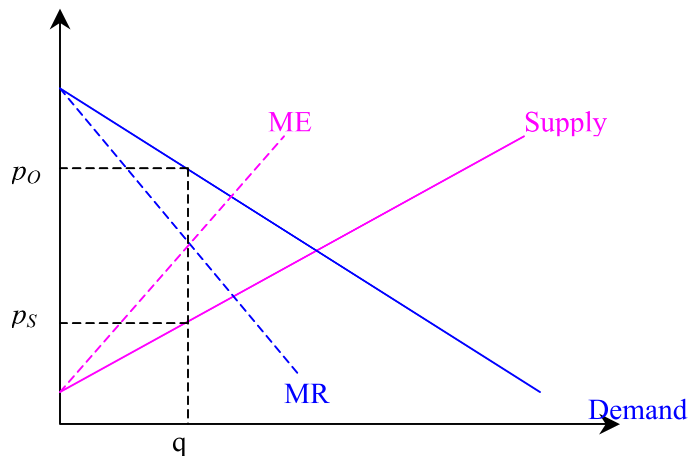
Economics 370
Microeconomic Theory
Problem Set 7
Answer Key

1) *A firm is a monopoly in its output market and a monopsony in its input market. Its only input is the finished good, which it buys from a competitive market with an upward sloping supply curve. The firm sells the same good to competitive buyers in the output market. Determine its profit-maximizing output. What price does it charge in the output market? What price does it pay to its suppliers?*

The firm optimizes profits like any other firm. Profits, in this case, are best expressed as:
Π = Revenue – Expenditure.
So profit maximization requires MR = ME.

Mathematically, if the inverse demand function is p(q), and the inverse supply function is w(q), then profits are:
Π(q) = p(q) × q – w(q) × q.
Profit Maximization then requires
MR = ME or p′(q) × q + p(q) = w′(q) × q + w(q)

Graphically, the problem is solved as:



Here, q is the amount produced, p_o is the price charged in the output market, and p_s is the price it pays its suppliers.

2) *Southwest Airlines' cost to fly one seat one mile is 7.5¢ compared to 15¢ for USAir. Assume that Southwest and USAir are the only two firms competing on a single route. The distance traveled on the route is 500 miles. If the market demand function is q = 600 – p, what is the Cournot equilibrium (where p is in dollars)?*

Let c_s represent the cost per seat for Southwest for a single trip, and
Let c_A represent the cost per seat for USAir for a single trip.
Then $c_s = 0.075 \times 500 = \$37.5$, and $c_A = 0.15 \times 500 = \$75$.

A Cournot competitor maximizes profits by choosing the optimal *production level*, taking into account what its competitor produces. For Southwest, this means:

$$\Pi = \max_{q_s}\{p(q_A + q_s)q_s - c_s q_s\} = \max_{q_s}\{(600 - q_A - q_s)q_s - c_s q_s\}.$$

Profit maximization implies that: $600 - q_A - c_s - 2q_s = 0$; or $q_s = 300 - \frac{q_A + c_s}{2}$.

By symmetry, we also have $q_A = 300 - \frac{q_s + c_A}{2}$.

Substituting the second expression into the first gives us:
$q_s = 300 - \frac{c_s}{2} - \frac{300}{2} - \frac{1}{2}\left(-\frac{q_s + c_A}{2}\right)$, or $\frac{3}{4}q_s = 150 - \frac{c_s}{2} + \frac{c_A}{4}$, or $q_s = 200 - \frac{2c_s}{3} + \frac{c_A}{3}$.

By symmetry, we have $q_A = 200 - \frac{2c_A}{3} + \frac{c_s}{3}$.

Inserting the numbers from above gives us $q_s = 200 - \frac{2}{3}(37.5) + \frac{1}{3}(75) = 200$, and
$q_A = 200 - \frac{2}{3}(75) + \frac{1}{3}(37.5) = 162.5$.
So $p = 600 - q_s - q_A = 600 - 200 - 162.5 = \$237.5$.

3) *Suppose that identical duopoly firms have constant marginal costs of \$10 per unit. Firm 1 faces a demand function of*

$$q_1 = 100 - 2p_1 + p_2$$

Where q_1 is firm 1's output, p_1 is firm 1's price, and p_2 is firm 2's price. Similarly, the demand firm 2 faces is:

$$q_2 = 100 - 2p_2 + p_1$$

a) *Solve for the Bertrand equilibrium.*

Let c_1 represent the marginal cost for Firm 1, and
Let c_2 represent the marginal cost for Firm 2.
Here $c_1 = c_2 = \$10$.

A Bertrand competitor maximizes profits by choosing the optimal *price*, taking into account its competitor's price. For Firm 1, this means:
$\Pi = \max_{p_1}\{p_1 q_1(p_1, p_2) - c_1 q_1\} = \max_{q_s}\{(p_1 - c_1)(100 - 2p_1 + p_2)\}$.
Profit maximization implies that $\frac{d}{dp_1}\{(p_1 - c_1)(100 - 2p_1 + p_2)\} = 0$, or
$(100 - 2p_1 + p_2) - 2(p_1 - c_1) = 0$, or $4p_1 = 100 + p_2 + 2c_1$.
By symmetry, we also have $4p_2 = 100 + p_1 + 2c_2$.

Substituting the second expression into the first gives us $p_1 = \frac{1}{4}\left[100 + 2c_1 + \frac{1}{4}(100 + 2c_2 + p_1)\right]$,
or $\frac{15}{16}p_1 = \frac{125}{4} + \frac{c_1}{2} + \frac{c_2}{8}$, or $p_1 = \frac{500 + 8c_1 + 2c_2}{15}$.
By symmetry, $p_2 = \frac{500 + 8c_2 + 2c_1}{15}$.
Substituting the marginal costs above gives: $p_1 = p_2 = \$40$, and $q_1 = q_2 = 60$.

b) *Solve for the Bertrand equilibrium if both firms have a marginal cost of \$0 per unit.*

Substituting the appropriate marginal costs into the expression above gives
$p_1 = p_2 = \$33.33$, and $q_1 = q_2 = 66.67$.

c) *Solve for the Bertrand equilibrium if firm 1 has a marginal cost of \$30 and firm 2 has a marginal cost of \$10 per unit.*

Substituting the appropriate marginal costs into the expression above gives
$p_1 = \$50.67$, $p_2 = \$42.67$, $q_1 = \$41.33$, and $q_2 = \$65.33$.

4) *Suppose that Panasonic and Zenith are the only two firms than can produce a new type of high-definition television. The payoffs (in millions of dollars) from entering this product market are shown in the following payoff matrix. Be sure to consider both pure and mixed strategies below as appropriate.*

		Panasonic	
		Enter	Do not Enter
Zenith	Enter	-40 (Zenith), -40 (Panasonic)	250 (Zenith), 0 (Panasonic)
	Do not Enter	0 (Zenith), 250 (Panasonic)	0 (Zenith), 0 (Panasonic)

a) *If both firms move simultaneously, what is/are the Nash equilibrium(s)?*

There are two Nash equilibria in pure strategies:
Zenith Enters and Panasonic does not Enter, and
Zenith does not Enter and Panasonic Enters.

There is also one Nash mixed-strategy Nash Equilibrium.
Assume Panasonic plays Enter with probability α, and Zenith plays Enter with probability β.
A mixed strategy equilibrium requires that
$\alpha(-40) + (1 - \alpha)(250) = 0$, and
$\beta(-40) + (1 - \beta)(250) = 0$.

Solving these for α, and β, gives $\alpha = \beta = \frac{25}{29}$. Expected profit for both firms is zero.

b) *If the US government commits to paying Zenith a lump-sum subsidy of \$50 million if it enters this market, what is/are the Nash equilibrium(s)?*

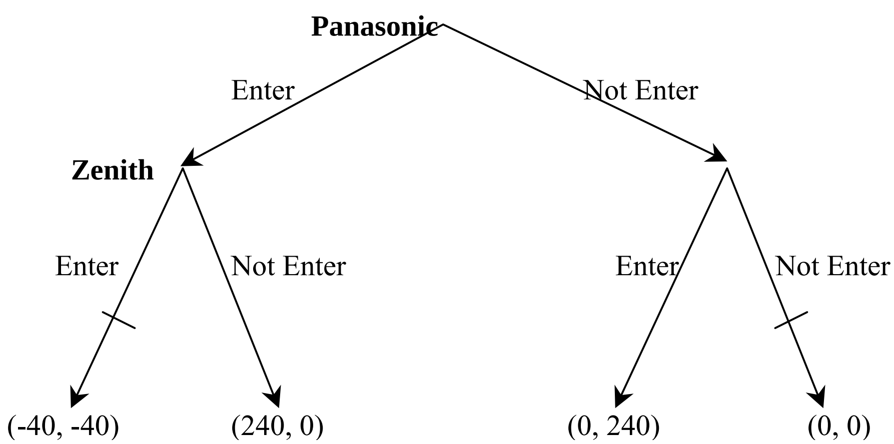
That will produce the following altered game:

		Panasonic: Enter	Panasonic: Do not Enter
Zenith	Enter	***10***, -40	***300***, 0
	Do not Enter	0, 250	0, 0

Now there is only one Nash Equilibrium
Zenith Enters, and Panasonic does not Enter.

c) *If Panasonic is first-mover (and Zenith does not have the subsidy), what is/are the Nash equilibrium(s)?*

Model this game in extensive form:



We can see that Zenith will not take the marked branches. Put differently, Zenith will not choose an outcome that is not a Nash Equilibrium from part a. So Panasonic is essentially choosing between the Nash Equilibria from part a.

Clearly, in this case, Panasonic will choose "Enter" and Zenith will choose "Not Enter", which in this version of the game becomes the sole Nash Equilibrium.

5) *There are many buyers who value high-quality used cars at the full-information market price p_1, and lemons at p_2. There are a limited number of potential sellers who value high-quality cars at $v_1 \leq p_1$ and lemons at $v_2 \leq p_2$. Everyone is risk-neutral. The share of lemons among all the used cars that might be sold is θ.*

a) *Under what conditions are all cars sold?*

In order for all cars to be sold, two conditions must hold. The buyers must be willing to buy the cars, and the sellers must be willing to sell them.

Since sellers are limited, the price will be set by the buyers' willingness to pay. Since the buyers cannot distinguish good cars from lemons, a market in which all cars get sold would have to have

a price equal to the expected value of the cars to the buyers: $p_1(1-\theta)+p_2\theta$. For all cars to be sold, the sellers also have to be willing to participate. In this case, Lemon owners are clearly willing to participate, so we have to check the good car owners. That requires that
$p_1(1-\theta)+p_2\theta \geq v_1$,
or $\theta \leq \dfrac{p_1 - v_1}{p_1 - p_2}$.

b) *Under what conditions are only lemons sold?*

First, Good car owners must be unwilling to participate, which requires $\theta > \dfrac{p_1 - v_1}{p_1 - p_2}$.
In this case, the market price will be p_2, but at this price, lemon owners will always be willing to participate (since $p_2 \geq v_2$). So the sole criterion is that $\theta > \dfrac{p_1 - v_1}{p_1 - p_2}$.

c) *Under what conditions are no cars sold?*

There are no such conditions in this model.

Suppose that buyers incur a transaction cost of $200 to purchase a car. This transaction cost is the value of their time to find a car. What is the

d) *Under what conditions are all cars sold?*

The market price, again, is determined by the buyers' willingness to pay. In this case, that is $p_1(1-\theta)+p_2\theta-200$. As before, good car owners must be willing to participate, which means:
$p_1(1-\theta)+p_2\theta-200 \geq v_1$, or $\theta \leq \dfrac{p_1 - v_1 - 200}{p_1 - p_2}$.

e) *Under what conditions are only lemons sold?*

As before, we reverse the requirement above: $\theta > \dfrac{p_1 - v_1 - 200}{p_1 - p_2}$.
Since good car owners are willing to participate (a fact the buyers will rapidly become aware of) the used car price will be $p_2 - 200$. Participation by the lemon owners requires $p_2 - 200 \geq v_2$.

f) *Under what conditions are no cars sold?*

When even the lemon owners are unwilling to participate. That occurs when $p_2 - 200 < v_2$.