# Economics 370
# Microeconomic Theory
# Problem Set 2
# Answer Key

1) *Jack starts with $500,000 to spend on housing and all other goods. Housing costs $100 per square-foot. Jack chooses to purchase a 2000 square-feet home. Assume that it costs nothing (in terms of real estate fees, lawyer fees et cetera) to buy and sell houses.*

   a) *Draw an indifference curve between "size of house" and "all other goods". Draw the budget line and indicate Jack's most preferred point.*

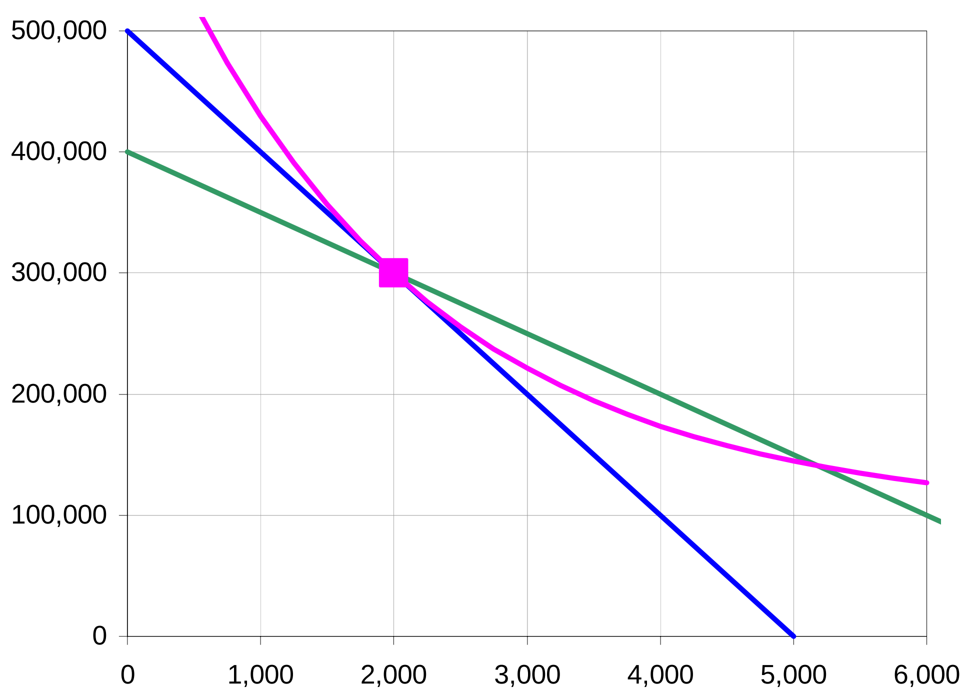
   (see below, in blue)

   b) *Suppose that **after** Jack has bought his 2,000 square-feet house, the price of housing falls to $50 per square-foot. Show his new budget line.*

   (below, in green)

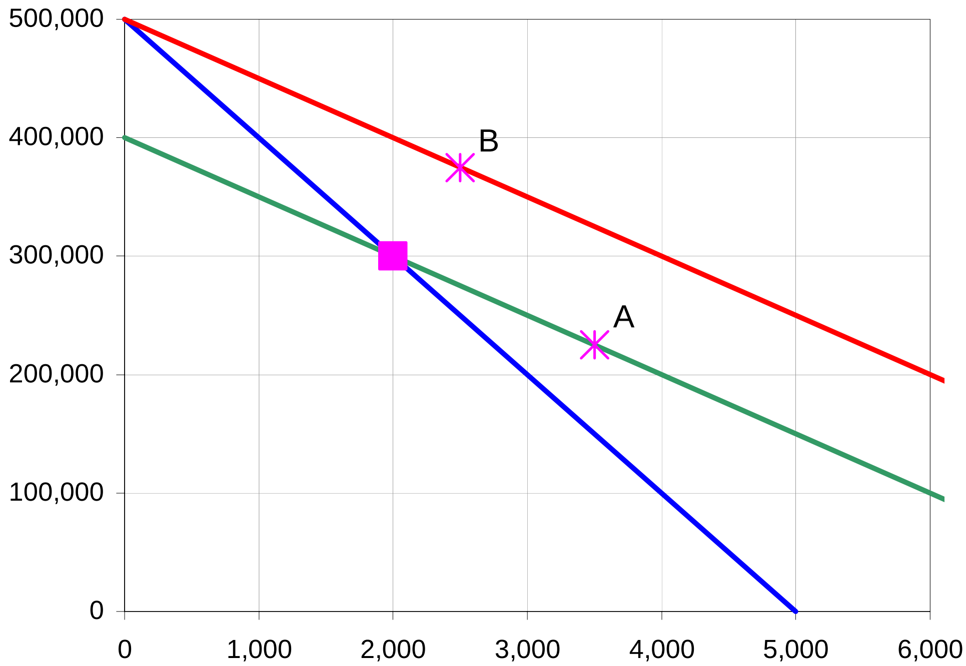
   c) *Does the fall in the price of housing make Jack better or worse off than he would have been if the price had not fallen?*

   It cannot make him worse off since he still has the bundle he bought before the price change. In other words, while it changed the cash value of the house, it is still the same house, and he still owns it. If his preferences do not have a kink, then he will be better off (see the indifference curve in magenta).



d) *Suppose that after the price of housing falls Jack moves from the 2,000 square-feet house to a house of size A. If the price of housing had fallen before he bought his house, he would have chosen a house of size B. If A > B, what (if anything) can be said about Jack's income elasticity of demand for housing?*

The counterfactual budget line is shown below in red. From the graph above, the point A must be greater than 2,000 SF (look at the preferences shown above). Points A and B consistent with the setup of the problem are shown below.




Given the description, housing must be an **inferior** good for Jack. The income elasticity of an inferior good is:

$$\eta = \frac{m}{h}\frac{\partial h}{\partial m} < 0\,,$$

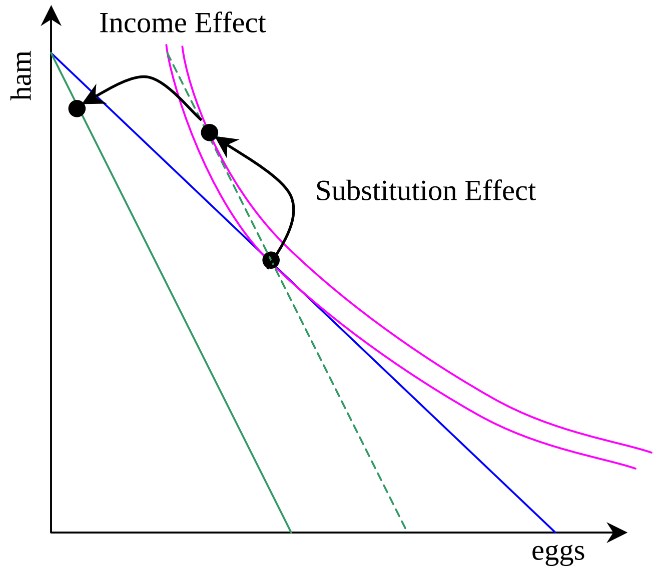
since m >0, h >0 and $\frac{\partial h}{\partial m} < 0$ (by the definition of an inferior good).

2) *Anne consumes only eggs and ham. Ham is an inferior good for her. One day the price of eggs goes up.*

a) *Show Anne's old and new optimum points and show both the substitution and income effects. How does this graph reflect the fact that ham is an inferior good?*

Since the question does not specify whether to use the Slutsky or Hicks method, you can use either one. I choose to use Slutsky.

Notice that the income effect for ham shown on the graph is negative, which is consistent with the statement above that ham is an inferior good.




b) ***True or false**: When the price of eggs goes up, Anne certainly buys more ham than before. Justify your answer carefully by considering the directions of both the substitution and income effects.*

**True.** The substitution effect is always negative, which is to say that consumption of eggs goes down (and consumption of ham goes up) as the price of eggs goes up. Since price is increasing, effective income is decreasing. Since ham is an inferior good, the income effect serves to further increase the consumption of ham. Since both income and substitution effects increase consumption of ham, consumption of ham certainly must go up.

c) ***True or false:** Ham and eggs must be (gross) substitutes. That is, an increase in the price of eggs will shift Anne's Marshallian demand curve for ham to the right.*

**True.** Consider the example above. The price of ham has not changed, yet demand has increased. That is shown on a supply-demand graph as the shift of the demand curve for ham to the right. Since an increase in the price of eggs **always** increases demand for ham (assuming the price of ham remains constant) then ham and eggs **must** be gross substitutes.

3) *Abe consumes guns (x) and butter (y). His utility function is U(x) = x – 3 / y.*

a) *Suppose $p_x = 9$, $p_y = 16$, and I = 900. Find the utility maximizing quantities of x and y.*

Plugging in to the demand functions (part b below) gives:

$$x^0 = \frac{m - \sqrt{3p_x^0 p_y}}{p_x^0} = \frac{900 - \sqrt{3 \cdot 9 \cdot 16}}{9} = 100 - \frac{4\sqrt{3}}{3}, \text{ and } y^0 = \sqrt{\frac{3p_x^0}{p_y}} = \sqrt{\frac{3 \cdot 9}{16}} = \frac{3\sqrt{3}}{4}$$

b) *Find the demand functions for x and y.*

$$MU_x = \frac{\partial U}{\partial x} = 1, \text{ and } MU_y = \frac{\partial U}{\partial y} = \frac{3}{y^2}.$$

I solve this by setting $MRS = -\frac{MU_x}{MU_y} = -\frac{p_x}{p_y}$, or $\frac{y^{*2}}{3} = \frac{p_x}{p_y}$, or $y^* = \sqrt{\frac{3p_x}{p_y}}$.

Substituting into the budget constraint gives x: $p_x x + p_y y = m$ or $x^* = \frac{m - p_y y^*}{p_x}$, or

$x^* = \frac{m}{p_x} - \frac{p_y}{p_x}\sqrt{\frac{3p_x}{p_y}} = \frac{m}{p_x} - \sqrt{\frac{3p_y}{p_x}}$. Probably the simplest form of this is: $x^* = \frac{m - \sqrt{3p_x p_y}}{p_x}$.

c) *What are the price-elasticities of demand for x and y? What are the income elasticities of demand for x and y?*

$$\varepsilon_x = \frac{p_x}{x}\frac{\partial x}{\partial p_x} = \frac{p_x^2}{m - \sqrt{3p_x p_y}}\frac{\partial}{\partial p_x}\left[\frac{m}{p_x} - \sqrt{\frac{3p_y}{p_x}}\right] = -\frac{p_x^2}{m - \sqrt{3p_x p_y}}\left[\frac{m}{p_x^2} - \frac{1}{2}\sqrt{\frac{3p_y}{p_x^3}}\right]$$

This "simplifies" to

$$\varepsilon_x = -\frac{p_x^2}{m - \sqrt{3p_x p_y}}\left[\frac{m - 0.5\sqrt{3p_x p_y}}{p_x^2}\right] = -\frac{m - \sqrt{3p_x p_y} + 0.5\sqrt{3p_x p_y}}{m - \sqrt{3p_x p_y}} = -1 - \frac{0.5\sqrt{3p_x p_y}}{m - \sqrt{3p_x p_y}}.$$

$$\varepsilon_y = \frac{p_y}{y}\frac{\partial y}{\partial p_y} = p_y\sqrt{\frac{p_y}{3p_x}}\frac{\partial}{\partial p_x}\sqrt{\frac{3p_x}{p_y}} = -p_y\sqrt{\frac{p_y}{3p_x}}\frac{1}{2p_y}\sqrt{\frac{3p_x}{p_y}} = -\frac{1}{2}$$

$$\eta_x = \frac{m}{x}\frac{\partial x}{\partial m} = \frac{mp_x}{m - \sqrt{3p_x p_y}}\frac{\partial}{\partial m}\left[\frac{m}{p_x} - \sqrt{\frac{3p_y}{p_x}}\right] = \frac{mp_x}{m - \sqrt{3p_x p_y}}\frac{1}{p_x} = \frac{m}{m - \sqrt{3p_x p_y}} > 1$$

$$\eta_y = \frac{m}{y}\frac{\partial y}{\partial m} = m\sqrt{\frac{p_y}{3p_x}}\frac{\partial}{\partial m}\sqrt{\frac{3p_x}{p_y}} = m\sqrt{\frac{p_y}{3p_x}}[0] = 0$$

Your book describes this as a quasi-linear utility function. One of its characteristics is that income elasticity for the non-linear good is usually zero.

d) *What happens to Abe's utility-maximizing bundle if all prices and his income increase by the same percentage?*

Nothing. His budget constraint does not change, and neither does his preferences.

e) *If the price of butter falls to $p_x = 8$ [oops], what are the (Slutsky) income and substitution effects?*

I am going to solve this assuming $p_x$ drops to 8 (if you make the opposite assumption, I will grade it accordingly).

$$x^1 = \frac{m - \sqrt{3p_x^1 p_y}}{p_x^1} = \frac{900 - \sqrt{3 \cdot 8 \cdot 16}}{8} = 112.5 - \sqrt{6}\text{, and } y^1 = \sqrt{\frac{3p_x^1}{p_y}} = \sqrt{\frac{3 \cdot 8}{16}} = \frac{\sqrt{6}}{2}$$

Now we need to calculate $x^s$ and $y^s$. Note that since y does not depend on income, $y^s = y^1$.

To calculate $x^s$, we calculate the income that will allow Abe to exactly afford the old bundle at the new prices. That is

$$m^s = p_x^1 x^0 + p_y y^0 = 8\left(100 - \frac{4\sqrt{3}}{3}\right) + 16\left(\frac{3\sqrt{3}}{4}\right) = 800 - \left(\frac{32\sqrt{3}}{3}\right) + 12\sqrt{3} = 800 + \frac{4\sqrt{3}}{3}$$

Then $x^s = \dfrac{m^s - \sqrt{3p_x^1 p_y}}{p_x^1} = \dfrac{800}{8} + \dfrac{4\sqrt{3}}{3 \cdot 8} - \dfrac{\sqrt{3 \cdot 8 \cdot 16}}{8} = 100 + \dfrac{\sqrt{3}}{6} - \sqrt{6}$

Therefore,

$$SE_x = x^s - x^0 = 100 + \frac{\sqrt{3}}{6} - \sqrt{6} - \left(100 - \frac{4\sqrt{3}}{3}\right) = \frac{3\sqrt{3}}{2} - \sqrt{6}$$

$$SE_y = y^s - y^0 = \frac{\sqrt{6}}{2} - \frac{3\sqrt{3}}{4} = \frac{2\sqrt{6} - 3\sqrt{3}}{4}$$

$$IE_x = x^1 - x^s = 112.5 - \sqrt{6} - \left(100 + \frac{\sqrt{3}}{6} - \sqrt{6}\right) = 12.5 + \frac{\sqrt{3}}{6}$$

$$SE_y = y^1 - y^s = 0$$

*f) What are the Hicksian income and substitution effects?*

Again, we need to calculate $x^h$ and $y^h$. Note that since $y$ does not depend on income, $y^h = y^1$.

To calculate $x^h$, we calculate the income that will allow Abe to exactly attain the old utility level at the new prices. That is

$$x^0 - \frac{3}{y^0} = x^h - \frac{3}{y^h}, \text{ or } 100 - \frac{4\sqrt{3}}{3} - \frac{4}{\sqrt{3}} = 100 - \frac{8\sqrt{3}}{3} = \frac{m^h}{p_x^1} - \frac{\sqrt{3p_x^1 p_y}}{p_x^1} - \frac{3p_y}{\sqrt{3p_x^1 p_y}}.$$

So $m^h = \left(100 - \dfrac{8\sqrt{3}}{3}\right)p_x^1 + \sqrt{3p_x^1 p_y} + \dfrac{3p_x^1 p_y}{\sqrt{3p_x^1 p_y}} = \left(100 - \dfrac{8\sqrt{3}}{3}\right)p_x^1 + 2\sqrt{3p_x^1 p_y}$,

Or $m^h = \left(100 - \dfrac{8\sqrt{3}}{3}\right)8 + 2\sqrt{3 \cdot 8 \cdot 16} = 800 + 16\sqrt{6} - \dfrac{64\sqrt{3}}{3}$

Then $x^h = \dfrac{m^h - \sqrt{3p_x^1 p_y}}{p_x^1} = \dfrac{1}{8}\left(800 + 16\sqrt{6} - \dfrac{64\sqrt{3}}{3}\right) - \dfrac{\sqrt{3 \cdot 8 \cdot 16}}{8} = 100 + \sqrt{6} - \dfrac{8\sqrt{3}}{3}$

Therefore,

$$SE_x = x^h - x^0 = 100 + \sqrt{6} - \frac{8\sqrt{3}}{3} - \left(100 - \frac{4\sqrt{3}}{3}\right) = \sqrt{6} - \frac{4\sqrt{3}}{3}$$

$$SE_y = y^h - y^0 = \frac{\sqrt{6}}{2} - \frac{3\sqrt{3}}{4} = \frac{2\sqrt{6} - 3\sqrt{3}}{4}$$

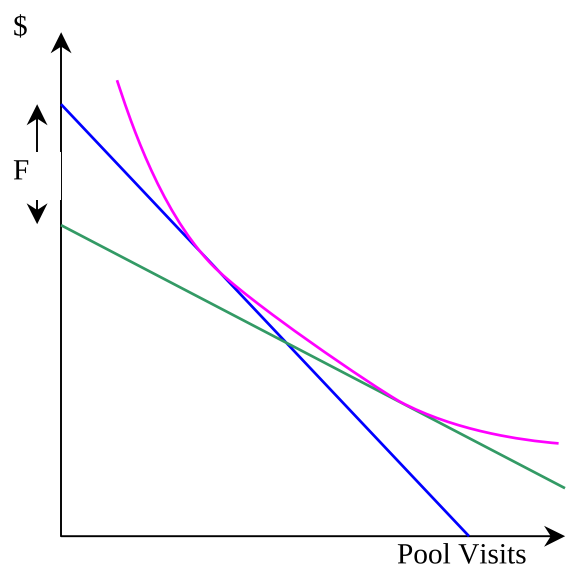
$$IE_x = x^1 - x^h = 112.5 - \sqrt{6} - \left(100 + \sqrt{6} - \frac{8\sqrt{3}}{3}\right) = 12.5 + \frac{8\sqrt{3}}{3} - 2\sqrt{6}$$

$$SE_y = y^1 - y^h = 0$$

4) *The local swimming pool charges nonmembers \$10 per visit. If you join the pool, you can swim for \$5 per visit, but you have to pay an annual fee of \$F.*

a) *Use and indifference curve diagram to find the value of F that would make it just worthwhile for you to join the pool.*

Since I can choose whether or not to join the pool, I will only do so if joining leaves me at least as well off as not joining. That is, I will join only if by joining I can reach the same or a better indifference curve.




b) *Suppose that the pool charged you exactly that value. Would you swim more or less than you did before joining? Use income and substitution effects to explain your answer.*

If the pool charged exactly the value described above, the choice to join the pool results in a pure (Hicksian) substitution effect. That is, there is no income effect here. By decreasing the marginal price of swimming, they induce me to swim more often, as clearly illustrated on the graph above.

5) *Below you see Marsha's demand curve for housing, d. Note that the graph is not drawn to scale. She can purchase all the housing she desires at the market price of $p_1$ per square foot. Marsha is also eligible for housing in a government project. If she chooses this option, she only needs to pay $p_2$ per square foot, but she must consume an apartment of $x_2$*

*square feet. Which option does she choose? Explain how you would determine the answer.*

In the first case, Marsha will choose to buy $x_1$ amount of housing. In the second case, although she would prefer a different amount of housing at that price than is offered, she cannot choose how much housing she wants—she is faced with a take-it-or-leave-it proposition. She will accept the government housing if (and only if) she is better off, on net, in the government project. That is, in economic terms, if her consumer welfare is greater in the government project.

If she takes obtains her own housing on the market, her net welfare will be the area marked as *A* on the graph below.

If she takes the government project, her welfare will be A + B – C. To see that the area marked C is a net loss for her, notice that for any amount of housing greater than $x^*$, the amount she is paying for the housing exceeds the benefit she receives from the housing by precisely the amount represented by the height of the area C at that point.

She will take the project housing if B > C.