Economics 370
Microeconomic Theory
Midterm 2
Answer Key

1) *Suppose anyone with a driver's license is capable of supplying one trip from the airport to the downtown business center on any given day. The long-run supply curve of such trips is horizontal at p = \$50, which is the average cost of such trips. Suppose daily demand is Q(p) = 1000 – 10p.*

a) *What's the competitive equilibrium? Calculate the consumer surplus and producer surplus respectively. Give a numerical answer.*

Since there are no restrictions on market entry, P = \$50. Q = 1000 – 10 × 50 = 500. Producer Surplus = \$0.

Consumer Surplus is the area under the demand curve and above the price.
The area of that triangle is ½ × Q × (P_0 – P) = ½ × 500 × (100 – 50) = \$12,500.

b) *If the city government requires those people supplying such trips to possess a special license, and the government issues only 300 licenses (that is, only 300 trips can be made), what will be the new price? Calculate the change in consumer surplus, producer surplus and social welfare.*

If only 300 trips can be made, the price solves: 300 = 1000 – 10 × P, so P = \$70.

Producer Surplus is (P – MC) × Q = (70 – 50) × 300 = \$6,000.
Consumer Surplus is ½ × 300 × 30 = \$4,500.
Social Welfare = \$10,500.

	Consumers	Producers	Government	Society
Free Market	\$12,500	\$0	\$0	\$12,500
Licenses	\$4,500	\$6,000	\$0	\$10,500
Change	- \$8,000	+ \$6,000	\$0	(DWL =) - \$2,000

2) *You are asked to predict the behavior of an industry given that:*

I. *The industry is perfectly competitive.*

II. *The industry is composed of 50 firms which have identical cost curves.*

III. *To begin with, the industry is in short-run and long-run equilibrium.*

The ***short-run*** *supply function is:* $q = \begin{cases} 0 & p < 2 \\ 50p + 100 & p \geq 2 \end{cases}$.

Demand is q = 700 – 100p, where supply and demand is in terms of units per week.

There is a permanent rise in market demand, so that at each price 300 additional units are demanded per week.

a) *What will industry supply and price be in the short run?*

Price and supply will be where the short-run supply curve crosses the new demand curve.
The new demand curve is $q = 300 + 700 – 100p = 1{,}000 – 100p$

The short-run result solves $q = 1{,}000 - 100p$ and $q = 100 + 50p$.
So, $q = 400$; $p = 6$.

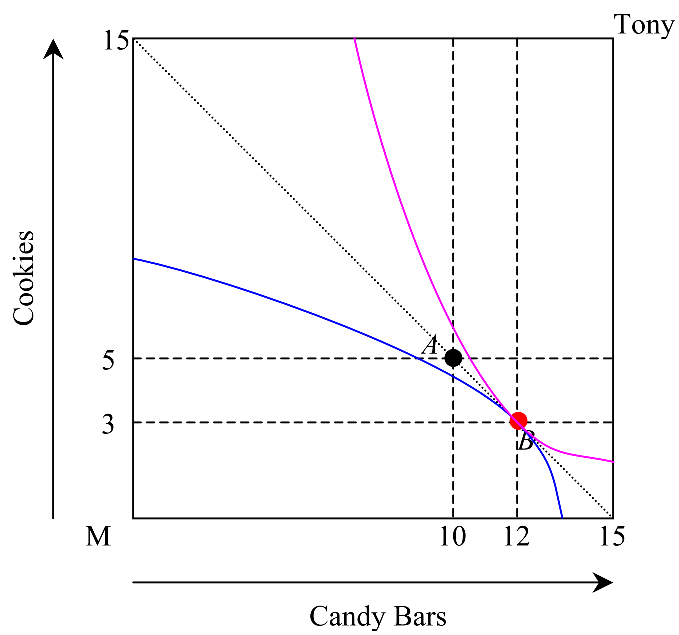
b) *Assume that the industry has constant costs in the long run, and that free-entry and free-exit is possible (that is, assume that any new firms entering the industry have cost curves identical with that of established firms). What will the long-run price and quantity be in this problem?*

With free-entry and –exit, eventually prices will be reduced back down to long-run marginal cost. Since the market was originally in long-run equilibrium, long-run marginal cost must be the original equilibrium price.
The original equilibrium solved: $q = 700 - 100p$ and $q = 100 + 50p$.
$P_0 = 4$; $Q_0 = 300$.
So, the new long-run cost will be $P_1 = 4$; $Q_1 = 600$.

c) *How many firms will there be in the long-run equilibrium?*

Originally, there were 50 firms producing 300 units. Sine we end up in long-run equilibrium and are producing 600 units (twice as much product) we must have twice as many firms.
n = 100.

3) *Initially, Michael has 10 candy bars and 5 cookies, and Tony has 5 candy bars and 10 cookies. After trading, Michael has 12 candy bars and 3 cookies. In an Edgeworth box, label the initial Allocation 'A' and the new allocation 'B'. Draw some indifference curves that are consistent with this trade being optimal for both Michael and Tony.*



4) *Each firm in a competitive market has a cost function of $c(y) = 16 + y^2$. The market demand function is $y = 24 - p$. Determine the equilibrium price, quantity supplied per firm, market quantity, and number of firms.*

In general, there will be enough firms in the market so that if one more firm entered the market, profits would be negative.

Assume, initially, that profits are zero. Afterward, we will check to see if this gives us a reasonable answer, and adjust accordingly.

Zero profits, in this case, requires that MC = ATC.

$$\frac{\partial}{\partial y}\left(16+y^2\right)=2y=\frac{16+y^2}{y}=\frac{16}{y}+y.$$

$y^2 = 16$, or $y = 4$.
At $y = 4$, MC = $2y$ = 8. On our assumption of zero profits, price = MC.
Market demand q = 24 – p = 24 – 8 = 16.
So the following results satisfy our requirements:
Price = 8. Market Quantity = 16. Per Firm Production = 4. Firms = 4. Per Firm Profits = 0.

Clearly, any additional firms entering the market would result in all firms earning negative profits. Similarly, any fewer firms would leave the firms in the market earning positive profits, **but** (clearly) an additional firm could enter the market and earn non-negative profits. So, the results above solve our problem.

5) *A competitive firm has a production function described as follows: "Weekly output is the square root of the minimum of the number of units of capital and the number of units of labor employed per week." Suppose that in the short run this firm must use 16 units of capital but can vary its amount of labor freely.*

a) *Write down a formula that describes the marginal product of labor in the short run as a function of the amount of labor used (be careful at the boundaries).*

The long-run formula would be: $y=\sqrt{\min\{K,L\}}$.
In the short run, labor greater than 16 has no effect on output. Therefore, the short-run production function is: $y = \begin{matrix}\sqrt{L} & L \le 4 \\ 4 & L > 4\end{matrix}$.

Therefore, the marginal product of labor is: $MP_L = \begin{matrix}(4L)^{-1/2} & L \le 4 \\ 0 & L > 4\end{matrix}$

b) *If the wage is w = \$1 and the price of output is p = \$4, how much labor will the firm demand in the short run?*

Optimal labor demand is where $p_y MP_L = w$.
In this case that means $4\frac{1}{2\sqrt{L}}=1$, or $L = 4$.

c) *Write down an equation for the firm's short-run demand for labor as a function of w and p.*

Optimal labor demand is where $p_y MP_L = w$.
In this case that means $\frac{p_y}{2\sqrt{L}}=w$, provided $L \le 4$.

So, we have $L=\min\left\{\left(\frac{p_y}{2w}\right)^2,4\right\}$.

6) **(Extra Credit)**: *A firm has a Constant Elasticity of Substitution production function. That is, its production function is:* $y = [ax_1^\rho + ax_2^\rho]^{1/\rho}$. *If prices of inputs* x_1 *and* x_2 *are* p_1 *and* p_2 *respectively, what is the minimum-cost method for producing y units of output?*

Cost minimization requires that $TRS = \frac{MP_1}{MP_2} = \frac{p_1}{p_2}$.

$$MP_1 = \frac{\partial}{\partial x_1}\left[ax_1^\rho + ax_2^\rho\right]^{1/\rho} = \frac{1}{\rho}\left[ax_1^\rho + ax_2^\rho\right]^{(1/\rho)-1} a\rho x_1^{\rho-1} = ax_1^{\rho-1}\left[ax_1^\rho + ax_2^\rho\right]^{(1-\rho)/\rho}.$$

To simplify the notation, I rewrite this as: $MP_1 = ax_1^{\rho-1}y^{1-\rho}$.
Similarly, we get $MP_2 = ax_2^{\rho-1}y^{1-\rho}$.

Therefore, we have $\frac{ax_1^{\rho-1}y^{1-\rho}}{ax_2^{\rho-1}y^{1-\rho}} = \left(\frac{x_1}{x_2}\right)^{\rho-1} = \frac{p_1}{p_2}$,
That gives us: $x_1 = x_2\left(\frac{p_1}{p_2}\right)^{1/(\rho-1)}$.

Submitting back into the original production function:

$$y = \left[ax_1^\rho + ax_2^\rho\right]^{1/\rho} = \left[ax_2^\rho\left(\frac{p_1}{p_2}\right)^{\rho/(\rho-1)} + ax_2^\rho\right]^{1/\rho} = \left[a\left(\frac{p_1}{p_2}\right)^{\rho/(\rho-1)} + a\right]^{1/\rho} x_2 = \frac{\left[ap_1^{\rho/(\rho-1)} + ap_2^{\rho/(\rho-1)}\right]^{1/\rho}}{p_2^{1/(\rho-1)}} x_2$$

$$x_2 = \frac{p_2^{1/(\rho-1)}}{\left[ap_1^{\rho/(\rho-1)} + ap_2^{\rho/(\rho-1)}\right]^{1/\rho}} y.$$

Then $x_1 = \frac{p_1^{1/(\rho-1)}}{\left[ap_1^{\rho/(\rho-1)} + ap_2^{\rho/(\rho-1)}\right]^{1/\rho}} y$.