Math 102 Spring 2008: **Solutions: HW #9**

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1. section 10.7, #4

   The sequence $\{\frac{n}{3n^2+2}\}$ is monotonically decreasing with limit 0, so the given series meets both criteria of the alternating series test.

   Therefore this series converges.

2. section 10.7, #12

   The series diverges by the nth-term test for divergence because $(\frac{n\pi}{10})^{n+1}$ goes to $+\infty$ as n goes to $+\infty$.

3. section 10.7, #26

   The ratio test yields $\rho = \lim_{n\to\infty} \frac{3^{n+1}n!n}{3^n(n+1)!(n+1)} = \lim_{n\to\infty} \frac{3n}{(n+1)^2} = 0$.

   Therefore the given series converges absolutely.

4. section 10.7, #32

   Because $\lim_{n\to\infty} \frac{2^{3n}}{7^n} = \lim_{n\to\infty} \frac{8^n}{7^n} = \lim_{n\to\infty}(\frac{8}{7})^n = +\infty$, the series diverges by the nth-term test for divergence.

5. section 10.7, #40

   The ratio test yields

   $$\rho = \lim_{n\to\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)\cdot(2n+1)\cdot 1\cdot 4\cdot 7\cdots(3n-2)}{1\cdot 3\cdot 5\cdots(2n-1)\cdot 1\cdot 4\cdot 7\cdots(3n-2)\cdot(3n+1)} = \lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3}.$$

   Because $\rho < 1$, the original series converges absolutely.

6. section 10.7, #52

   The condition $\frac{1}{(2n)!} < 0.00001$ leads to $4 < n < 5$, so the sum of the terms through $n = 4$ will provide five-places accuracy. The sum of the first terms of the series is

   $$\sum_{n=0}^{4} \frac{(-1)^n}{(2n)!} = \frac{4357}{8064} \cong 0.5403025$$

   so to five places, the sum of the infinite series is 0.54030.

7. section 10.7, #60

   Part(a), The ratio test yields

   $$\rho = \lim_{n\to\infty} \frac{(n+1)|r|^{n+1}}{n|r|^n} = \lim_{n\to\infty} \frac{(n+1)|r|}{n} = |r| < 1.$$

   Therefore the series in question converges.

   Part(b), Let S denote the sum of the series in Part(a). Then we have

$$
\begin{aligned}
(1-r)S &= (1-r)\sum_{n=0}^{\infty} nr^n \\
&= \sum_{n=0}^{\infty}(nr^n - nr^{n+1}) \\
&= \lim_{k\to\infty}\sum_{n=0}^{k}(nr^n - nr^{n+1}) \\
&= \lim_{k\to\infty}(r - r^2 + 2r^2 - 2r^3 + 3r^3 - 3r^4 + \cdots + (k-1)r^{k-1} - (k-1)r^k + kr^k - kr^{k+1}) \\
&= \lim_{k\to\infty}(1 + r + r^2 + r^3 + \cdots + r^k - kr^{k+1} - 1) \\
&= \lim_{k\to\infty}(\frac{1-r^{k+1}}{1-r} - kr^{k+1} - 1) \\
&= \frac{r}{1-r} = \sum_{n=0}^{\infty} r^n.
\end{aligned}
$$

Solving for S gives $\sum_{n=0}^{\infty} nr^n = S = \frac{r}{(1-r)^2}$.

8. section 10.8, #8

   The ratio test yields

$$
\lim_{n\to\infty}\frac{(2n+1)^{1/2}4^{n+1}|x|^{n+1}}{(2n+3)^{1/2}4^n|x|^n} = 4|x|.
$$

   So the series converges if $-\frac{1}{4} < x < \frac{1}{4}$. When $x = 1/4$,the series converges by the alternating series test.When $x = -1/4$,the series diverges by limit-comparison test with the p-series for which $p = 1/2$,hence $(-1/4, 1/4]$.

9. section 10.8, #26

   The ratio test yields

$$
\lim_{n\to\infty}\frac{n\cdot 10^n\cdot |x-2|^{n+1}}{(n+1)\cdot 10^{n+1}\cdot |x-2|^n} = \frac{|x-2|}{10},
$$

   and therefore this series converges if $-10 < x - 2 < 10$,that is,if $-8 < x < 12$. If $x = 12$ it converges by the alternating series test;if $x = -8$ it diverges because it becomes harmonic series,hence $(-8, 12]$.

10. section 10.8, #28

    Notice that the given series is actually geometric with first term 1 and ratio $r = \frac{x^2+1}{5}$. Hence it converges if $-1 < r < 1$,that is,$-2 < x < 2$. So its interval of convergence is $(-2, 2)$ and its sum is $\frac{5}{4-x^2}$.