

Math 102 Spring 2008: Solutions: HW #4

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1. section 10.2, #6

The denominators are always one more than a square. So the n th denominator is $n^2 + 1$ so the general term in the sequence is

$$a_n = \frac{1}{n^2 + 1}.$$

2. section 10.2, #12

Dividing top and bottom by n^2 we get $\frac{n}{10+1/n}$. Now $10 + 1/n \rightarrow 10$ as $n \rightarrow \infty$ so we get that $\frac{n}{10+1/n}$ must diverge.

3. section 10.2, #14

We have $(-1/2)^n = (-1)^n \frac{1}{2^n} \rightarrow 0$ as $n \rightarrow \infty$. Thus

$$\lim_{n \rightarrow \infty} 2 - (-1/2)^n = 2.$$

4. section 10.2, #20

We have $-1 \leq \cos n \leq 1$ so $1 \leq 2 + \cos n \leq 3$ and hence

$$\frac{1}{n} \leq \frac{2 + \cos n}{n} \leq \frac{3}{n}$$

Thus

$$\frac{1}{\sqrt{n}} \leq \sqrt{\frac{2 + \cos n}{n}} \leq \frac{3}{\sqrt{n}}$$

Since $\frac{1}{\sqrt{n}} \rightarrow 0$ this means that

$$\sqrt{\frac{2 + \cos n}{n}} \rightarrow 0$$

as $n \rightarrow \infty$.

5. section 10.2, #30

Using L'hôpital's rule four times we have

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3}{e^{x/10}} &= \lim_{x \rightarrow \infty} \frac{3x^2}{e^{x/10}/10} \\ &= \lim_{x \rightarrow \infty} \frac{6x}{e^{x/10}/100} \\ &= \lim_{x \rightarrow \infty} \frac{6}{e^{x/10}/1000} \\ &= 0 \end{aligned}$$

This means that the sequence converges and the limit is zero.

6. section 10.2, #34

Let's look at $b_n = \ln a_n = \frac{1}{n} \ln(2n + 5)$. Now by L'hôpital's rule

$$\lim_{x \rightarrow \infty} \frac{\ln(2x + 5)}{x} = \lim_{x \rightarrow \infty} \frac{2/(2x + 5)}{1} = 0$$

So $b_n \rightarrow 0$ which means $a_n \rightarrow 1$.

7. section 10.2, #50

We have

$$\lim_{n \rightarrow \infty} \frac{3n - 1}{4n + 1} = \lim_{n \rightarrow \infty} \frac{3 - 1/n}{4 + 1/n} = \frac{3}{4}$$

where we divided the top and bottom by n to get the first equality. Thus

$$\lim_{n \rightarrow \infty} 3 \sin^{-1} \sqrt{\frac{3n - 1}{4n + 1}} = 3 \sin^{-1} \left(\lim_{n \rightarrow \infty} \sqrt{\frac{3n - 1}{4n + 1}} \right) = 3 \sin^{-1} \sqrt{\frac{3}{4}} = \pi.$$

8. section 10.2, #56

Since $a_{n+1} = 1 + (1/a_n)$ and the limit $\lim_{n \rightarrow \infty} a_n = L$ exists we can take the limit of both sides to get

$$L = 1 + 1/L.$$

This gives $L^2 - L - 1 = 0$ which means (by the quadratic formula) that

$$L = \frac{1 \pm \sqrt{5}}{2}$$

Since L is clearly non-negative this means

$$L = \frac{1 + \sqrt{5}}{2}.$$