Math 102 Spring 2008: **Solutions: HW #2**

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1. section 7.4, #2 Use the identity $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ to get,

$$\begin{aligned}
\int \cos^2(5x)dx &= \frac{1}{2}\int 1+\cos(10x)dx \\
&= \frac{1}{2}(x+\frac{1}{10}\sin(10x))+C \\
&= \frac{1}{2}x+\frac{1}{20}\sin(10x)+C
\end{aligned}$$

2. section 7.4, #10 Use that $\cot x = \frac{\cos x}{\sin x}$ and the identity $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$\begin{aligned}
\int \sin^2 x\cot^2 x dx &= \int \sin^2 x\frac{\cos^2 x}{\sin^2 x}dx \\
&= \int \cos^2 x dx \\
&= \frac{1}{2}\int 1-\cos(2x)dx \\
&= \frac{1}{2}(x-\frac{1}{2}\sin(2x))+C \\
&= \frac{1}{2}x+\frac{1}{4}\sin(2x)+C
\end{aligned}$$

3. section 7.4, #24 Use that $\tan^2 x = \sec^2 x - 1$

$$\begin{aligned}
\int \tan^3 x dx &= \int (\sec^2 x-1)\tan x dx \\
&= \int \sec^2 x\tan x dx + \int \tan x dx \\
&= \frac{\tan^2 x}{2} + \ln|\sec x| + C
\end{aligned}$$

4. section 7.4, #26

   Use the identity $\sec^2\theta = \tan^2\theta + 1$ and then set $u = \tan\theta$, then $du = \sec^2\theta d\theta$ we have:

$$\begin{aligned}
\int \tan\theta \sec^4\theta d\theta &= \int \tan\theta(\tan^2\theta+1)\sec^2\theta d\theta \\
&= \int u(u^2+1)du \\
&= \int u^3 + udu \\
&= \frac{1}{4}u^4 + \frac{1}{2}u^2 + C \\
&= \frac{1}{4}\tan^4\theta + \frac{1}{2}\tan^2\theta + C
\end{aligned}$$

5. section 7.4, #34

   Use the identities $\sec\theta = \frac{1}{\cos\theta}$ and $\sec^2\theta = \tan^2\theta + 1$ and then set $u = \tan 2x$, then $du = 2\sec^2 2xdx$. So we find

$$\begin{aligned}
\int \frac{1}{\cos^4 2x}dx &= \int \sec^4 2xdu \\
&= \int (\tan^2 2x + 1)\sec^2 2xdx \\
&= \frac{1}{2}\int (u^2+1)du \\
&= \frac{1}{6}u^3 + \frac{1}{2}u + C \\
&= \frac{1}{3}\tan^3 2x + \frac{1}{2}\tan 2x + C
\end{aligned}$$

6. section 7.4, #48

   To find the area between to curves we graph them and see which is above the other on the interval over which we are integrating, here $\sin^3 x$ is above $\cos^3 x$, so the integral is:

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin^3 x - \cos^3 xdx$$

   Also the area is symmetric about $x = \frac{3\pi}{4}$ so this is the integral:

$$2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^3 x - \cos^3 xdx$$

   Now we break it into two different integrals use the identity $\cos^2 + \sin^2 x = 1$ and do substitution on both of them.

$$2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^3 x - \cos^3 x dx \quad = \quad 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1-\cos^2 x)\sin x dx - 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1-\sin^2 x)\cos x dx$$

Let $u = \cos x$ so $du = -\sin x dx$ for the first integral, and let $v = \sin x$ so $du = \cos x dx$ for the second. Now $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ then we have:

$$\begin{aligned}
&= 2\int_{\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}} 1 - u^2(-du) - 2\int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} 1 - v^2 dv \\
&= -2(u - \frac{u^3}{3})|_{\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}} - 0 \\
&= -2(-\frac{\sqrt{2}}{2} - \frac{(-\frac{\sqrt{2}}{2})^3}{3} - [\frac{\sqrt{2}}{2} - \frac{(\frac{\sqrt{2}}{2})^3}{3}]) \\
&= -2(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{12} - [\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{12}]) \\
&= 2\sqrt{2} - \frac{\sqrt{2}}{3} = \frac{5\sqrt{2}}{3}.
\end{aligned}$$

7. section 7.4, #60

   Use the identity $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ so we have,

$$\begin{aligned}
\int \frac{1}{2}(\cos(2x) - \cos(6x))dx &= \frac{1}{2}(\frac{1}{2}\sin(2x) - \frac{1}{6}\sin(6x)) + C \\
&= \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x) + C
\end{aligned}$$

8. section 7.5, #2

   The degree of x^3 is grater than the degree of $2x-1$ so we must first divide, to get,

$$\frac{x^3}{2x-1} = \frac{1}{2}x^2 + \frac{1}{4}x + \frac{1}{8} + \frac{1}{8}\frac{1}{2x-1}.$$

   Then

$$\int \frac{1}{2}x^2 + \frac{1}{4}x + \frac{1}{8} + \frac{1}{8}\frac{1}{2x-1}dx \quad = \quad \frac{1}{6}x^3 + \frac{1}{8}x^2 + \frac{1}{8}x + \frac{1}{16}\ln|2x-1| + C$$

9. section 7.5, #6

The degree of x^3 is grater than the degree of x^2+x-6 so we must first divide, to get,

$$\frac{x^3}{x^2+x-6} = x-1+\frac{7x-6}{x^2+x-6}.$$

Now

$$\frac{7x-6}{x^2+x-6} = \frac{7x-6}{(x+3)(x-2)} = \frac{A}{x+3}+\frac{B}{x-2}.$$

Multiple to get the same denominator and set the coefficients of the numerators equal to get:

$$\begin{aligned} 7 &= A+B \\ -6 &= -2A+3B \end{aligned}$$

Solving the equations gives $A=\frac{27}{5}$ and $B=\frac{8}{5}$.

$$\int x-1+\frac{27}{5}\frac{1}{x+3}+\frac{8}{5}\frac{1}{x-2}dx \quad = \quad \frac{1}{2}x^2+x+\frac{27}{5}\ln|x+3|+\frac{8}{5}\ln|x-2|+C$$

10. section 7.5, #12

The degree of $2x^3-1$ is grater than the degree of x^2+1 so we must first divide, to get,

$$\frac{2x^3-1}{x^2+1} = 2x+\frac{-2x-1}{x^2+1}.$$

In $\frac{-2x-1}{x^2+1}$ the denominator is an irreducible quadratic.