Math 102 Spring 2008: **Solutions: HW #1**

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1. section 7.2, #2

   Substitute $u = 1 + 2x$ so that $du = 2dx$ and we get

$$\int \frac{1}{u^2} du/2 = -\frac{1}{2u} + C = -\frac{1}{2(1+2x)} + C.$$

2. section 7.2, #8

   Substitute $u = \pi(2x+1)$ so that $du = 2\pi dx$ and we get

$$\int \sin(u) \frac{du}{2\pi} = -\frac{\cos(u)}{2\pi} + C = -\frac{\cos(\pi(2x+1))}{2\pi} + C.$$

3. section 7.2, #16

   Substitute $u = 1 + e^{2x}$ so that $du = 2e^{2x}dx$ and we get

$$\int \frac{e^{2x}}{u} \frac{du}{2e^{2x}} = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln(1 + e^{2x}) + C.$$

   Notice that $1 + e^{2x} > e^{2x} > 0$ so we can take out the absolute value signs around $|u|$.

4. section 7.2, #34

   If $u = x - 1$ then $x = u + 1$ and $du = dx$ and we get

$$\begin{aligned} \int (u+1)u^{1/3} du &= \int u^{4/3} + u^{1/3} du \\ &= \frac{3}{7}u^{7/3} + \frac{3}{4}u^{4/3} + C \\ &= \frac{3}{7}(x-1)^{7/3} + \frac{3}{4}(x-1)^{4/3} + C \end{aligned}$$

5. section 7.2, #52

   We substitute $u = x + 2$ so $du = dx$. We get

$$\begin{aligned} \int \frac{1}{(x+2)^2 + 1} dx &= \int \frac{1}{u^2+1} du \\ &= \tan^{-1}(u) + C \\ &= \tan^{-1}(x+2) + C. \end{aligned}$$

6. section 7.3, #2

Using integration by parts we have

$$\begin{aligned}\int x^2e^{2x}dx &= x^2e^{2x}/2-\int(2x)e^{2x}/2dx\\ &= x^2e^{2x}/2-\int xe^{2x}dx\\ &= x^2e^{2x}/2-(xe^{2x}/2-\int e^{2x}/2dx)\\ &= x^2e^{2x}/2-xe^{2x}/2+e^{2x}/4+C.\end{aligned}$$

7. section 7.3, #20

   Using integration by parts we have

$$\begin{aligned}\int \sin(\ln(t))dt &= t\sin(\ln t)-\int t\cos(\ln t)\frac{1}{t}dt\\ &= t\sin(\ln t)-\int \cos(\ln t)dt\\ &= t\sin(\ln t)-(t\cos(\ln t)-\int t(-\sin(\ln t))\frac{1}{t}dt)\\ &= t\sin(\ln t)-t\cos(\ln t)-\int \sin(\ln t)dt\end{aligned}$$

   Thus we get

$$\int \sin(\ln(t))dt=\frac{1}{2}(t\sin(\ln t)-t\cos(\ln t)).$$

8. section 7.3, #38

   Substitute $t=x^{3/2}$ so that $x=t^{2/3}$ and $dt=3/2x^{1/2}dx$ and we get

$$\begin{aligned}\int x^2\sin x^{3/2}dx &= \int x^2\sin(t)\frac{2/3}{x}^{-1/2}dt\\ &= \int \frac{2}{3}x^{3/2}\sin(t)dt\\ &= \int \frac{2}{3}t\sin(t)dt\\ &= \frac{2}{3}(-t\cos(t)-\int(-\cos(t))dt)\\ &= \frac{2}{3}(-t\cos(t)+\sin(t))+C\\ &= \frac{2}{3}(-x^{3/2}\cos(x^{3/2})+\sin(x^{3/2}))+C\end{aligned}$$

   where we also used integration by parts.

9. section 7.3, #50

   We use $u = x^{n-1}$ and $v' = xe^{-x^2}$. Then $v = -\frac{1}{2}e^{-x^2}$ and we get

$$\int x^n e^{-x^2}dx = x^{n-1}(-\frac{1}{2}e^{-x^2}) - \int (n-1)x^{n-2}(-\frac{1}{2}e^{-x^2})dx = -\frac{1}{2}x^{n-1}e^{-x^2} + \frac{n-1}{2}\int x^{n-2}e^{-x^2}dx$$

   which is what we wanted to show.

10. section 7.3, #56

    Using the recursion from problem 50 repeatedly we get

$$\begin{aligned}
\int_0^1 x^5 e^{-x^2}dx &= -\frac{1}{2}x^4 e^{-x^2}|_0^1 + \frac{4}{2}\int x^3 e^{-x^2}dx \\
&= -\frac{1}{2}e^{-1} + 2(-\frac{1}{2}x^2 e^{-x^2}|_0^1 + \frac{2}{2}\int x^1 e^{-x^2}dx) \\
&= -\frac{1}{2e} - e^{-1} + 2(e^{-x^2}/(-2))|_0^1 \\
&= -\frac{1}{2e} - \frac{1}{e} - (e^{-1} - e^0) \\
&= -\frac{3}{2e} - \frac{1}{e} + 1 \\
&= -\frac{5}{2e} + 1.
\end{aligned}$$