

Rice University
Physics 332

HOLOGRAPHIC INTERFEROMETRY

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I. Introduction

Optical interferometry is a powerful tool for measuring displacements of the order of a wavelength of light. One significant limitation of common interferometric methods is that they require specular reflectors, so that one is limited to measuring the displacement of a mirror. This limitation can be removed by utilizing holography, allowing very small motions of arbitrary, diffusely reflecting, objects to be detected.

Holography, or wavefront reconstruction, is a process by which the amplitude and phase variation across a wavefront may be recorded and subsequently reproduced. Recall that if monochromatic, coherent light enters the eye or any other optical instrument, the only information furnished is the amplitude and phase variation across the input aperture. It follows, therefore, that the holographically reconstructed wavefront is completely equivalent to the original optical wave. Any measurement that could be made on the original optical signal can also be made with the reconstruction.

In double-exposure holographic interferometry this principle is applied to obtain an interference pattern between two reconstructed images of the same object. A holographic recording is first made with the object in some reference state. The object is then moved or deformed as desired, and a second hologram is made on the same photographic film. During reconstruction two waves are created and interfere in the space beyond the hologram. The resulting interference fringes record the local displacement of the surface of the object.

The experiment to be done here consists of making a double-exposure hologram of a cantilever beam and using the fringe pattern to deduce the deflection under load. The measured deflection can be compared with a simple theory. If desired, the deformation of more interesting objects can be examined as well.

II. Theoretical considerations

A. Hologram formation

Complete discussions of holography can be found in the references. Here we provide a plausibility argument for holographic imaging, largely following the treatment in Hecht.

Consider first the geometry of Fig. 1a. Laser light enters from the right and reflects off the two mirrors to reach the photographic film. Since the beams are monochromatic and coherent, an interference pattern will form at the film plane. If the incident illumination is approximately a plane wave, the pattern will simply be vertical fringes, which will produce a regular pattern of dark lines after the film is developed. When the developed film is illuminated again with laser light, as in Fig. 1b, the pattern will act as a diffraction grating, producing three beams: one undeflected and one to either side of the incoming beam. Although it is not very interesting, we can consider the deflected beams to be reconstructions of the beam from the mirror.

Now look at Fig. 1c, where the mirror has been replaced by a small object. Light reflected from the object reaches the film and forms an interference pattern. The pattern is now very complicated, but not in principle different than the uniform vertical fringes produced by the mirror. When the film is developed and illuminated by laser light there will again be three beams. What is not so obvious is that two of those beams will be reproductions of the "object beam" originally reflected from the object.

A full analytic treatment of the processes of formation and reconstruction of the hologram is quite complex, but we can get some idea of what happens with a simple calculation. In the arrangement of Fig. 1c, let the film define an xy plane. Denote the light from the mirror as the reference beam, with electric field E_R , and the light from the object as the object beam, E_O . We assume that the reference beam is a plane wave, so

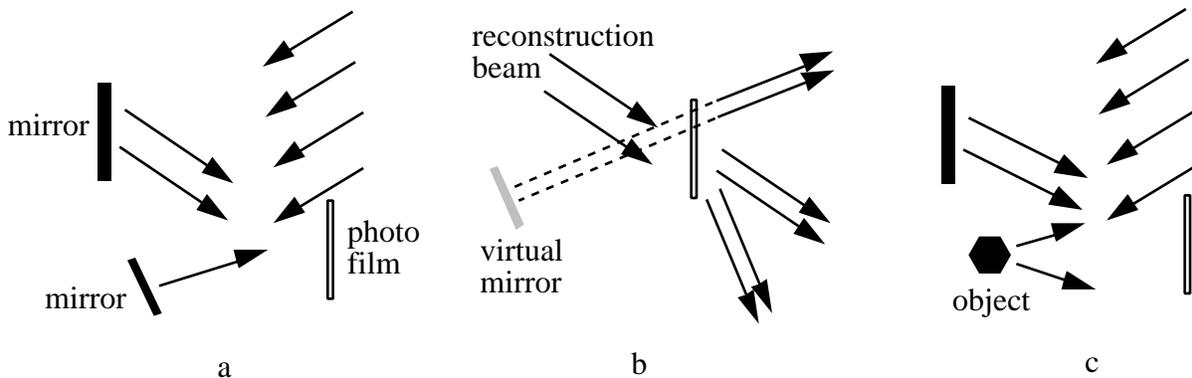


Fig. 1 Geometry for (a) making a transmission hologram of a simple mirror, (b) viewing a transmission hologram of a mirror, and (c) making a transmission hologram of a diffusely reflecting object.

$$E_R = E_{0R} \cos[\omega t + \phi_R(x, y)] \quad (1)$$

The amplitude is constant, while the phase varies across the film because the wave does not arrive at normal incidence. The object wave can similarly be expressed as

$$E_O(x, y) = E_{0O}(x, y) \cos[\omega t + \phi_O(x, y)] \quad (2)$$

but now both the amplitude and phase are complicated functions of position because the wavefront is quite irregular. The photographic film records the time-average intensity of the combined beams

$$I(x, y) = \langle (E_R + E_O)^2 \rangle = \frac{E_{0R}^2}{2} + \frac{E_{0O}^2}{2} + E_{0R}E_{0O} \cos(\phi_R - \phi_O) \quad (3)$$

as a pattern of light and dark fringes.

To reconstruct the image we illuminate the developed film with a replica of the reference beam,

$$E_r = E_{0r} \cos[\omega t + \phi_R(x, y)] \quad (4)$$

The transmitted wave is proportional to the product $I(x, y)E_r(x, y)$, which can be written out as

$$\begin{aligned} E(x, y) &= \frac{1}{2} E_{0r} (E_{0R}^2 + E_{0O}^2) \cos[\omega t + \phi_R(x, y)] \\ &\quad + \frac{1}{2} E_{0r} E_{0R} E_{0O} \cos[\omega t + 2\phi_R - \phi_O] \\ &\quad + \frac{1}{2} E_{0r} E_{0R} E_{0O} \cos[\omega t + \phi_O] \end{aligned} \quad (5)$$

The first term is the undeflected reference beam, while the other two contain information about the object beam. The last is, in fact, the desired replica of the object beam, except for the constant $E_{0R}E_{0r}/2$. The middle term is similar, except that the phase factor has the wrong sign.

Although it is not obvious from the equations, an analogy with the simple grating discussed above suggests that the beams carrying object information are deflected to either side of the reconstructing beam, and this is in fact the case. It is also true that the last term describes a virtual image of the original object, located at the object position. The phase-reversed term

describes a real image which is inverted in a peculiar way. In practice, we use the virtual image, which can be examined by looking *through* (not *at*) the hologram as though it were a window.

Making and viewing a plane transmission hologram, as described above, demands excellent stability of all the components and fairly careful placement of the reconstruction beam. By going to the geometry shown in Fig. 2a, both these problems are eased. The reference and object beams now mix in such a way that interference fringes are formed in the volume of the photographic emulsion, more or less parallel to the surfaces. The hologram is therefore a three-dimensional grating, more like a crystal than the familiar flat pattern. Since there are fewer components and they can be closer together, alignment and vibration during hologram exposure are less critical. During reconstruction, shown in Fig. 2b, the hologram is viewed by reflected light, which need not be coherent or even monochromatic. Like a crystal, the three-dimensional grating will only reflect wavelengths that meet the Bragg condition, so it acts as a monochromator at the same time as it forms the image.

Finally, we note that the photographic process can be further manipulated to enhance the hologram. Exposure to light sensitizes some of the silver halide grains in the emulsion in such a way that they can be chemically reduced to metallic silver. This process is called "development", and yields the familiar black photographic image. (Small grains of silver appear black, not shiny.) For a hologram the blackened regions absorb light, and serve as the barriers of a grating, but it would obviously be more efficient to use all the incident light. This can be accomplished by converting the metallic silver to a different silver halide compound. The new compound formed in the exposed and developed regions has a different index of refraction than the original silver halide left in the unexposed areas, so there is a differential phase shift for light which traverses different parts of the emulsion. Interference still occurs as it does in the absorptive grating, but with greater intensity because no light is actually lost.

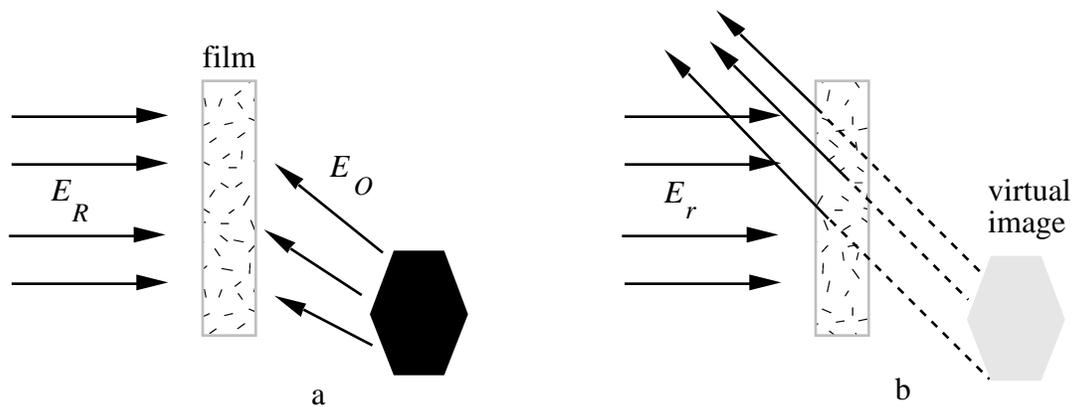


Fig. 2 Arrangement for (a) making and (b) viewing a reflection hologram.

B. Double-exposure interferometry

In double-exposure interferometry, one makes two successive holograms on the same film. For the first exposure the object is in some reference state, and for the second it is moved or deformed. During reconstruction, the two images interfere as though there were two copies of the object present. Where the displacement between the two "copies" is such that the reflected light would be shifted by an integral multiple of half wavelengths there will be a dark fringe in the combined image. Conversely, bright fringes occur when the displacement is an integral number of wavelengths.

The geometry is shown in Fig. 3. The change in optical path length ΔO due to the shift in position Δy of the surface is

$$\Delta O = (z_1 + z_4) - (z_2 + z_3) \quad (6)$$

Using the given angles this becomes

$$\Delta O = \Delta y(\cos \alpha + \cos \beta) \quad (7)$$

The conditions for interference can be written as

$$\Delta O = \frac{n\lambda}{2} \quad (8)$$

where odd integers imply dark fringes and even integers imply bright fringes. Equating these two expressions, the formula to be used in analysis is

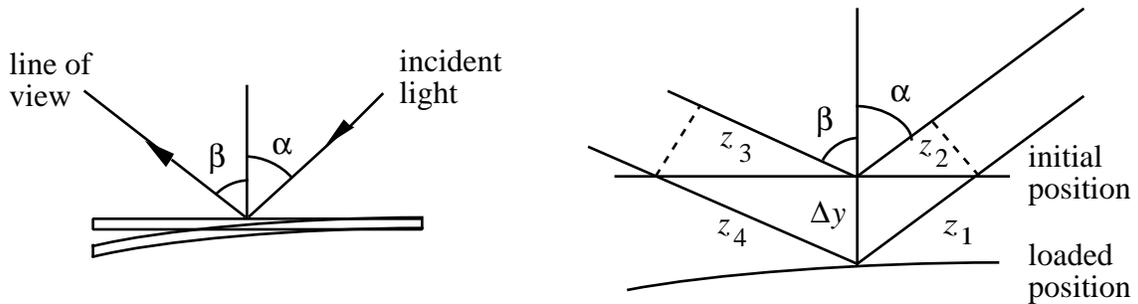


Fig. 3 The left diagram shows the geometry during reconstruction of the double-exposure hologram. The right diagram is an enlarged view of a portion of the bar, showing the displacement in more detail.

$$\Delta y = \frac{n\lambda}{2(\cos \alpha + \cos \beta)} \quad (9)$$

where n counts both dark and light fringes from some undeflected reference point.

C. Beam deflection

Because of engineering interest, the deflection of various beams and plates has been calculated for many different load conditions. The results can be found in the references. Our system is a uniform rectangular bar of length ℓ , width w and thickness t , clamped at one end. The total weight of the bar is W' and there is a small additional weight W at the end farthest from the clamp. The deflection $y(x)$ is given by

$$EIy = \frac{-W}{6}(\ell - x)^3 - \frac{W'(\ell - x)^4}{24\ell} - \frac{W\ell^2 x}{2} - \frac{W'\ell^2 x}{6} + \frac{W\ell^3}{6} + \frac{W'\ell^3}{24} \quad (10)$$

where x is the distance measured from the clamped edge. E is Young's modulus for the material, and I is the moment of area, $wt^3/12$ for a rectangular bar deflected normal to the thickness. In our experiment we are actually interested in the additional deflection produced by the added weight W , so we need only consider Δy ,

$$EI\Delta y = \frac{-W}{6}(\ell - x)^3 - \frac{W\ell^2 x}{2} + \frac{W\ell^3}{6} \quad (11)$$

This is the formula we need to compare with the measured deflections deduced from Eq. 9.

III. Methods and Measurements

Several steps are needed to construct the holographic interferogram and measure the beam deflection. A laser beam must be expanded and directed to uniformly illuminate the object. The photographic film must be positioned and exposed to record the holograms of the bar with and without load. The latent image in the film must then be developed and viewed under appropriate conditions to visualize the interference fringes. The fringe positions can then be measured, the deflection as a function of position deduced, and a comparison made with the expected variation. We consider each step in turn.

A. Optical set up

The beam emerging from the laser is non-uniform because of inhomogeneities in the lasing medium, and is too narrow to illuminate the object. These problems are cured with a spatial filter. The expanded beam can then be directed onto the subject with mirrors. A typical arrangement is shown in Fig. 4.

The spatial filter consists of a microscope objective, which is effectively a short focal length lens, and a pin hole in a piece of metal. The objective focuses the plane-wave portion of the laser output to a very small diameter beam which can pass through the pin hole. The remainder of the light from the laser is intercepted by the metal, so that the output of the spatial filter assembly is a spherically diverging wave.

The spatial filter is usually left in position. If not, you will need to align it. Put a white screen in the laser beam, and note where the beam strikes it. Remove the pin-hole from the filter assembly, and position the filter in the beam so that the expanding light beam is centered on the previously noted position. Use the focusing screw to move the objective away from the pin hole position, and install the pin hole. Roughly align the aperture with the focused spot by looking at

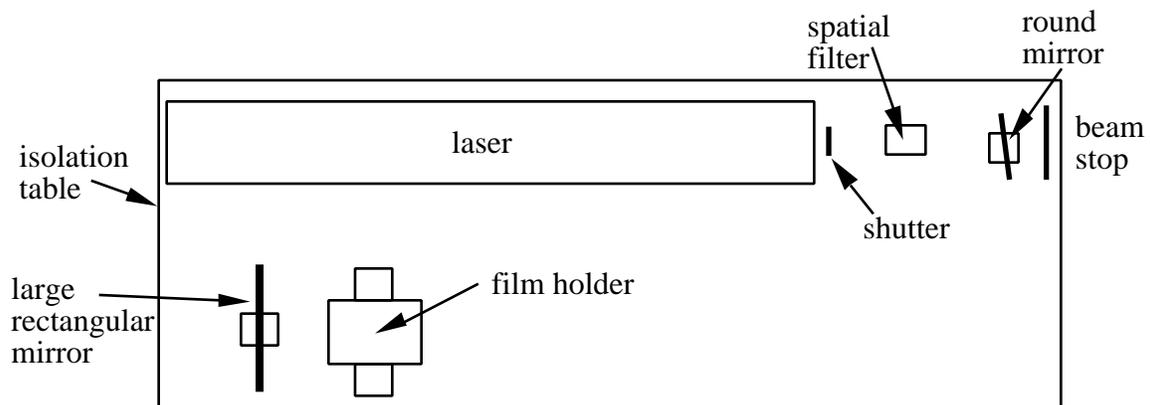


Fig. 4 Typical layout of the optical table for making a reflection hologram.

the pin hole and using the x - y controls to scan the holder for maximum brightness. At some position you will see a very bright red flash, indicating that the pin hole is somewhere in the focal spot of the objective. To fine tune the alignment, observe the output on the white screen. You will see a red blob surrounded by faint rings, the diffraction pattern of a circle. Adjust the x - y controls to center the spot in the rings and maximize the brightness. Then adjust the focusing screw to bring the objective closer to the pin hole, adjusting the x - y alignment as needed. Continue the adjustments until you have a large bright disk and the rings have essentially disappeared. At this point, the pin hole is exactly at the focal point of the lens. (With practice, this ritual will take a few minutes, but it can be excruciatingly tedious the first time. Get help if you seem to be having problems.)

After aligning the spatial filter, arrange the mirrors so that the light beam expands enough to uniformly illuminate the bar. You can check the variation of intensity along the cantilever position with the light meter. The incident beam should be at approximately 30° away from the normal (vertical) ($\alpha = 30^\circ$) to generate the best fringes in the film.

B. Cantilever beam

The beam is a piece of steel, firmly clamped at one end. It can be loaded by placing a piece of lead at the free end. By design, the clamp and support bar deflect negligibly under the required load. The needed parameters are:

length	11.4 cm
width	2.54 cm
thickness	0.32 cm
Young's modulus	2.0×10^{11} nt/m ²
load	10.0 gm

The length and the distance along the bar, x , are both measured from the edge of the clamp.

C. Making holograms

A hologram is made by properly exposing and processing a photographic emulsion. Exposure can be estimated with an electronic light meter. The double-exposure hologram is made on a glass plate, rather than flexible film, to insure dimensional stability between exposures. Processing involves a special developer and bleach to convert the latent image to a phase hologram. After processing, the final image can be viewed and measured.

The required materials are relatively expensive (Plates are \$10.00 each, and a batch of developer is about \$1.00.), so use what you need, but do not be wasteful.

1. General procedures

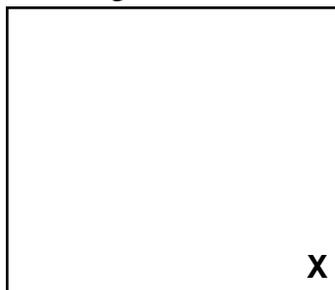
The holographic material is most sensitive to red light, so we can use a dim green "safe light" to facilitate handling. Even this will eventually fog the emulsion, however, so limit total exposure to a few minutes. Keep the plate at least three feet from the safe light whenever it is out of the box.

It is essential that the plate not move a significant fraction of a wavelength during exposure. To accomplish this, it is held between two glass plates with heavy paper clamps. When ready for an exposure, you simply center the plate on the lower glass and then gently hold down the upper glass while you position the clamps. To minimize loss of object beam by reflection on glass surfaces, the emulsion side of the plate should face the object. Figure 5 will help you identify the emulsion side of the material. After making any disturbance on the optical table you should allow several seconds for vibrations to damp out before starting an exposure.

An approximate exposure time can be found by measuring the light intensity with the photodiode light meter. (Be sure the battery is connected and the meter is on the X1 scale. The voltage reading is then proportional to light intensity.) For Slavich PFG-03M plates, processed as directed, optimum exposure is obtained when the product of incident intensity (read in volts) x exposure time (seconds) is about 18 volt-seconds. As is typical for photographic materials, variations of 20-30% are not significant. The laser intensity drifts with time, even after 20 minutes of warm-up, so you should take the photometer reading immediately before loading and exposing the plate.

Processing consists of development, a rinse to stop the process, bleaching to convert the metallic silver, and a final rinse to remove residual chemicals. Detailed instructions are posted in the darkroom. At this stage, water has expanded the emulsion so much that the grating is not effective at visible wavelengths. After several hours of drying it should return to its original dimensions, and the hologram will be visible. You can speed the drying by using a fan to blow air across the plate.

When fully dried, the reflection hologram can be viewed in white light against a black



4x5 plate

Fig. 5 When positioned as shown, the emulsion (sensitive) side is facing you.

background. A slide projector makes a convenient source of bright, collimated light. Arrange the projector and a mirror so that the reconstruction beam is at the same angle as the laser beam was during exposure. It is also necessary to orient the hologram correctly, that is emulsion side down and with the reconstruction beam coming from the proper direction.

2. Measurements to be made

Since you need to expose the plate twice, each interval should be about $2/3$ of the optimum time you estimated. This will be long enough to give a good image without grossly over exposing the plate.

To get the interference pattern, put the lead weight on the end of the cantilever, and put a plate in the holder. Allow the vibrations to die out and expose for the needed time. Using the string, gently remove the weight, while creating minimal disturbance to the table. Let the vibrations die out for a few seconds, and then do the second exposure. If everything works, the finished hologram will show a series of straight dark fringes across the invisibly-bent bar.

D. Analysis

To analyze the fringe pattern, set up the hologram for viewing as explained above. Lay a ruler on top of the plate, and estimate the center of each dark fringe, relative to the edge of the clamp. Note that there will be some parallax because the ruler is not in the plane of the fringes. Alternatively, you can support the plate above the viewing surface and hold the ruler in the plane of the cantilever. In either case, it is probably most convenient to look straight down on the image so that $\beta = 0$.

Convert your fringe count to a displacement using Eq. 9, and compare with that expected on the basis of Eq. 11. A detailed error analysis is probably not justified, but you should quantitatively estimate major uncertainties for both the position and deflection measurements. Discuss any deviations you do (or don't) see between measurements and calculation.

IV. Further Reading

N. Abramson, *Making and Evaluation of Holograms*, Academic Press, NY, (1981); Practical advice from a practicing engineer.

E. Hecht, *Optics*, Addison-Wesley, Reading, MA, (1987), Section. 14.3; Simplified treatment of holographic imaging theory.

G. Saxby, *Practical Holography*, Prentice Hall, NJ, (1987); Lots of details about making pictorial holograms, aimed at the aspiring amateur.

K. R. Symon, *Mechanics*, Addison-Wesley, Reading, MA, (1971), Section. 5.10; An elementary but entirely adequate discussion of beam bending.

C. M. Vest, *Holographic Interferometry*, Wiley, NY, (1979); Detailed, very formal treatment of holographic fringe formation in many geometries.