THE FRANCK-HERTZ EXPERIMENT

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Introduction

By the early part of the twentieth century it was clear that isolated atoms absorb and emit electromagnetic radiation at characteristic frequencies. Bohr's atomic model suggested that this was due to an inherent quantization of the allowed energies of atomic electrons. Given these ideas, it is reasonable to ask if other modes of energy transfer would also exhibit quantization. This was the question posed, and answered, by James Franck and Gustav Hertz (a nephew of Heinrich Hertz, of electromagnetic fame) in 1914.

Franck and Hertz bombarded gaseous mercury with electrons and showed that the electrons lost discrete amounts of energy. Further, they were able to show that electron bombardment at an appropriate energy led to optical emission at the wavelength corresponding to that energy. Their results could be interpreted within the Bohr model as demonstrating excitation of one of the discrete energy levels, followed by a transition back to the ground state with emission of light. This is obviously a classic experiment, in the sense of being of key importance, but the adjective is otherwise unfortunate in that the experiment provided strong evidence against classical mechanics and in favor of the nascent quantum mechanics.

In our laboratory we will repeat Franck and Hertz's energy-loss observations, using mercury, and try to interpret the data in the context of modern atomic physics. We will not attempt the spectroscopic measurements, since the characteristic emission is weak and in the ultraviolet portion of the spectrum.
Theoretical considerations

It is well known that electrons can be boiled out of a hot metal filament. By applying a positive potential between the filament and a nearby electrode one can give the electrons any desired kinetic energy. If the electron subsequently hits a gas atom it may transfer energy to the atom. In this experiment we seek to measure that energy transfer. Because of the large mass difference between the electron and any atom, collisions which do not excite internal motions in the atom result in very small changes in electron energy. The energy loss we observe, therefore, is essentially a measure of the energy changes internal to the atom.

A. Simple model

To a first approximation, the experiment can be understood by examining the idealized sketch in Fig. 1. Electrons are emitted from the filament with near-zero energy and then accelerated toward a grid through a sealed bulb containing a small amount of mercury. If the accelerating voltage is large enough, and there is no loss of energy in collisions, most electrons will pass through the grid and continue up the retarding gradient to the collector electrode. The current meter measures this flow of electrons. Alternatively, electrons which have made an inelastic collision and reach the grid with small kinetic energy will be captured there, rather than at the collector. If we measure the collector current as a function of grid-filament voltage we can infer the probability of an inelastic collision as a function of electron energy.

With this geometry and our assumption of quantized atomic energy levels we should expect to see several dips in the collector current as we increase the accelerating voltage. Ignoring the retarding potential for the moment, the first dip will occur when electrons reach the

Fig. 1 Schematic representation of the geometry and voltages used for the Franck-Hertz experiment. The graph shows the mechanical potential energy of an electron as a function of position within the tube. Note that both accelerating and retarding regions are present.
grid with just enough energy to excite a mercury atom. The next dip will occur at twice this voltage, since an electron can then lose its energy midway from filament to grid and then be accelerated enough to collide again near the grid. At still higher voltages there can be additional collisions leading to more dips. The important point is that the whole array of equally-spaced current dips should be due to the lowest energy transition that the electron can excite.

Since the applied electric field is assumed to be uniform, all electrons will reach the excitation threshold at the same place. This creates a population of excited Hg atoms which will emit light from a disk-shaped region between cathode and grid. The expected UV emission is absorbed by the glass envelope, but it can excite other atomic levels, and transitions between those could produce emission in the visible range. From our analysis of the current dips, we expect to see the first emission disk form near the grid. At higher accelerating potentials an additional luminous disk should appear with each current dip, and they should move towards the cathode with further increases in potential.

At sufficiently large accelerating voltages a discharge may occur because electron impact can ionize the atom, as well as induce the intra-atomic transitions we have discussed so far. The free electron produced by the ionization event can then be accelerated and ionize another atom, eventually leading to creation of a conducting plasma in the tube. The exact accelerating voltage at which breakdown occurs depends on the tube geometry, gas density and electron current, as well as the type of gas present. Once the gas has broken down the electrons are in a conducting medium and we can no longer interpret the collector current as a measure of the collision probability.

The final feature of this model is that the voltage of the initial current dip is unlikely to correspond to an atomic excitation energy. Most obviously, the applied retarding potential will shift the whole pattern. Also, the emitter and collector are made of different materials so their work functions will differ, resulting in shifts of order 1-2V. Finally, temperature gradients in the apparatus cause thermoelectric voltage differences, amounting to a few tenths of a volt. Fortunately, these constant offsets do not affect the voltage differences we want to measure.

B. More realistic models

Measurements immediately show that the simple model is incomplete at best. In particular, the voltage interval between dips does not correspond to an atomic transition energy in mercury. Worse, the observed interval depends on temperature and perhaps other parameters. Hanne [1] and Rapior et al [2] try to explain some of these anomalies by modifying the model. Their arguments are summarized here.

Figure 2 is a simplified energy level diagram for Hg and corresponding cross-sections for collisional excitation to the low-lying triplet levels. The original model assumes current dips will
occur when the electrons have just enough energy to excite the lowest 4.67 eV state. However, the cross-section for that transition is relatively small so it is plausible that some electrons will not collide until they gain enough energy to excite one of the higher levels. This would tend to raise the observed voltage of the dip and increase the range of voltages over which the current drops.

The energy levels and cross-sections are, of course, temperature independent, but the number density of Hg atoms increases rapidly with temperature because the gas is in equilibrium with a droplet of liquid mercury. This means that the distance the electrons can travel between inelastic collisions, the mean free path $\lambda$, becomes shorter as temperature increases. Rapior et al [2] claim that this leads to a decrease in the apparent dip spacing with increasing temperature and number density.

Their argument can be understood by reference to Fig. 3. The assumption is that the electron will be accelerated to the effective excitation energy $E_a$ and then travel an average distance $\lambda$ before actually colliding and losing energy. The electron continues to gain energy, shown by $\delta$, as it moves the distance $\lambda$ in the constant field. This tends to increase the dip voltage by an amount that depends on $\lambda$ and therefore the number density.

Fig. 3a shows the situation for the first minimum, where the collision occurs at the grid when the electron reaches energy $E_a + \delta_1$. At the second minimum, Fig. 3b, the electron will have gained energy $E_a + \delta_2$ twice, with $\delta_2$ somewhat bigger than $\delta_1$ because of the stronger field. The energy $E_n$ for the $n$'th dip is therefore
\[ E_n = n\left(E_a + \delta_n\right) \]  

(1)

If the cathode-grid distance \(L\), is much less than \(\lambda\) then \(\delta_n\) is given by

\[ \delta_n = n\frac{\lambda}{L}E_a \]  

(2)

Combining (1) and (2) we arrive at

\[ E_n = nE_a + n^2\frac{\lambda}{L}E_a + E_{\text{offset}} \]  

(3)

for the energy of the \(n\)'th minimum. An extra term, \(E_{\text{offset}}\), has been added to account for the retarding potential, work function difference and thermoelectric voltages noted above.

A second-order polynomial fit to \(E_n\) vs \(n\) directly yields model estimates of \(E_a\) and \(\lambda\). The mean free path \(\lambda\) is, in turn, related to the number density of mercury atoms, \(N\), and the scattering cross-section \(\sigma\) by

\[ \lambda = \frac{1}{N\sigma} = \frac{k_B T}{p\sigma} \]  

(4)

where \(p\) and \(T\) are the vapor pressure and temperature of the Hg gas. It is, therefore, possible to quantitatively compare the model parameters with the atomic data in Fig. 2. (A spreadsheet on the lab page will calculate \(p\) and \(N\) as a function of \(T\).)

Fig. 3 Electron kinetic energy vs position in the uniform field region between filament and grid. Inelastic electron-mercury collisions are assumed to occur at energy \(E_a + \delta\), on average, reducing the electron energy to zero. (a) Single collision near grid. (b) \(U_B\) just large enough to allow two collisions. Adapted from [2]
C. Even more complications

On the occasion of the hundredth anniversary of the Franck and Hertz publication, Robson, White and Hildebrandt [3] published a detailed review of the physics underlying the experiment. They show that the standard explanations are inadequate and, surprisingly, that it is not entirely clear what actually happens.

The main complication is that the elastic scattering cross-section is much larger than the inelastic cross-section, as shown in Fig. 2. Elastic collisions randomize the directions of electron velocities and, if numerous, significantly spread the energy about the mean. The result is that the electrons form as a loose cloud that moves from filament to grid diffusively, rather than as a unidirectional beam of well-defined energy.

Robson et al [3] go on to show that modern plasma physics techniques can account for most of the features observed in the Franck-Hertz regime. In particular, their models predict that the oscillations in electron density, which give rise to the current dips and optical phenomena, will occur for a restricted range of accelerating voltage and number density. The details are beyond the scope of this exercise, but you may observe that current dips vanish for the largest $U_B$ at the higher temperatures.
Measurement and analysis

The laboratory is equipped with a mercury-filled tube in a small oven, power supplies and appropriate meters. Using this apparatus it is possible to plot the collector current as a function of filament to grid voltage, and to observe the effect of varying both the electron current and the gas density.

Most of the equipment is contained in two boxes. An electric oven is used to raise the temperature of the tube to 150-200 °C to produce the desired vapor pressure of mercury. The other box contains the voltage sources and a current-to-voltage amplifier for the collector signal. (Contrary to the diagram on the oven, it does not contain any electronics.) The manufacturer's operating instructions are available (in translation) for reference in the lab.

The apparatus should be connected as in Fig. 4. The filament heater current is set with the Heizung control, and monitored with the AC ammeter. A voltmeter allows precise calibration of the accelerating voltage scale. A digital interface device reads the scaled accelerating voltage and a voltage proportional to the collector current. These can be displayed with LoggerPro using the FHPPlotter.cmbl startup file.

![Connection diagram for the NEVA Franck-Hertz apparatus](image)

Fig. 4 Connection diagram for the NEVA Franck-Hertz apparatus. The circuitry inside the dashed lines is contained in the electronics box. The tube is housed in an oven, which is not shown.
Familiarize yourself with the apparatus as follows:

1. Stabilize the oven temperature at about 180 °C. The thermostat provided allows large temperature swings so it is better to turn the control all the way up and use the variable transformer to set the input power level. Transformer settings in the range 50-70 will usually give the desired temperature. You will not see dips much below 150 °C, and the apparatus will be damaged above about 200 °C.

2. Set the accelerating voltage scale to read actual voltage, using the LoggerPro calibration procedure. There is no need to calibrate the current scale, as we only need relative values.

3. Set the amplifier gain (Verstärkung) to mid-range and adjust the zero-offset control (0-Punkt) until you get a definite positive reading on the signal channel. (The system will not read negative voltages and may indicate a small positive offset.) The filament current (Heizung) should also be at mid-range initially. The accelerating voltage control is labeled $U_B$. The switch should be set for DC (straight line) operation, not sweep (ramp line). Be sure the unit is switched on.

4. If you now sweep $U_B$ slowly through its range you should get a curve with several relatively abrupt dips, perhaps terminated by an abrupt jump in the current when ionization occurs. Extended operation in the discharge regime can cause overheating and damage the tube, so immediately turn down the accelerating voltage.

5. Darken the room and closely observe the tube as you vary the voltage $U_B$. Does the pattern of light emission you see correspond to the current dips, as predicted by the simple model discussed above?

6. Determine $E_n$ from the dip plot by positioning the cursor at the chosen point and reading the voltage from the lower-left corner of the graph window.

Once you have verified that the apparatus is working, you can explore the effect of varying the electron emission current (by varying the filament heating current) and the Hg number density (by varying temperature). Because ionization breakdown will limit the range of accelerating voltages you can use at each temperature and heater current, it is important to plan your measurement strategy carefully in order to separate the various effects.
The first issue is the choice of $E_n$. Ideally, your curves would show a sharp sawtooth pattern, dropping abruptly at each excitation point. For various reasons, there will be some spread in the curves so the assignment of $E_n$ is somewhat arbitrary. The models suggest that the minimum current point is the best estimate of $E_n$, but it might be equally reasonable, and more precise, to take the half-way point on the drop as an average. You should test for systematic effects by analyzing one or two data sets both ways and comparing the results.

You should then try to separate the effects of number density and emission current by obtaining current-voltage plots at various temperatures for one or more fixed filament currents, and for a range of filament currents at one or more fixed temperatures.

Analyze your data by finding the set of $E_n$ for each plot, using your chosen criterion. If there are at least 7-8 minima you can fit the $E_n$ to Eq. 3 and estimate both $E_a$ and $\sigma$ for that number density and emission current. With fewer minima you can still do a linear fit to the first 3-4 to get an estimate of $E_a$ alone.

In your report you should present your data for the excitation voltage, inelastic cross section and any other observations you consider pertinent. Discuss the extent to which your data are or are not consistent with the claims made in section B above. Also, be sure to explain the tests you have made for systematic errors due to your method of picking $E_n$. 
References

