I. Introduction

The energy levels of atoms are largely determined by the Coulomb interaction between the electrons and the nucleus, but other interactions may also be present. These additional interactions manifest themselves as shifts or splittings of the observed spectral lines relative to the lines which would occur if the electrons moved in a pure central potential. For example, the fine structure splitting arises from electron-electron Coulomb interaction in multi-electron atoms and from the spin-orbit interaction of the moving electrons with the nuclear electric field. Smaller splittings, called hyperfine structure, arise from non-electrostatic interactions with the nucleus. For example, if the nucleus has a spin there can be magnetic interactions which perturb and split some of the electronic levels.

Differences in the nuclear mass and volume can also shift the atomic energy levels between isotopes of the same element. As an example, we will observe the differences between the visible-light emission spectra of hydrogen and deuterium. The observed shifts are attributed to the different masses of the nuclei, yielding an estimate of the mass ratio.

The next section outlines the pertinent atomic physics. Section III describes the measurements and their analysis. Additional information on the spectrometer and the lock-in system can be found in the instruction manuals, available in the lab.
II. Theoretical considerations

The energy levels of the hydrogen atom, including the effect of a finite nuclear mass $M$, are given by

$$E_n = \frac{hcR}{n^2} \left(1 + \frac{m}{M}\right)^{-1}$$

(1)

where $R$ is the Rydberg constant for an infinite-mass nucleus, $m$ is the mass of the electron, $M$ is the mass of the nucleus, and $n = 1, 2, 3, \ldots$. The Balmer series of lines, which fall in the visible spectrum, are due to transitions from $n = 3, 4, 5, 6\ldots$ to $n = 2$. The corresponding wavelengths are given by

$$\lambda = \frac{1}{R} \left(1 + \frac{m}{M}\right) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}$$

(2)

where $n_i, n_f$ are the initial and final level numbers of the transition. Evidently a measurement of the wavelength difference between spectral lines from hydrogen and deuterium would determine the relative mass of the isotopes. (Deuterium was actually discovered this way. See Phys. Rev. 40,1 (1932))
III. Measurements

The objective is to measure as accurately as possible the wavelength difference between several of the Balmer lines of hydrogen and deuterium. Since the difference is small, of order 0.1 nm, it is not feasible to measure the individual wavelengths to sufficient accuracy.

The apparatus is sketched in Fig. 1. Briefly, light from a gas-discharge tube containing both hydrogen and deuterium is analyzed by a scanning spectrometer. Light intensity as a function of wavelength is measured by a photomultiplier tube connected to a lock-in amplifier system and recorded on a paper chart. Using the spectrometer and chart recorder calibrations and the known parameters of the hydrogen atom, one can determine the deuterium mass from the measured difference in wavelength.

Since one of the main purposes of this exercise is to accurately measure a small shift, it is important to establish the errors in the measurement. The basic measurements should be done in such a way as to obtain a realistic standard deviation. If there are uncontrolled effects which might lead to systematic errors they should also be discussed and the likely effects estimated, if possible.

For reference, we list below the accepted values of the wavelengths involved in this experiment and some useful physical constants. Your data should stand on their own, however, and it is not intended that you compare your results with the accepted value to define an "error".

<table>
<thead>
<tr>
<th>Table I. Constants for the hydrogen-deuterium system</th>
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<tbody>
<tr>
<td>Infinite-mass Rydberg constant: $R = 1.0973731 \times 10^7 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>Mass of electron: $9.109534 \times 10^{-31} \text{ kg}$</td>
</tr>
<tr>
<td>Mass of hydrogen nucleus: $1.672649 \times 10^{-27} \text{ kg}$</td>
</tr>
<tr>
<td>Mass of deuterium nucleus: $3.343637 \times 10^{-27} \text{ kg}$</td>
</tr>
</tbody>
</table>

Fig. 1 Block diagram of apparatus for H/D spectroscopy.
Table II. Wavelengths of the hydrogen spectrum

<table>
<thead>
<tr>
<th>label</th>
<th>n_i</th>
<th>( \lambda_{\text{H}} ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>3</td>
<td>656.285</td>
</tr>
<tr>
<td>( \beta )</td>
<td>4</td>
<td>486.133</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>5</td>
<td>434.047</td>
</tr>
<tr>
<td>( \delta )</td>
<td>6</td>
<td>410.174</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>397.007</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>388.906</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>383.540</td>
</tr>
</tbody>
</table>

A. Setup

The discharge lamp is started by turning on the high voltage supply. The light output will increase for the first 3-4 minutes of operation and then slowly decline. You should, therefore, allow the tube to warm up briefly before starting a measurement and turn it off if there will be an interval between measurements.

The lamp and a lens are placed on the optical bench in front of the spectrometer approximately as indicated in Fig. 2. By using a lens we increase the amount of light entering the spectrometer and separate the electrically noisy arc discharge from the sensitive detection electronics. The maximum amount of light is transferred into the spectrometer when the lens forms an image of the discharge on the entrance slit. Adjust the positions of the lamp and lens as needed to get a clear image centered on the slit. You will probably need to remove the chopper from the optical path so that you can see the slits and locate the image properly.

Install the chopper near the slit, with the outer ring of holes in the chopper disk vertically centered on the spectrometer slit. Since the holes are fairly short, set the spectrometer slit height to 1 cm (not the maximum of 2 cm) to ensure that the chopper blocks all the incoming light.

High voltage for the PMT comes from a precision supply. Start with a setting of about 400 V negative, and do not exceed the tube rating of 1250V. The signal output from the PMT goes to the current input of the lock-in.

Set the lock-in controls, following the instructions in the lock-in manual as needed. A chopping frequency of about 400 Hz is convenient and safely away from harmonics of 60Hz.

Fig. 2 Input optics for H/D spectroscopy. All components except slits are mounted on an optical bench.
You will probably find it helpful to use the bandpass, 60 Hz and 120 Hz filters to minimize interference.

The last step in the optical alignment is to maximize the actual signal. Open up the spectrometer slits to about 40 µm width and check for 1 cm height setting. Set the wavelength controls near the red Balmer line. With the lamp on and the chopper running you should get some signal on the lock-in, which you should roughly maximize by adjusting the wavelength control. Increase the sensitivity on the lock-in to get a reasonable level and then adjust the phase to maximize it. You can also increase the gain of the PMT by increasing the PMT voltage and decreasing the lock-in sensitivity until the lock-in overloads, but do not exceed 1250V. Once you have a definite signal, you can try very small lateral adjustments of the discharge tube to maximize the output by centering the image on the slit. When you have found the best conditions, be sure the optical components are firmly secured for subsequent measurements.

**B. Data acquisition and analysis**

Operating conditions must be optimized for each pair of lines you measure. The red lines near 656 nm are the brightest, so it is best to start there. Scan through the region near 656 nm and plot the output. You should find two distinct peaks, corresponding to the transitions in hydrogen and deuterium. By reducing the input and output slit widths you will improve the resolution at the expense of intensity. A rapid wavelength sweep is convenient, but if it is too fast the lock-in and chart recorder response times will distort the shapes of the peaks. Try varying all these parameters until you get clearly resolved lines of consistent shape.

When you have found satisfactory settings you should record 3-4 scans of the pertinent wavelength region so that you can check for random variation in the apparent peak separation. Be sure to record the sweep rate settings on the spectrometer and chart recorder so that you can convert your peak separations to wavelength differences. Carry out the optimization and measurement process for all of the Balmer lines that you can detect. Measure the plots to find the wavelength differences between the hydrogen and deuterium lines, and the uncertainty in the differences.

An explicit expression for the wavelength difference can be found from Eq. 2,

\[ \Delta \lambda = \frac{m}{M_H R} \left( 1 - \frac{M_H}{M_D} \right) \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)^{-1} \]

where \( M_H \) and \( M_D \) are the hydrogen and deuterium masses. From this result, there are at least two ways to extract \( M_H/M_D \) and its uncertainty.
A plot of $\Delta \lambda$ vs $\left( n_j^2 - n_i^2 \right)^{-1}$ should be a straight line whose slope will give a value of $M_H/M_D$. The uncertainty can be derived from the slope uncertainty determined by the linear least squares fitting algorithm, and the adequacy of the fit can be checked with $\chi^2$ as usual. A disadvantage of this method is that all the data are weighted equally even though some $\Delta \lambda$ may be more precise than others.

A more direct approach is to vary $M_H/M_D$ in Eq. 3 to minimize $\chi^2$ between the theoretical and experimental $\Delta \lambda$, using the experimental standard deviation for each $\Delta \lambda$. This is easily done by systematically changing $M_H/M_D$ in a Matlab or Excel program to search for the minimum. The uncertainty can be estimated by changing $M_H/M_D$ until the total $\chi^2$ increases by one. The minimum $\chi^2$ per degree of freedom can be used to test the fit.