Polarization

‘You damn sadist,’ said Mr Cummings, ‘you try to make people think.’

Ezra Pound (1885-1972)

OBJECTIVES
To learn about polarization states and how to manipulate them.

THEORY
Light is a transverse electromagnetic wave and thus its interaction with matter can depend on the orientation of the electric field vector. Such phenomena are called polarization effects. In this experiment you will study some of the basic polarization effects and how optical elements can change the polarization of a light beam.

1. Notation
We can represent the two orthogonal components of a plane electromagnetic wave traveling in the $z$ direction in the form

$$\vec{E}_x(z,t) = \hat{x} E_{0x} \cos(kz - \omega t)$$

and

$$\vec{E}_y(z,t) = \hat{y} E_{0y} \cos(kz - \omega t + \epsilon)$$

where $\epsilon$ is the relative phase of the two components. The total field is the vector sum of the two components, $\vec{E} = \vec{E}_x + \vec{E}_y$. The state of polarization depends on $\epsilon$ and the relative magnitudes of $E_{0x}$ and $E_{0y}$.

For most values of $\epsilon$ and the two amplitudes $E_{0x}$, $E_{0y}$ the light beam is said to be elliptically polarized. This means that the end of the electric field vector traces out an ellipse as the wave moves through a fixed plane normal to the propagation direction. Linear polarization occurs when $\epsilon = 0$ or is an integral multiple of $\pm \pi$. The tip of the field vector follows a sine curve in a plane containing the direction of propagation, and therefore traces a straight line in a fixed plane perpendicular to the direction of propagation. Left-hand circular polarization (LHC) occurs when $\epsilon = \pi/2 + 2m\pi$ (where $m = 0, \pm 1, \text{etc}$) and $E_{0x} = E_{0y}$. Now the tip of the field vector follows a circular helix around the direction of propagation, and traces out a circle in the perpendicular
Looking into the beam the vector rotates counter-clockwise around its circle. Similarly, right-hand circular polarization (RHC) occurs when \( \epsilon = -\pi/2 + 2m\pi \) (where \( m = 0, \pm 1, \text{ etc} \)) and \( E_{ox} = E_{oy} \) which looks just like LHC except that the sense of rotation is reversed to be clockwise looking into the beam.

Detectors in the optical frequency range generally do not respond to the time variation or phase of the input signal, but only the intensity. The measured detector output is proportional to the light intensity defined as

\[
I \propto \langle |\mathbf{E}|^2 \rangle \tag{3}
\]

where the angle bracket indicates a time average. We will therefore want to calculate the intensity resulting from the particular combination of optical elements we are studying.

2. Polarizers

Several physical phenomena can select a particular linear polarization direction from an incident beam. We will be using the common Polaroid™ material, which is dichroic. In its broadest sense the term dichroism refers to the selective absorption of one of the two orthogonal components of an incident beam. Polaroid is a plastic sheet that has been stretched to align its long chain polymeric molecules in the plane of the sheet and along a preferred axis. The material is then impregnated with an iodine compound which makes the molecules conductive so they strongly absorb light whose electric field is parallel to the molecular chains. The resultant sheet blocks waves with electric fields along the molecular axes, and passes waves with perpendicular electric fields. The output is a beam linearly polarized along the preferred axis. There is also some absorption of the transmitted wave, so the intensity is reduced even if the input is completely polarized already.

To calculate the transmission of a polarizer, assume that the input beam is linearly polarized along the x-axis so the incident wave is

\[
\mathbf{E}_x = \hat{x}E_0 \cos(kz - \omega t) \quad \mathbf{E}_y = 0 \tag{4}
\]

and the incident intensity is \( I_0 = E_0^2 / 2 \). The polarizer axis can be set at any chosen angle \( \theta \) with respect to the x-axis, which aligns it parallel to a unit vector

\[
\hat{r}_p = \hat{x} \cos \theta + \hat{y} \sin \theta \tag{5}
\]
We expect the polarizer to transmit the component of the input beam parallel to \( \hat{r}_p \), and absorb the perpendicular component, leading to an output wave

\[
\vec{E}_p = (\vec{E}_x \cdot \hat{r}_p) \hat{r}_p = (E_0 \cos(kz - \omega t) \cos \theta) \hat{r}_p \tag{6}
\]

Computing the transmitted intensity from the definition, Eq. 3, gives

\[
I(\theta) = I_0 \cos^2 \theta \tag{7}
\]

which is known as Malus’ Law. This expression needs to be multiplied by an additional factor \( T_p \) to account for the extra absorption in the Polaroid. As expected, the transmitted intensity should be maximum when the polarization axis is aligned with the input polarization, \( \theta = 0 \), and minimum when at right angles to the input.

A more interesting result occurs if we examine the transmission through two polarizers in sequence. The second polarizer, closest to the detector, is set perpendicular to the input polarization, so the transmitted intensity would be zero in the absence of the first polarizer. However, as noted above, the first polarizer transmits the component of the input beam along \( \hat{r}_p \). The second polarizer then transmits any component of \( \hat{r}_p \) along the y-axis. In effect, adding a polarizer can increase the transmission because it projects the plane of polarization of the beam onto the y-axis, allowing some to go through the second polarizer.

A quantitative analysis starts with the output of the first polarizer as given by Eq. 6. The second polarizer is oriented along the y-axis, so the wave after the second polarizer is

\[
\vec{E}_2 = (\vec{E}_p \cdot \hat{y}) \hat{y} = E_0 \cos \theta \sin \theta \cos(kz - \omega t) \hat{y} \tag{8}
\]

Including a factor of \( T_p \) for each polarizer, the transmitted intensity is predicted to be

\[
I(\theta) = T_p^2 I_0 \cos^2 \theta \sin^2 \theta = \frac{T_p^2 I_0}{8} (1 - \cos 4\theta) \tag{9}
\]

This shows peaks at \( \theta = \pi/4, 3\pi/4 \ldots \), whenever the first polarizer axis is at \( \pi/4 \) to the input beam polarization.
3. Wave plates

It is also possible to manipulate the relative phase shift of the different components of an input beam with a device called a wave plate or retarder. Many substances, especially crystals, exhibit the property of double refraction or birefringence. The details can be quite complicated but, in the simplest terms, the velocity of propagation of an electromagnetic wave with $\mathbf{E}$ along one axis of a birefringent material is different from that with $\mathbf{E}$ along another axis. Due to the different velocities of the two polarization components, the optical path lengths are different, so that the relative phase of the two components can be changed. The phase difference is given by

$$\varepsilon = \frac{2\pi d}{\lambda_0} (n_o - n_e)$$  \hspace{1cm} (10)

where $n_e$ is the index of refraction for the wave with $\mathbf{E}$ along the optic axis, $n_o$ is the index of refraction for the wave with $\mathbf{E}$ perpendicular to the optic axis, and $\lambda_0$ is the wavelength in free space. Using the definitions of the polarization states given above you can readily see how this effect can be used to manipulate the various states.

A quantitative analysis is straightforward but messy. The vector component of the input parallel to the optical axis will propagate as the extraordinary ray, while the perpendicular component will be the ordinary ray. If the optical axis of the wave plate makes an angle $\theta$ with the input, polarized along the x-axis, the output rays will be polarized along the unit vectors

$$\hat{\mathbf{r}}_e = \hat{x} \cos \theta + \hat{y} \sin \theta \hspace{0.5cm} \hat{\mathbf{r}}_o = -\hat{x} \sin \theta + \hat{y} \cos \theta$$  \hspace{1cm} (11)

The output waves are

$$\mathbf{E}_e = (\mathbf{E}_x \cdot \hat{\mathbf{r}}_e) \hat{\mathbf{r}}_e = E_0 \left[ \hat{x} \cos^2 \theta + \hat{y} \sin \theta \cos \theta \right] \cos (kz - \omega t + \varepsilon)$$

$$\mathbf{E}_o = (\mathbf{E}_x \cdot \hat{\mathbf{r}}_o) \hat{\mathbf{r}}_o = E_0 \left[ \hat{x} \sin^2 \theta + \hat{y} \sin \theta \cos \theta \right] \cos (kz - \omega t)$$  \hspace{1cm} (12)

The total output wave is of course the sum of these components, $\mathbf{E}_w = \mathbf{E}_e + \mathbf{E}_o$. Note that the retarder has produced a phase shift, as expected. If the retarder transmits the extraordinary and ordinary components equally, the output intensity is independent of $\theta$, so an intensity detector shows no effect of the wave plate.

Intensity variation can be observed if the beam goes through a polarizer after the wave plate. The beam transmitted by a polarizer at angle $\phi$ is given by
\[ \mathbf{E}_2 = \left[ (\mathbf{E}_e + \mathbf{E}_o) \cdot \mathbf{\hat{r}}_p \right] \mathbf{\hat{r}}_p \]  

(13)

where the unit vector \( \mathbf{\hat{r}}_p \) is given by

\[
\mathbf{\hat{r}}_p = \mathbf{i} \cos \phi + \mathbf{j} \sin \phi
\]

(14)

Evaluating the dot product and collecting the terms with the same time dependence gives

\[
(\mathbf{E}_e + \mathbf{E}_o) \cdot \mathbf{\hat{r}}_p = E_o \cos(kz - \omega t) \left[ \cos \epsilon \cos \theta(+) + \sin \theta(-) \right] \\
- E_o \sin(kz - \omega t) \sin \epsilon \cos \theta(+)
\]

(15)

with

\[
(+) = \cos \theta \cos \phi + \sin \theta \sin \phi
\]

\[
(-) = \sin \theta \cos \phi - \cos \theta \sin \phi
\]

(16)

The intensity is found by squaring and then time-averaging, which eventually yields

\[
\frac{I_2}{I_0} = \cos^2 \theta(+) + \sin^2 \theta(-)^2 + 2 \cos \epsilon \cos \theta \sin \theta(+)(-)
\]

(17)

A general evaluation is best done numerically, but some special cases can be checked easily.

If \( \theta \) is fixed at 0 or \( \pi/2 \) and the polarizer is rotated, the intensity is

\[
\frac{I_2}{I_0} = \cos^2 \phi
\]

(18)

exactly as if the wave plate were absent. This should not be surprising, because when the input beam polarization is parallel or perpendicular to the optic axis of the retarder the output is phase shifted but the direction of polarization is unchanged.

If \( \theta \) is fixed at \( \pi/4 \) and the polarizer is rotated, the intensity variation is

\[
\frac{I_p}{I_0} = \frac{1}{2} (1 + \cos \epsilon \cos 2\phi)
\]

(19)
which is more informative because it depends on the phase shift $\varepsilon$. If $\varepsilon$ is $\pm \pi/2$ the output is circularly polarized, and there is no variation in the intensity. In general, the observed variation will give a measure of the actual phase shift produced by the plate.

**EXPERIMENTAL PROCEDURE**

Figure 1 shows the general layout of the light source and detector. The laser diode, aperture, lens and first polarizer create a monochromatic, polarized test beam. The components to be tested are placed in one or more holders between the source polarizer and the detector. We will study the effects of various combinations of retarders and polarizers to verify that they work as advertised, and to measure some of their characteristics.

The light source is a laser diode in a cylindrical holder. It produces a partially-polarized beam which diverges in an elliptical pattern and has a wavelength of 650 nm. An aperture and a 5 cm focal length lens are mounted in a double holder. By positioning the holder a few centimeters from the diode you can produce a beam of roughly constant diameter and uniform intensity. Then install a polarizer on a holder so that the $0^\circ$ mark is in a convenient orientation, perhaps vertically down. Rotate the diode holder to align the diode polarization with the Polaroid, as indicated by maximum intensity. For best results try to laterally center all elements on the beam and orient them perpendicular to the beam axis. Once everything is aligned you can secure the holders to the optical bench with the supplied screws. This arrangement provides the polarized test beam for all subsequent measurements.

The LoggerPro file *Polarization.cmbl* uses the output from the rotary holder and light detector to create graphs of intensity vs angle. You will need to calibrate the rotary holder so that the voltage divider output corresponds to the rotation angle in radians. Do this by going to Experiment > Calibrate > LabQuest Mini:1 CH 1: Voltage (+/-10V) and then Calibrate Now. Align the zero mark on the rotor with the mark on the holder and enter 0 (radians) in the first calibration box. Advance the rotor one full turn so that the marks are again aligned and enter 6.28 (radians) in the calibration box. You will need to repeat this procedure periodically to compensate for slippage in the belt drive.

![Diagram](image.png)

Fig. 1 General layout for polarization experiments. The aperture, lens and first polarizer form a polarized beam. The optical element to be rotated is mounted to the rotary holder, and any additional elements are placed in holders located as needed.
The light detector used here is a semiconductor device called a photodiode. When light falls on the semiconductor it creates a current which is converted to a voltage signal by an amplifier circuit. By measuring the voltage we can determine the current, and infer the amount of light falling on the detector. We won’t be concerned with the constant of proportionality, but simply take the measured voltage as an estimate of the light intensity.

Connect the DMM and LabQuest to the photocell output and set both to read voltage. (The LabQuest will be configured when you start Polarization.cmbl.) The absolute maximum photocell output is slightly less than 7.5V, so the maximum reading should be somewhat less than 7V. If your laser diode has excessive output you can rotate it slightly to decrease the intensity coming through the polarizer. You should also zero the LabQuest photocell channel with no light input before taking data. Note that the DMM reading is more stable than the LabQuest, so you should always use the DMM readings when making adjustments and limit the LabQuest to plotting.

To minimize background signal you will need to take data in dim light, usually with the room lights off. Further, silicon photodiodes are more sensitive to near infra-red light than to the visible red from the laser, so there should be no incandescent lamps operating in the room when you are taking data. Distant fluorescent lamps, computer screen backlights and LED lamps are acceptable as long as the detector input is not directly exposed. To see if your precautions are adequate, block the input beam and note the intensity reading on the DMM. If this is more than a few millivolts, turn off lights or shield the detector input as needed.

Please handle all the optics by the edges. Do not put optical elements on top of each other or slide them across the bench, as they scratch easily.

1. Single polarizer

The first exercise is to measure the transmission of the polarized beam through a linear polarizer as it is rotated about the propagation direction. We expect a maximum transmission when the polarizer axis is aligned with the polarization of the beam, and zero transmission when it is perpendicular.

Start by making some auxiliary measurements to determine the polarization efficiency and transmission factor of the Polaroid. Mount the Polaroid on the rotary holder and adjust the angle to obtain DMM readings for maximum and minimum transmitted intensity. The polarization effectiveness is then defined as

\[
\text{Percent polarization} = \frac{100(I_{\text{max}} - I_{\text{min}})}{I_{\text{max}}} \quad (20)
\]
You will probably find that Polaroid is a rather effective polarizer. Next, determine the transmission factor $T_p = I_{\text{max}} / I_0$ by removing the Polaroid and measuring the intensity of the incident beam. Note that a significant fraction of the input beam is absorbed.

Put one of the polarizers on the rotary holder, obtain a plot of intensity vs angle and fit the data to the expected form, derived from Eq. 7

$$A \cos(x-C)^2$$

Parameter $A$ should equal $T_p I_0$. The angular shift $C$ accounts for possible misalignment between the polarizer axis and input beam orientations, but is not otherwise significant. Is your data consistent with the theory?

2. Vector Decomposition of E field

To test the prediction for two polarizers, Eq. 9, align a holder and Polaroid between the rotary holder and the detector. Orient the second Polaroid at right angles to the input polarization by setting it for minimum transmission with the rotary holder empty. You can then replace the rotating polarizer and obtain a plot of intensity vs angle. Fit the data to

$$A(1-\cos(4x-C))$$

Are the angular dependence and fitted value of $A$ consistent with Eq. 9? Note that $I_0$ may drift, so it should be checked regularly.

3. Wave Plates and Elliptically Polarized Light

The retarder is a piece of plastic (polymerized polyvinyl alcohol) which has been stretched to give it a preferred axis and create the desired birefringence. The material is formed into a thin sheet, with the optical axis in the plane of the sheet. The thickness is chosen so that the optical path length difference $d(n_o - n_e)$ for polarizations parallel or perpendicular to the axis is $140 \pm 20$ nm, approximately independent of wavelength. This retardation corresponds to $\epsilon = \pi/2$ at 560 nm $\pm 15\%$. As with the Polaroid, there may be some absorption of the incident light, and it may be slightly different for polarization parallel or perpendicular to the optic axis.

Characterize the absorption of the retarder by measuring $T_r$ for light polarized along the $0^\circ$ and $90^\circ$ lines on the retarder mount. These directions should be approximately parallel and perpendicular to the optical axis. The transmission may differ somewhat, but you can use the average as your estimate of $T_r$. 

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To check Eq. 19 you need to pass the input beam through the retarder and then through a polarizer on the rotary mount. The retarder axis must be set accurately to 45°, which can be done by using the fact that the input polarization is unchanged if the optical axis is parallel, 0°, or perpendicular, 90°, to the input polarization. With the retarder out of the beam, rotate the polarizer to minimize transmission, thereby setting its axis perpendicular to the input. Now put the retarder plate in the input beam, and find the angles for minimum transmission near 0° and 90° on the retarder mount. The true 45° angle will be halfway between these measured angles.

With the retarder correctly aligned, obtain a plot of intensity vs polarizer angle and fit it to

\[ A^* \left( 1 + B \cos(2\times-C) \right) \]  
(23)

If this form adequately describes the data you can compare the fitted parameters with the expected values from Eq. 19 and Eq. 10

\[ A = \frac{I_0 T_r T_p}{2} \quad B = \cos \epsilon \]  
(24)

Do your results agree?

4. Tricks With a Circular Polarizer

The last component we will study is a left-hand circular polarizer. These are made by gluing a sheet of linear polarizer to a \( \lambda/4 \) wave plate with an angle of +45°. The LHC polarizers have been labeled so that unpolarized light incident on the polarizer side, labeled IN, will give LHC polarized light exiting the side labeled OUT. Since they actually contain the same nominal 140 nm retarder you have been using, the resultant may not be exactly circular, but it is close enough for our purposes.

The known LHC polarizer can be used to determine if the circularly polarized light coming out of a retarder is left- or right- hand circular. Set up the optics as in Fig. 2 with the retarder at ±45° and the LHC polarizer side labeled IN facing the photocell, as shown, so that the beam travels opposite to the usual direction. Record the intensity. Now rotate the retarder by 90° and again note the intensity. Report your observations and draw conclusions about the handedness of the two cases. Reverse the LHC polarizer and repeat. How do you explain what you observe? Figure 3 shows circular polarized light created from an unpolarized source, incident "backwards" on a circular polarizer, which should give you some hints.
As a final trick, place a coin or other metallic object on a piece of white paper, put the LHC polarizer on it, and illuminate the scene with white light. Record your observations. Turn the LHC polarizer over, and record what you see. How do you explain this? (We can get away with using white light because 560 nm is pretty much in the center of the visible spectrum.) The LHC polarizers are actually manufactured and marketed to reduce reflections in this manner.

**REPORT**

Your report should include a discussion of your observations, along with answers to the questions in the text, and the required plots.

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![Fig. 2 Layout for finding handedness of near-circular polarization.](image1)

![Fig. 3 Vector diagram showing the effect of a LHC polarizer on circularly polarized light.](image2)