Particle Counting

To be uncertain is uncomfortable but to be certain is ridiculous.
J. W. v. Goethe (1749-1832)

OBJECTIVES
To learn about particle counting technology and verify a statistical property of radioactive decay.

THEORY
This laboratory exercise is designed to give you some experience in particle-counting methods and in the statistical treatment of experimental data. Using typical instrumentation, you can determine the gamma energies emitted in the decay of $^{22}$Na and the number of gamma rays emitted in a chosen time interval. The time-interval data will be compared to a Poisson distribution, to test the idea that decays occur randomly in time.

1. Radioactive source

Figure 1 shows the decay scheme for $^{22}$Na. The isotope decays to an excited state of $^{22}$Ne by capturing an atomic electron into the nucleus (10% of decays) or by emitting a positron (90%). The $^{22}$Ne promptly (.00524 ns) decays to its ground state by emitting a 1.275 MeV gamma in a random direction. The positron usually stops in the plastic source container and annihilates with an electron, producing two .511 MeV gammas. The gammas can be emitted in any direction but the pair must have zero total momentum so they move in opposite directions. Several predictions of this decay scheme can be tested in the lab.

Fig. 1. Decay of $^{22}$Na to $^{22}$Ne, showing fractions via electron capture and beta decay.
2. Counting statistics

If decays occur at a constant average rate but randomly in time, the number of counts in a fixed time interval should follow a Poisson distribution. If the average number of events in a time interval $\Delta t$ is $\mu$, it is convenient to define a counting rate $r$ by

$$r = \mu / \Delta t \quad (1)$$

The probability of observing exactly $v$ events in $\Delta t$ is then given by the Poisson distribution function with mean $\mu = r\Delta t$

$$P_\mu(v) = \frac{\mu^v}{v!} e^{-\mu} = \left(\frac{r\Delta t}{v!}\right)^v e^{-r\Delta t} \quad (2)$$

From the Poisson distribution function, one can show that the standard deviation $\sigma$ is related to the mean by

$$\sigma^2 = \mu \quad (3)$$

In order to compare data with Eq. 2 and 3 we must estimate $\mu$ and $\sigma$ from the measurements. The estimate of the mean is found from the usual formula

$$\bar{v} = \frac{1}{N} \sum_i v_i f_i \quad (4)$$

where $v_i$ is the value of the random variable (the number of counts/time interval in this case), $f_i$ is the number of measurements yielding a result of $v_i$, and $N$ is the total number of measurements. Similarly, $\sigma$ is estimated by

$$s^2 = \frac{1}{N} \sum_i f_i (v_i - \bar{v})^2 \quad (5)$$

where the symbols have the same meanings.

It will also be useful to estimate the standard deviations of our estimates. For $\bar{v}$ this is just the standard deviation of the mean,

$$\sigma_{\bar{v}} = \sigma / \sqrt{N} = s / \sqrt{N} \quad (6)$$
The standard deviation of $s^2$, $\sigma_{s^2}$, is found from the general formula

$$\left(\sigma_{s^2}\right)^2 = \sum_i \left[ \frac{\partial (s^2)}{\partial \nu_i} \right]^2 \sigma_{\nu_i}^2$$  \hspace{1cm} (7)

which reduces to

$$\left(\sigma_{s^2}\right)^2 = \frac{4}{N} \sigma^2 s^2 = \frac{4}{N} s^4$$  \hspace{1cm} (8)

when all the $\sigma_{\nu_i}$ are equal, as is the case here.

3. Particle detector

The detector used here is a scintillator, a crystal of sodium iodide doped with a small amount of thallium. In a scintillating material the incident radiation excites some atoms to emit light, usually visible or near UV. The average number of photons emitted is, to a good approximation, directly proportional to the energy absorbed in the material.

Conversion of the weak burst of scintillation photons to a strong electrical signal is done by a photomultiplier tube (PMT). As sketched in Fig. 2, the scintillator is optically bonded to a light guide which conveys the light to the PMT. The entire assembly is, of course, shielded to eliminate outside light. The light guide is only used when it is necessary to locate the tube in a more favorable area, perhaps away from a large magnetic field or excessive temperature.

A typical PMT structure is also shown in Fig. 2. An applied voltage of 500 - 1000 V is divided so that the potential between adjacent dynodes is 50-100V, increasing from the negative cathode end to the anode output. Photons incident on the front plate cause emission of electrons (photoelectric effect) from the cathode. These electrons are accelerated to the closest dynode, where their impact releases additional electrons. This process is repeated at each of the 12-14 dynode stages in the tube, resulting in a current pulse of $10^6$-$10^7$ electrons per incident photon at

![Fig. 2 Schematic of a scintillator crystal coupled to a photomultiplier tube](image)
the anode. Very little noise is added in the process, so it is often possible to detect individual photons incident at the photocathode surface. Obviously, such a sensitive device should never see daylight when the high voltage is on.

4. Pulse measurement instrumentation

The current pulse from the PMT is usually converted to a voltage pulse for further processing. The pulse contains two pieces of information: Its presence indicates that a particle was detected; Its amplitude is proportional to the energy absorbed.

The most basic instrument, called a counter or scaler, simply counts the number of pulses from the detector. A slightly more elaborate device, called a multi-channel scaler (MCS), can count for a preset time interval, store the result, and then repeat until it runs out of memory. You will use such a device to generate a histogram of counts to compare with Eq. 2.

A multi-channel analyzer (MCA) measures the amplitude of each incoming pulse and forms a histogram of number of events vs pulse amplitude. When used with an energy-sensitive detector, such as a scintillator, this provides an energy spectrum for the detected particles.

EXPERIMENTAL PROCEDURE

Figure 3 shows the detector and initial signal processing electronics used for the measurements. The 3" diameter NaI-Tl scintillation crystal is coupled to a photomultiplier tube with appropriate voltage-divider base. A separate power supply provides high voltage for the PMT.

The pulse of current from the phototube is fed to a preamplifier which converts it to a voltage pulse and then to an amplifier/single-channel-analyzer. The amplifier provides a bipolar analog pulse for each event detected, while the SCA supplies a logic pulse for each pulse amplitude within the selected window. The analog pulse goes to the input of a computer

Fig. 3 Basic detector circuit. Use +900 V for both PMTs
controlled MCA/MCS unit. The logic pulse is not used in this exercise.

1. Radiation safety

The guiding principle of radiation protection is to keep the dose as low as reasonably attainable (ALARA). This can be accomplished with a combination of shielding, distance, and avoidance of contamination. Standard procedures include:

- Store sources in a shielded enclosure whenever possible.
- Minimize exposure time and maximize distance from unshielded sources.
- Do not handle food or drink while using sources.
- Wash hands after using sources.

Federal regulations limit the annual whole-body dose of radiation allowed for the general public to 100 mrem, and to 500 mrem for occupational exposure. This should be compared to the unavoidable annual dose of approximately 300 mrem from natural sources including cosmic rays, potassium 40 decay in the body, radon from rocks, etc. Doses from common medical procedures, such as a CT scan, can easily exceed 1 rem.

The initial strength of our gamma sources is 10 µCi or less, giving a measured dose from an unshielded source of less than 0.1 mrem per hour at 30 cm. It would, therefore, require 40 days of continuous exposure at this distance to reach the allowable dose for the general public. You will not receive a significant whole-body dose in the course of running an experiment in this lab unless you are in contact with the source capsule for an extended period.

2. Initial set up

Verify the connections as shown in Fig. 3, and turn on power to the HV supply, the NIM bin (upper one in the rack) containing the amplifier module, and the Spectrum Techniques MCA/MCS unit. The computer should already be running.

When the HV standby light comes on, set the dials to total 900 V, positive output, and switch the HV on. Turn off the HV if you ever need to make changes to the PMT connection.

Connect the analog output of the amplifier to the scope and observe the positive-going pulses when the $^{22}$Na source is nearby. (All cables into the oscilloscope should be terminated with their characteristic 50 Ω impedance to get meaningful results. Attach a terminator and the desired cable to a BNC "T" connector before attaching to the scope input.) Try varying the gain and observe that the amplifier will not produce pulse heights above a maximum limiting voltage. Amplifier-limited pulses are not of much use since their pulse height is not related to the gamma
ray energy. Set the gain on the amplifier such that the brighter band of pulses (corresponding to 0.511 MeV γ-rays) is at about 1 V.

3. Energy spectrum

The first exercise is to verify that the $^{22}$Na source emits gamma rays of two distinct energies. The amplitude of the analog pulses from the amplifiers is proportional to the energy absorbed in the scintillator, so a histogram of these pulse amplitudes is equivalent to an energy spectrum from the source.

The Spectrum Techniques UCS-30 can be used to measure the desired histogram. Connect the analog output from the 3" detector amplifier to the input on the rear panel of the UCS-30 and start the UCS-30 software from the desktop icon. Choose Mode > Pulse Height Analysis (Direct In), Settings > Amp/HV/ADC to set conversion gain 2048. The two triangles under the histogram display define the range of pulse amplitudes plotted. Move them to the ends of the bar and then click Go to start data collection. After a minute or so you should see a spectrum with two distinct peaks, corresponding to the two gamma energies expected.

Adjust the Coarse Gain and Fine Gain controls on the amplifier to put the .511 Mev peak at about channel 500. Run long enough to obtain a clear spectrum of the gamma emissions, and print the result for your report. Identify the two peaks on your plot, and use the fact that the channel number is proportional to absorbed energy to check the ratio of gamma energies. If you look carefully, you may be also able to see a third peak above the 1.275 MeV peak. This one is due to the simultaneous detection of the two main gammas, which is an indication that they are emitted nearly simultaneously. You can check this claim by estimating its energy as well.

You will find that the spectrum has many broad features in addition to the principal peaks you identified. Most of this structure is due to gammas which scatter out of the crystal, leaving only a fraction of their energy to be converted to photons. For the gammas that are totally absorbed, the number of photons produced is subject to statistical fluctuations and the photon collection efficiency depends on where the gamma hits the scintillator. These effects result in broadening of the main peaks.

4. Distribution of counts

The next goal is to verify that the frequency distribution of counts from a radioactive source follows a Poisson distribution. For this part you will use the multi-channel scaler function of the UCS-30. The UCS-30 has a maximum of 2048 channels, allowing you to measure the count rate 2048 times without manual intervention. You can then analyze the data to show that the numbers of counts in the time intervals follow a Poisson distribution.
A. Measurements

To ensure that counts are only accepted from a single physical process, use the SCA feature of the UCS-30 to pick out the .511 MeV annihilation radiation. This is done by placing the triangular sliders below the channel axis at either side of the peak. A short run should demonstrate that only those pulses are present in the spectrum.

Switch to MCS operation by selecting Mode > MCS (Internal) and Settings > MCS > Dwell Time to pick a dwell time (time interval to count in each channel). Click Go and you should see a display of counts in each period as they are collected. Adjust the source to detector distance and dwell time as needed to obtain mean values of about 2, 10 and 100 for three separate runs of 2048 channels. Save your data as a .tsv file for analysis.

B. Analysis

The analysis consists of histogramming the data sets and overlaying a Poisson distribution function over each histogram. A $\chi^2$ test is used to decide if the Poisson is a good description of the data. A more detailed explanation of the required steps follows.

1. Calculate the mean and standard deviation of your data set, and their errors, from Eq. 4, 5, and 6. These are your best estimates of the actual (unknown) $\mu$ and $\sigma$. Do they satisfy the Poisson relation, $\mu = \sigma^2$, within errors?

2. Prepare a histogram of the number of occurrences versus the number of counts, combining channels as needed to get a usable plot. Try to pick a reasonable compromise between small bins, which represent the function accurately, and large bins, which have lower relative error.

3. Using the estimates of $\mu$ and $\sigma$ calculated above, plot the normalized distribution functions on the same graph.

Steps 1, 2, 3 can be done with a MatLab program, poiss_plot.m, available on the lab computer. Enter the name of the saved .tsv data file and other parameters as requested, and the program will return summary statistics, the binned histogram entries and the corresponding values for the normalized Poisson distribution. You can print the histogram plot directly, and copy the histogram entries to another program to do the $\chi^2$ analysis.

4. Use the histogram entries to calculate $\chi^2$ for each data set, and compare the values with a table of the $\chi^2$ distribution. In doing the comparison, note that the $\chi^2$ distribution tables tacitly assume that many observations are expected in each bin. This condition will be adequately
satisfied when all the expected values used in calculating $\chi^2$ are greater than about 5. You can achieve this by eliminating small bins, and their degrees of freedom, from the $\chi^2$ calculation.

Can you conclude that your data statistically support the hypothesis that radioactive decay is a Poisson process?

REPORT

Your report should include the measured $^{22}$Na energy spectrum and your comments on the peak locations. For the distribution of counts, show that $\mu = \sigma^2$ is, or is not, correct within uncertainties for your three data sets. Also, provide the three plots showing the Poisson distribution superimposed on your data, and your statistical tests of the goodness-of-fit.