Optical Spectroscopy

Twinkle, twinkle little star,
I don't wonder what you are,
For by spectroscopic ken,
I know that you are hydrogen.

Anonymous

OBJECTIVES

To learn how to operate an optical spectrometer and to measure the Rydberg constant for atomic hydrogen.

THEORY

Starting with Newton, it slowly became apparent that much could be learned by dispersing light into its components with a prism and measuring the relative intensity of the various colors. Most spectra were continuous bands of color, but by the late 19th century various scientists had shown that hot gases emitted only a few colors, and those colors were characteristic of the chemical elements in the gas. That observation led to considerable advances in chemical analysis and fundamental physics.

1. Hydrogen spectrum

Of all the elements studied this way, the emission from hydrogen was found to be the simplest, consisting of a few colors with an apparently regular pattern. Balmer deduced an empirical formula for the four wavelengths known in 1885, and suggested that there would be other groups outside the visible range. His generalized expression is

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (1)$$

where λ is the observed wavelength, n_f and n_i are integers, and R_H, the same for all groups, is now called the Rydberg constant, in honor of a Swedish spectroscopist. By 1924 the groups corresponding to n_f from 1 to 5 had all been observed, and agreed with Balmer's generalized formula. Bohr's atomic model identified the integers n with successive energy levels of hydrogen, thereby providing a physical explanation for the Balmer formula.

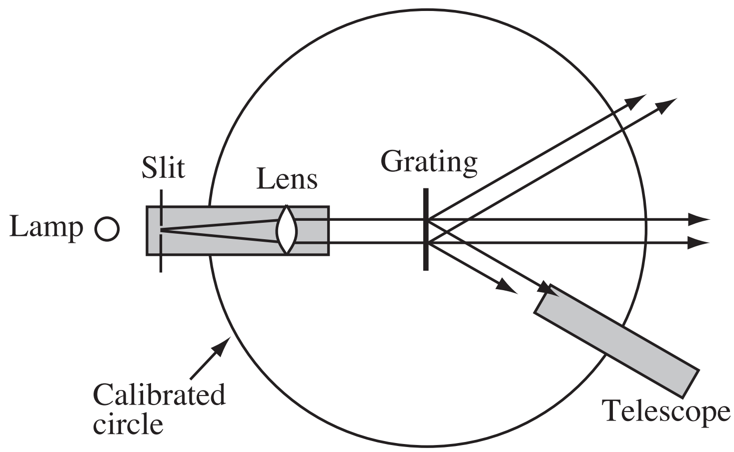

Fig. 1 Schematic diagram showing principal components of a grating spectrometer.

2. Spectroscope principles

Figure 1 is a schematic of a simple spectroscope that can be used to study optical emission spectra. The lamp is a tube filled with the desired gas. A few kilovolts applied to electrodes inside the tube causes the gas to conduct, exciting the atoms to emit light. Some of the emitted light goes through a slit and is collimated into a parallel beam by the lens. Upon striking the diffraction grating the various wavelengths are scattered at different angles. A telescope, focused at infinity, will then form an image of the slit from the rays emerging at any particular angle. A circular scale is provided to measure the diffraction angle.

If the grating is perpendicular to the incoming beam, the angle of diffraction is given by the usual relation

$$m\lambda = a \sin \theta_m \tag{2}$$

where a is the grating spacing. Determining θ_m for each color allows us to compute the wavelengths needed to find R_H from Eq. 1. Unfortunately, it is not easy to align the grating perpendicular to the input beam, and a is not known with sufficient precision. It is, however, possible to directly compare certain atomic emission wavelengths with primary length standards by making clever use of a Michelson interferometer. Those particular wavelengths can then be used to determine both a and the angle of incidence for a particular setup.

The diffraction geometry for off-normal incidence is shown in Fig. 2. When the angle of incidence θ_i is not zero, Eq. 2 becomes

$$m\lambda = a(\sin \theta_m - \sin \theta_i) \tag{3}$$

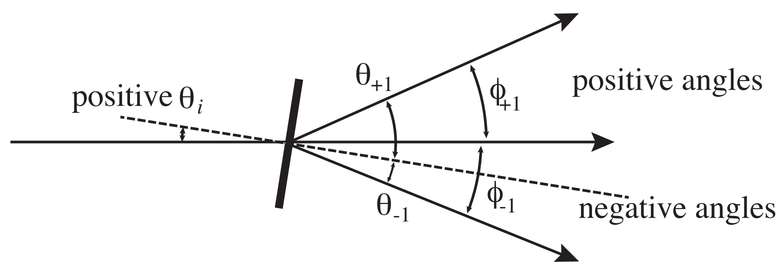

Fig. 2. Angles and sign convention for diffraction at non-normal incidence.

where θ_m is defined relative to the normal to the grating surface but cannot be measured directly. The angles ϕ_m are defined relative to the incident beam direction and are related to θ_m by

$$\phi_m = \theta_m - \theta_i \tag{4}$$

so Eq. 3 can be written in terms of the observable quantities as

$$m\lambda = a(\sin(\phi_m + \theta_i) - \sin\theta_i) \tag{5}$$

with both a and θ_i still unknown. If ϕ_m is measured for a $\pm m$ pair at a standard wavelength, Eq. 5 can be solved for the two unknowns.

With reasonable care it is possible to make θ_i small. Using a trig identity for the sum of angles, setting $\sin\theta_i = \theta_i$ and rearranging slightly yields

$$\frac{\lambda}{a} = \frac{1}{m}(\sin\phi_m + \theta_i \cos\phi_m - \theta_i) \tag{6}$$

Equating the approximate expressions for λ/a for $\pm m$ and solving for θ_i gives

$$\theta_i = \frac{\sin\phi_{+m} + \sin\phi_{-m}}{2 - \cos\phi_{+m} - \cos\phi_{-m}} \quad \text{radians} \tag{7}$$

Putting the estimated θ_i into Eq. 5 gives an explicit value for λ/a and therefore a by using the known standard λ.

EXPERIMENTAL PROCEDURE

Several steps are needed in order to estimate the Rydberg constant from the hydrogen spectrum. The spectroscope collimator must be adjusted so that light coming through the slit forms a parallel beam at the grating surface. The telescope must focus that light into a clear image of the slit at the plane of the internal cross-hairs. Once that is done, the grating is installed, aligned perpendicular to the incoming light beam, and calibrated with a known wavelength from a mercury discharge lamp. Finally, the hydrogen lamp is put in place and wavelengths measured for the visible Balmer lines. These steps must be done methodically, since adjustments interact.

1. Spectroscope set-up and calibration

Basic manipulations: The circular scale and the telescope can be independently rotated relative to the collimator. A knurled knob located near the collimator, under the scale, unlocks the rotation. A knob directly under the telescope unlocks telescope motion, with another knob at the side providing fine control when the telescope is locked.

The diffraction grating is a polymer replica of a ruled grating, supported on a piece of glass set in a plastic frame. The delicate grating replica is on the deeper side of the frame, affording some protection. Do not touch the grating surface with anything, ever.

Focus adjustments: Remove the grating, if present, and swing the telescope around to look past the collimator. Move the instrument so that you can view some bright, distant, object, preferably out a window. Looking through the eyepiece of the telescope you should see crossed reference lines. Focus on the lines by gently sliding the black eyepiece in or out relative to the silver tube. Then focus the image of a distant object by twisting the black ring at the other end of the telescope body. These operations focus the telescope at infinity.

Turn on the mercury lamp and place it in front of the slit. (To light, flip switch to Start and push the red button. After a few seconds, flip the switch to Operate.) Swing the telescope around to look directly into the collimator. Move the light source sideways to get the brightest image, and widen the slit if necessary. (Small knob on the side of the collimator tube.) Now focus the collimator by turning the black ring near the center to get the sharpest image of the slit in the telescope. This sets the collimator to produce parallel light out, and completes the focusing.

Grating alignment: Install the grating on the smaller table with the shallow side of the holder against the short posts, and gently clamp in place. Rotate the scale table so that the shallow side of the grating holder faces the collimator. Lock the table rotation, and move the telescope to center the slit image on the telescope cross-hairs. Using the angular scale, rotate the telescope exactly 90° so it points perpendicular to the incoming light, and clamp the telescope in place. Release the table and rotate it until the reflection of the slit is centered on the telescope cross-hairs. Be sure you are looking at the slit image, not one of the diffracted images. The grating surface is now at 45° to the incoming light. Using the graduated scale, rotate the table by ± 45° so that the collimated beam enters the shallow side of the grating mount perpendicular to the grating surface. This ensures that the diffracted beams do not pass through the supporting glass. Lock the table rotation.

Grating calibration: If you now look through the grating without the telescope you should see several lines of different colors, corresponding to the various diffracted m for each emission color. Gently move the telescope into the line of sight until the green line closest to the incident beam is in view. Narrow the slit to more precisely define the angular position.

To measure angles, use the fine adjustment on the telescope to center the vertical cross-hair on the slit image. You may need to add some external light to clearly see the cross-hair. Determine ϕ_m relative to the incident beam direction for $m = \pm 1$. You will need to use the vernier scale, as described at https://faraday.physics.utoronto.ca/PVB/Harrison/Vernier/Vernier.html, to get the best precision. Repeat the measurements 3-4 times, without changing the grating angle.

Use Eq. 7 and then Eq. 5 on each pair of ϕ_m measurements to find θ_i and a. The wavelength of mercury green emission is 546.0735 nm in air. You can specify the uncertainty as the standard deviation of the mean for your data.

2. Rydberg measurement

Put the hydrogen discharge lamp in front of the slit, and adjust the position to get the brightest image. Be careful not to hit the tube - it's fragile. Some dim background light will help you see the cross-hairs in the telescope. The hydrogen lamp has a short lifetime, so turn if off when not actually taking measurements.

Looking through the grating you should see first order lines in violet, turquoise-blue and red colors. You may also see a dark violet line, about 410 nm, but it is dim and close to the limit of vision. Some $m = 2$ lines may also be visible.

Measure ϕ_m relative to the incident beam direction for the $m = \pm 1$ order of as many colors as you can detect, again using the vernier scale to get the best precision. Repeat the set at least once to improve the average.

Use Eq. 5 with your calibration data for θ_i and a to estimate λ from each measurement. Eq. 1 will then give an estimate of R_H for each wavelength, assuming $n_f = 3, 4, 5, 6$ for the lines from red to dark violet and $n_i = 2$.

The final result is the average R_H, along with the standard deviation of the mean as an uncertainty estimate. How does your result compare with the accepted value R_H = $1.0973731 \times 10^7\ m^{-1}$?

Note: The standard deviation of the mean of your R_H estimates does not include the uncertainties in your estimates of a and θ_i. The effects are expected to be small, and a proper error propagation calculation would be messy for Eq. 5, so you can neglect these additional uncertainties in your report.

REPORT

Your report should include your measurements of the wavelengths of the hydrogen spectrum, and your estimate of R_H.