Experiment 7  AC Circuits

"Look for knowledge not in books but in things themselves."

W. Gilbert (1540-1603)

OBJECTIVES

To study some circuit elements and a simple AC circuit.

THEORY

All useful circuits use varying voltages, changing magnitude or even completely reversing polarity. In the present exercise we will study the behavior of some basic components and an elementary electrical filter, of the sort that might be used in audio equipment, when stimulated with a varying voltage. To keep things simple, we will consider only a sinusoidal voltage which oscillates at a steady frequency.

For purposes of analysis, circuits are usually considered to be made from resistors, capacitors and inductors. The components themselves are frequently characterized by the current that flows through them in response to a sinusoidal voltage at angular frequency $\omega$. As shown in your text, the current will then also be sinusoidal, with the ratio of peak voltage to peak current, the reactance, depending on frequency. There may also be a phase shift between the current and voltage, so that they peak at different times. The ideal relationships are given by

<table>
<thead>
<tr>
<th>Component</th>
<th>Reactance</th>
<th>Relative Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>resistor</td>
<td>$X_R = R$</td>
<td>in phase</td>
</tr>
<tr>
<td>capacitor</td>
<td>$X_C = 1/\omega C$</td>
<td>current leads by 90 degrees</td>
</tr>
<tr>
<td>inductor</td>
<td>$X_L = \omega L$</td>
<td>current lags by 90 degrees</td>
</tr>
</tbody>
</table>

Verifying the relations requires measuring the instantaneous amplitudes of current and voltage to determine the reactance and phase shift. A method for doing this will be described later.

As you might expect, real components are more complicated, but it is often possible to manufacture a reasonable approximation to these ideals. In fact, commercial resistors and capacitors are rather good, but inductors are not. The resistance of the coil turns out to be significant in most practical cases (superconductors are not practical), so we need to consider a more complex model for inductors. One approximation is to assume that the coil resistance is effectively in series with an ideal inductor, as shown in Fig. 7-1. Using the fact that the current flow through both components is the same, we can draw a phasor diagram showing the voltage...
drop across the resistor and across the ideal inductor. The total voltage drop across the model inductor is then the vector sum

\[ V_{lm} = I_p (\omega^2 L^2 + R_L^2)^{1/2} \quad (7-4) \]

from which it is easy to get the reactance and relative phase predicted for this model inductor

\[ X_{lm} = (\omega^2 L^2 + R_L^2)^{1/2} \quad \tan \phi_{lm} = \frac{\omega L}{R_L} \quad (7-5) \]

Later we will see if this accounts for the properties of a real inductor.

The circuit example is an RC filter, shown in Fig. 7-2. We are interested in finding the fraction of the input signal voltage \( V_s \) that appears across the resistor or the capacitor as a function of frequency. The phase could also be calculated, but it is not as important in the usual applications. Figure 7-2 provides the phasor diagram for the circuit, from which we see

\[ V_s = I_p \left( R^2 + \frac{1}{\omega^2 C^2} \right)^{1/2} \quad (7-6) \]
Solving for the current and substituting into the expressions for $V_C$ and $V_R$, we get

\[
V_C = V_s \frac{1}{(1 + \omega^2 \tau^2)^{1/2}}
\]

(7-7)

\[
V_R = V_s \frac{\omega \tau}{(1 + \omega^2 \tau^2)^{1/2}}
\]

(7-8)

where $\tau = RC$ is the time constant of the circuit.

The circuit in Fig. 7-2 is called a filter because the output signal is a modified version of the input, as graphed in Fig. 7-3. If we choose $V_C$ as our output, then low input frequencies are essentially unaffected, but high frequencies are strongly attenuated. This is called a low-pass filter, meaning the low frequencies are passed through but high frequencies are removed. Low-pass filters are often used in audio CD players to remove high-frequency digital artifacts from the desired audio signal. Conversely, if we choose $V_R$ as the output, we get a high-pass filter which is used, among other places, for the AC input setting of the oscilloscope. It lets us display small high-frequency signals that might be superimposed on a large DC component.

**EXPERIMENTAL PROCEDURE**

The experimental work will consist of measuring the current-voltage characteristic of a typical commercial capacitor and an inductor, and observing the filtering action of an RC low-

![](image)

Fig. 7-3 Ratio of output to input voltage for low and high-pass RC filters, as a function of scaled frequency.
A. Component characteristics

Figure 7-4 specifies the circuit used to measure the current and voltage for the component, Z, to be tested. The voltage across Z is measured with channel 1 of the oscilloscope. The current is found by measuring the voltage across R with channel 2, and then applying Ohm’s Law to deduce the current. We use the scope as a voltmeter because a DMM can’t measure phase and is inaccurate at frequencies above about a hundred Hertz. The voltage source is a function generator, set to a convenient amplitude and the desired frequency.

The oscilloscope causes some complications because, for safety reasons, it measures voltages with respect to the ground wire of the electric power system. The black terminal of the function generator is usually also connected to the power line ground, so we would not be able to measure both voltages in our circuit. The problem is circumvented by disconnecting the function generator from ground with a special plug. Both $V_R$ and $V_Z$ are then accessible, if the scope ground is connected between components as shown.

Wire the circuit as shown, using $R = 150 \, \Omega$, and another resistor for Z. (For later reference, use the DMM to get the exact value of the 150 $\Omega$ resistor before you connect it into the circuit.) Set the scope for AC input coupling, triggering on channel 1. Invert channel 2, so that positive voltages on R and Z will both result in upward deflections of the scope trace. Temporarily switch the inputs to ground and center the traces so both have the same zero point. Set the function generator for a reasonable amplitude and vary the frequency over the range 100 – 5000 Hz. You should see two sine waves, in phase, with a constant amplitude ratio. If not, you have probably made a wiring error.

When you are sure the circuit is correctly wired and the instruments are properly adjusted, replace the test resistor at Z with a 0.47 $\mu$F capacitor. Using the oscilloscope, measure the peak voltages across the resistor and capacitor and the phase shift between them at several frequencies from 20 Hz to 5 kHz. Use Ohm’s law and the previously measured resistance of the nominal 150 $\Omega$ resistor to get the peak current at each frequency. Use these data to estimate the

![Fig. 7-4 Circuit for measuring I-V characteristic of component Z.](image-url)
reactance and plot it vs inverse frequency. Is the graph a straight line with slope $1/2\pi C$, as expected for an ideal capacitor? (The factor $2\pi$ comes from converting frequency to angular frequency. The manufacturer claims the marked capacitance value is accurate to $\pm 20\%$.) Is the phase shift approximately constant at $\pi/2$ radians?

Note: To measure the relative phase of two sine waves on the scope screen, use the scope scales to determine $\Delta t$, as defined in Fig. 7-5. The phase difference is then

$$\phi = 2\pi f \Delta t$$

in radians. It is essential that the both sine waves have the same zero on the scope screen. You can achieve this by putting the traces on the middle line when the inputs are switched to ground.

Repeat the measurements of reactance and phase for the 10 mH inductor. The 150 $\Omega$ resistor is again suitable for current sensing. Plot the measured reactance vs frequency, and use the Graphical Analysis program to fit the model expression, Eq. 7-5 to your data. A convenient expression for the automatic curve fit is

$$(R^2 + (6.28 \times L)^2)^{0.5}$$

Are the fitted parameters reasonable? The inductance value should be within $\pm 20\%$, and you can check the DC resistance of the inductor with the DMM.

To analyze the phase shift for the inductor, plot $\tan \phi$ vs frequency, which should yield a straight line with slope $2\pi L/R_L$. In fact, the tangent function becomes extremely sensitive as the
argument approaches \( \pi/2 \), so it is difficult to find a good line from the data. An alternative approach is to use the parameters found before to draw a line of slope \( 2\pi L/R_C \), through the origin, on the plot and see if the data fall reasonably close to it. You can also verify directly that the phase shift approaches \( \pi/2 \) as the frequency increases.

B. RC Filter Circuit

The last exercise is to study the frequency response of an RC filter circuit, shown in Fig. 7-6. Component values of \( R = 1 \) k\( \Omega \) and \( C = 0.47 \) \( \mu \)F will give a convenient time constant. You should measure and plot the amplitude ratio \( V_C/V_s \) over the range 20 Hz – 5 kHz. Compare your results with the theoretical expectation, Eq. 7-7, using a fitting function like

\[
(1 + (6.28*x^2)^2)^{-0.5}
\]

Does our model accurately describe this low-pass filter?

REPORT

This exercise involves a lot of data. Be sure it is all clearly presented in labeled tables or graphs, as appropriate, and that your stated conclusions are clearly supported by your measurements.