COMP 412
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Top-down Parsing
Recursive Descent & LL(1)

Comp 412

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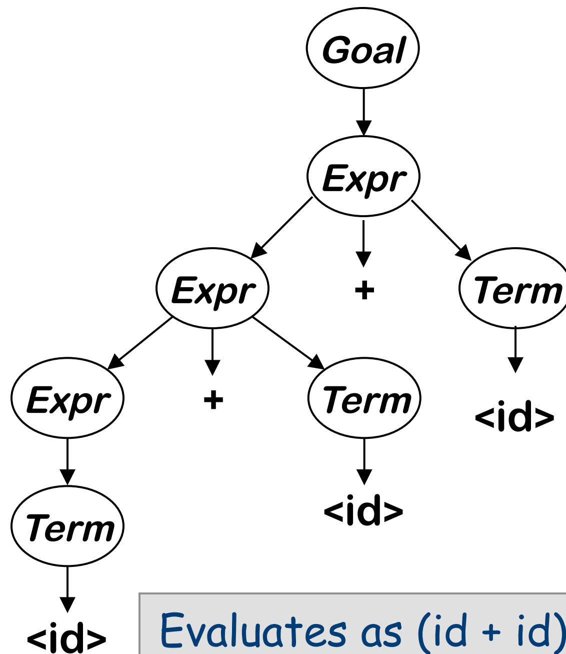
Question that baffled me last class:

Left Recursion Creates Left Associative Trees

A Trivial Expression Grammar

0	*Goal*	→	*Expr*
1	*Expr*	→	*Expr* + *Term*
2		|	*Term*
3	*Term*	→	id



Evaluates as (id + id) + id

Derivation of id + id +id

Rule	Sentential Form
—	*Goal*
0	*Expr*
1	*Expr* + *Term*
3	*Expr* +<*id*>
1	*Expr* + *Term* + <id>
3	*Expr* + <id> + <id>
2	*Term* + <id> + <id>
3	<id> + <id> + <id>

Roadmap (Where are we?)

We set out to study parsing

- Specifying syntax
  - Context-free grammars ✓
- Top-down parsers
  - Algorithm & its problem with left recursion ✓
  - Ambiguity ✓
  - Left-recursion removal ✓
- Predictive top-down parsing
  - The LL(1) condition ✓
  - Simple recursive descent parsers today
  - First and Follow sets today
  - Table-driven LL(1) parsers today

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$, the parser should be able to choose between α & β

FIRST sets

For some *rhs* $\alpha \in G$, define FIRST(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, $\underline{x} \in$ FIRST(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

The LL(1) Property

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ both appear in the grammar, we would like

$$\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \varnothing$$

This would allow the parser to make a correct choice with a lookahead of exactly one symbol !

This is almost correct
See the next slide

Predictive Parsing

What about ε-productions?

$\Rightarrow$ They complicate the definition of LL(1)

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ and $\varepsilon \in$ FIRST(α), then we need to ensure that FIRST(β) is disjoint from FOLLOW(A), too, where

FOLLOW(A) = the set of terminal symbols that can immediately follow A in a sentential form

Define FIRST$^+$($A \rightarrow \alpha$) as

- FIRST(α) $\cup$ FOLLOW(A), if $\varepsilon \in$ FIRST(α)
- FIRST(α), otherwise

Then, a grammar is *LL(1)* iff $A \rightarrow \alpha$ and $A \rightarrow \beta$ implies

FIRST$^+$($A \rightarrow \alpha$) $\cap$ FIRST$^+$($A \rightarrow \beta$) = $\varnothing$

Predictive Parsing

Given a grammar that has the *LL(1)* property

- Can write a simple routine to recognize each *lhs*
- Code is both simple & fast

Consider $A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3$, with

$$\text{FIRST}^+(A \rightarrow \beta_i) \cap \text{FIRST}^+(A \rightarrow \beta_j) = \emptyset \text{ if } i \neq j$$

```
/* find an A */
if (current_word ∈ FIRST+(A→β1))
   find a β1 and return true
else if (current_word ∈ FIRST+(A→β2))
   find a β2 and return true
else if (current_word ∈ FIRST+(A→β3))
   find a β3 and return true
else
   report an error and return false
```

Grammars with the *LL(1)* property are called *predictive grammars* because the parser can "predict" the correct expansion at each point in the parse.

Parsers that capitalize on the *LL(1)* property are called *predictive parsers*.

One kind of predictive parser is the *recursive descent* parser.

Of course, there is more detail to "find a β_i" (p. 103 in EaC, 1e)

Recursive Descent Parsing

Recall the expression grammar, after transformation

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	number
10		|	id
11		|	(*Expr*)

This produces a parser with six *mutually recursive* routines:

- *Goal*
- *Expr*
- *EPrime*
- *Term*
- *TPrime*
- *Factor*

Each recognizes one *NT* or *T*

The term *descent* refers to the direction in which the parse tree is built.

Recursive Descent Parsing (Procedural)

A couple of routines from the expression parser

```
Goal( )
    token ← next_token( );
    if (Expr( ) = true & token = EOF)
       then next compilation step;
       else
           report syntax error;
           return false;

Expr( )
  if (Term( ) = false)
     then return false;
     else return Eprime( );
```

looking for Number, Identifier, or "(", found token instead, or failed to find Expr or ")" after "("

```
Factor( )
  if (token = Number) then
     token ← next_token( );
     return true;
  else if (token = Identifier) then
     token ← next_token( );
     return true;
  else if (token = Lparen)
     token ← next_token( );
     if (Expr( ) = true & token = Rparen) then
        token ← next_token( );
        return true;
  // fall out of if statement
  report syntax error;
     return false;
```

EPrime, *Term*, & *TPrime* follow the same basic lines (Figure 3.7, EaC 1e)

Recursive Descent (Summary)

1. Massage grammar to have *LL(1)* condition
   a. Remove left recursion
   b. Build FIRST (and FOLLOW) sets
   c. Left factor it, as needed
2. Define a procedure for each non-terminal
   a. Implement a case for each right-hand side
   b. Call procedures as needed for non-terminals
3. Add extra code, as needed
   a. Perform context-sensitive checking
   b. Build an IR to record the code

Can we automate this process?

FIRST and FOLLOW Sets

FIRST(α)

For some $\alpha \in (T \cup NT)^*$, define FIRST(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, <u>x</u> $\in$ FIRST(α) *iff* $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

FOLLOW(A)

For some $A \in NT$, define FOLLOW(A) as the set of symbols that can occur immediately after A in a valid sentential form

FOLLOW(S) = {EOF}, where S is the start symbol

To build FOLLOW sets, we need FIRST sets …

Computing FIRST Sets

```
for each x ∈ T, FIRST(x) ← { x }
for each A ∈ NT, FIRST(A) ← Ø

while (FIRST sets are still changing) do
   for each p ∈ P, of the form A→β do
      if β is B1B2…Bk then begin;
         rhs ← FIRST(B1) - { ε }
         for i ← 1 to k-1 by 1 while ε ∈ FIRST(Bi) do
            rhs ← rhs ∪ ( FIRST(Bi+1) - { ε } )
            end  // for loop
      end        // if-then
      if i = k and ε ∈ FIRST(Bk )
         then rhs ← rhs ∪ { ε }
      FIRST(A) ← FIRST(A) ∪ rhs
      end  // for loop
   end     // while loop
```

Outer loop is monotone increasing for FIRST sets

→ | T ∪ NT ∪ ε | is bounded, so it terminates

Inner loop is bounded by the length of the productions in the grammar

For *SheepNoise:*

FIRST(*Goal*) = { baa }
FIRST(*SN*) = { baa }
FIRST(baa) = { baa }

Computing FIRST Sets

for each $x \in T$*, FIRST(x)* $\leftarrow \{x\}$
for each $A \in NT$*, FIRST(A)* $\leftarrow \emptyset$

while (FIRST sets are still changing) do
    for each $p \in P$*, of the form* $A \rightarrow \beta$ *do*
        if β *is* $B_1 B_2 \ldots B_k$ *then begin;*
            rhs $\leftarrow$ *FIRST*$(B_1) - \{\varepsilon\}$
            for $i \leftarrow 1$ *to* $k\text{-}1$ *by 1 while* $\varepsilon \in$ *FIRST*(B_i) *do*
                rhs $\leftarrow$ *rhs* $\cup$ *(FIRST*$(B_{i+1}) - \{\varepsilon\}$*)*
                end // for loop
        end // if-then
        if $i = k$ *and* $\varepsilon \in$ *FIRST*(B_k)
            then rhs $\leftarrow$ *rhs* $\cup \{\varepsilon\}$
        FIRST(A) $\leftarrow$ *FIRST(A)* $\cup$ *rhs*
        end // for loop
    end // while loop

Outer loop is monotone increasing for FIRST sets

$\rightarrow$ $| T \cup NT \cup \varepsilon |$ is ... ates

For *SN* $\rightarrow$ baa

Inner loop is bounded by the length of the productions in the grammar

For *SheepNoise:*
- FIRST(*Goal*) = { baa }
- FIRST(*SN*) = { baa }
- FIRST(baa) = { baa }

Computing FIRST Sets

for each $x \in T$, *FIRST(x)* $\leftarrow \{x\}$
for each $A \in NT$, *FIRST(A)* $\leftarrow \emptyset$

while (FIRST sets are still changing) do
  for each $p \in P$, *of the form* $A \rightarrow \beta$ *do*
    if β *is* $B_1B_2 \ldots B_k$ *then begin;*
      $rhs \leftarrow FIRST(B_1) - \{\varepsilon\}$
      for $i \leftarrow 1$ *to* $k-1$ *by 1 while* $\varepsilon \in FIRST(B_i)$ *do*
        $rhs \leftarrow rhs \cup (FIRST(B_{i+1}) - \{\varepsilon\})$
        end // for loop
    end // if-then
    if $i = k$ *and* $\varepsilon \in FIRST(B_k)$
      then $rhs \leftarrow rhs \cup \{\varepsilon\}$
    $FIRST(A) \leftarrow FIRST(A) \cup rhs$
    end // for loop
  end // while loop

Outer loop is monotone increasing for FIRST sets

→ | T ∪ NT ∪ ε | is

For *Goal* → *SN* es

Inner loop is bounded by the length of the productions in the grammar

For *SheepNoise:*
FIRST(*Goal*) = { baa }
FIRST(*SN*) = { baa }
FIRST(baa) = { baa }

Computing FOLLOW Sets

```
for each A ∈ NT, FOLLOW(A) ← Ø

FOLLOW(S) ← {EOF}

while (FOLLOW sets are still changing)
    for each p ∈ P, of the form A→B1B2 … Bk
        TRAILER ← FOLLOW(A)
        for i ← k down to 1
            if Bi ∈ NT then                              // domain check
                FOLLOW(Bi) ← FOLLOW(Bi) ∪ TRAILER
                if ε ∈ FIRST(Bi )                        // add right context
                  then TRAILER ← TRAILER ∪ ( FIRST(Bi) – { ε } )
                  else TRAILER ← FIRST(Bi)  // no ε => no right context
            else TRAILER ← {Bi}                          // Bi ∈ T => only 1 symbol
```

Classic Expression Grammar

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	number
10		|	id
11		|	(*Expr*)

FIRST+(A→β) is identical to FIRST(β) except for productiond 4 and 8

FIRST+(Expr'→ ε) is {ε,), eof}

FIRST+(Term'→ ε) is {ε,+,-,), eof}

Symbol	FIRST	FOLLOW
num	num	Ø
id	id	Ø
+	+	Ø
-	-	Ø
*	*	Ø
/	/	Ø
(	(	Ø
)	)	Ø
eof	eof	Ø
ε	ε	Ø
Goal	(,id,num	eof
Expr	(,id,num	), eof
Expr'	+, -, ε	), eof
Term	(,id,num	+, -,), eof
Term'	*, /, ε	+,-,), eof
Factor	(,id,num	+,-,*,/,),eof

Classic Expression Grammar

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	number
10		|	id
11		|	(*Expr*)

Prod'n	FIRST+
0	(,id,num
1	(,id,num
2	+
3	-
4	ε,), eof
5	(,id,num
6	*
7	/
8	ε,+,-,), eof
9	number
10	id
11	(

Building Top-down Parsers for LL(1) Grammars

Given an *LL(1)* grammar, and its FIRST & FOLLOW sets …

- Emit a routine for each non-terminal
  - Nest of if-then-else statements to check alternate rhs's
  - Each returns true on success and throws an error on false
  - Simple, working *(, perhaps ugly,)* code
- This automatically constructs a recursive-descent parser

Improving matters

- Nest of if-then-else statements may be slow
  - Good case statement implementation would be better
- What about a table to encode the options?
  - Interpret the table with a skeleton, as we did in scanning

I don't know of a system that does this …

Building Top-down Parsers

Strategy
- Encode knowledge in a table
- Use a standard "skeleton" parser to interpret the table

Example
- The non-terminal *Factor* has 3 expansions
  – (*Expr*) or Identifier or Number
- Table might look like:

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	number
10		|	id
11		|	(*Expr*)

Terminal Symbols

Non-terminal Symbols

	+	-	*	/	Id.	Num.	(	)	EOF
Factor	—	—	—	—	10	9	11	—	—

Cannot expand *Factor* into an operator ⇒ error

Expand *Factor* by rule 9 with input "number"

Building Top Down Parsers

Building the complete table

- Need a row for every *NT* & a column for every *T*
- Need an interpreter for the table (*skeleton parser*)

LL(1) Skeleton Parser

```
word ← NextWord()                  // Initial conditions, including
push EOF onto Stack                // a stack to track local goals
push the start symbol, S, onto Stack
TOS ← top of Stack
loop forever
  if TOS = EOF and word = EOF then
     break & report success        // exit on success
  else if TOS is a terminal then
     if TOS matches word then
        pop Stack                  // recognized TOS
        word ← NextWord()
     else report error looking for TOS  // error exit
  else                             // TOS is a non-terminal
     if TABLE[TOS,word] is A→ B1B2…Bk then
        pop Stack                  // get rid of A
        push Bk, Bk-1, …, B1       // in that order
     else break & report error expanding TOS
  TOS ← top of Stack
```

Building Top Down Parsers

Building the complete table

- Need a row for every *NT* & a column for every *T*
- Need a table-driven interpreter for the table
- Need an algorithm to build the table

Filling in TABLE[X,y], $X \in NT$, $y \in T$

1. entry is the rule $X \rightarrow \beta$, if $y \in$ FIRST$^+(X \rightarrow \beta)$
2. entry is *error* if rule 1 does not define

If any entry has more than one rule, *G* is not *LL(1)*

We call this algorithm the *LL(1)* table construction algorithm

Grammar and Sets LL(1) Construction

0	*Goal*	→	*Expr*
1	*Expr*	→	*Term Expr'*
2	*Expr'*	→	+ *Term Expr'*
3		|	- *Term Expr'*
4		|	ε
5	*Term*	→	*Factor Term'*
6	*Term'*	→	* *Factor Term'*
7		|	/ *Factor Term'*
8		|	ε
9	*Factor*	→	number
10		|	id
11		|	(*Expr*)

Right-recursive variant of the classic expression grammar

Prod'n	FIRST+
0	(,id,num
1	(,id,num
2	+
3	-
4	ε,), eof
5	(,id,num
6	*
7	/
8	ε,+,-,), eof
9	number
10	id
11	(

Augmented FIRST sets for the grammar

LL(1) Expression Parsing Table

	+	-	*	/	Id	Num	(	)	EOF
Goal	—	—	—	—	0	0	0	—	—
Expr	—	—	—	—	1	1	1	—	—
Expr'	2	3	—	—	—	—	—	4	4
Term	—	—	—	—	5	5	5	—	—
Term'	8	8	6	7	—	—	—	8	8
Factor	—	—	—	—	10	9	11	—	—