Baseball, Classical Mechanics, and Computer Graphics

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Outline

Part I: *Baseball Arithmetic*

Part II: *Archimedes' Law: Baseball Arithmetic in Classical Mechanics and Classical Mechanics in Computer Graphics*

Part III: *Pseudoperspective: Baseball Arithmetic in Computer Graphics*

Part IV: *Baseball Folklore* (If Time Permits)

Part I

Baseball Arithmetic

Themes

Baseball

Whoever wants to know the heart and mind of America had better learn Baseball.

Jacques Barzun

Next to religion, Baseball has furnished a greater impact on American life than any other institution.

Herbert Hoover

Arithmetic

There still remain three studies suitable for a free man. Arithmetic is one of them.

Plato -- The Laws

It is proper … to persuade those who are to share in the highest things in the city to go for and tackle the art of calculation, and not as amateurs.

Plato -- The Republic

Batting Average

Formula

- $$Batting\ \ Average\ =\ \frac{Total\ \ Hits}{Total\ \ at\ \ Bats}$$

Representation

- Overloaded Notation
- Lazy Evaluation

.400 Hitters

Immortals (1901– 1930)

Harry Heilmann (.403)
Shoeless Joe Jackson (.408)
Nap Lajoie (.426)
Bill Terry (.401)
Ty Cobb (.401, .409, .420)
Rogers Hornsby (.401, .403, .424)
George Sisler (.407, .420)

Ted Williams (1941)

$\frac{179}{448} = .39955 \approx .400$ -- *next to last day of season*

$\frac{6}{8} = .750$ -- *last day of season*

$\frac{179}{448} \oplus \frac{6}{8} = \frac{185}{456} \approx .406$

Fractions

Are there not two kinds of arithmetic, that of the people and that of philosophers? (Socrates -- Philebus)

Standard Arithmetic

1. Addition

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad bd \neq 0$$

2. Scalar Multiplication

$$n * \frac{a}{b} = \underbrace{\frac{a}{b} + \cdots + \frac{a}{b}}_{n \ terms} = \frac{na}{b} \qquad b \neq 0$$

3. Identity for Addition

$$\frac{0}{1}$$

Baseball Arithmetic

1. Addition

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a + c}{b + d}$$

2. Scalar Multiplication

$$n \otimes \frac{a}{b} = \underbrace{\frac{a}{b} \oplus \cdots \oplus \frac{a}{b}}_{n \ terms} = \frac{na}{nb}$$

3. Identity for Addition

$$\frac{0}{0}$$

Ordered Pairs

Standard Arithmetic

1. Addition
   $(a,b)+(c,d)=(ad+bc,bd) \qquad bd \neq 0$

2. Scalar Multiplication
   $n*(a,b)=(na,b) \qquad b \neq 0$

3. Identity for Addition
   $(0,1)$

Baseball Arithmetic

1. Addition
   $(a,b)\oplus(c,d)=(a+c,b+d)$

2. Scalar Multiplication
   $n\otimes(a,b)=(na,nb)$

3. Identity for Addition
   $(0,0)$

In Both Sets of Rules:

i. Addition is Associative and Commutative.
ii. Multiplication Distributes Through Addition.

Baseball Arithmetic

Equivalence Is Not Identity

$$\frac{1}{3} \equiv \frac{100}{300} \equiv .333\ldots$$

$$\frac{1}{3} \oplus \frac{0}{1} = \frac{1}{4} \equiv .250$$

$$\frac{100}{300} \oplus \frac{0}{1} = \frac{100}{301} \approx .332$$

Addition Is Weighted Averaging

$$\frac{2}{10} \oplus \frac{4}{10} = \frac{6}{20} \equiv \frac{3}{10}$$

$$\frac{1}{3} \oplus \frac{1}{3} = \frac{2}{6} \equiv \frac{1}{3}$$

$$\frac{1}{2} \oplus \frac{7}{10} = \frac{8}{12} \equiv \frac{2}{3}$$

The Laws of Averages

When I'm not hitting, I don't hit nobody.
But when I'm hitting, I hit anybody. (Willie Mays)

1. *Addition*

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$$

2. *Scalar Multiplication*

$$n \otimes \frac{a}{b} = \frac{na}{nb}$$

3. *Affect of Streaks and Slumps*

$$\frac{a}{b} \le \frac{c}{d} \quad \Rightarrow \quad \frac{a}{b} \le \frac{a}{b} \oplus \frac{c}{d} \le \frac{c}{d}$$

4. *Official Scorer's Correction Rule*

$$\frac{a}{b} \oplus \frac{h}{0} = \frac{a+h}{b}$$

Part II: Archimedes' Law

Baseball Arithmetic in Classical Mechanics
and
Classical Mechanics in Computer Graphics

Mass Points

Notation

$$P = point \ \ in \ \ affine \ \ space$$
$$m = mass$$
$$mP = mass \times point$$
$$(mP, m) = mass \ \ point$$
$$\frac{mP}{m} = mass \ \ point$$

Overloaded Notation

- Quotient = Position
- Denominator = Mass
- Numerator = Mass $\times$ Point

The Laws of the Lever

It's like deja vu all over again. (Yogi Berra)

1. *Archimedes' Law*

$$\frac{m_1 P_1}{m_1} \oplus \frac{m_2 P_2}{m_2} = \frac{m_1 P_1 + m_2 P_2}{m_1 + m_2}$$ (Center of Mass)

2. *Altering Mass Does Not Affect Position*

$$c \otimes \frac{mP}{m} = \frac{cmP}{cm}$$

3. *Fulcrum Lies Between the Masses*

$$\frac{m_1 P_1}{m_1} \prec \frac{m_1 P_1}{m_1} \oplus \frac{m_2 P_2}{m_2} \prec \frac{m_2 P_2}{m_2}$$

4. *Mass Has Inertia*

$$\frac{mP}{m} \oplus \frac{F}{0} = \frac{mP + F}{m}$$

<u>**The Laws of Averages**</u>

1. *Addition*

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$$

2. *Scalar Multiplication*

$$n \otimes \frac{a}{b} = \frac{na}{nb}$$

3. *Affect of Streaks and Slumps*

$$\frac{a}{b} \le \frac{c}{d} \quad \Rightarrow \quad \frac{a}{b} \le \frac{a}{b} \oplus \frac{c}{d} \le \frac{c}{d}$$

4. *Official Scorer's Correction Rule*

$$\frac{a}{b} \oplus \frac{h}{0} = \frac{a+h}{b}$$

The Laws of the Lever -- Revisited

1. *Archimedes' Law*

   $$(m_1P_1, m_1) \oplus (m_2P_2, m_2) = (m_1P_1 + m_2P_2, m_1 + m_2)$$

2. *Altering Mass Does Not Affect Position*

   $$c \otimes (mP, m) = (cmP, cm)$$

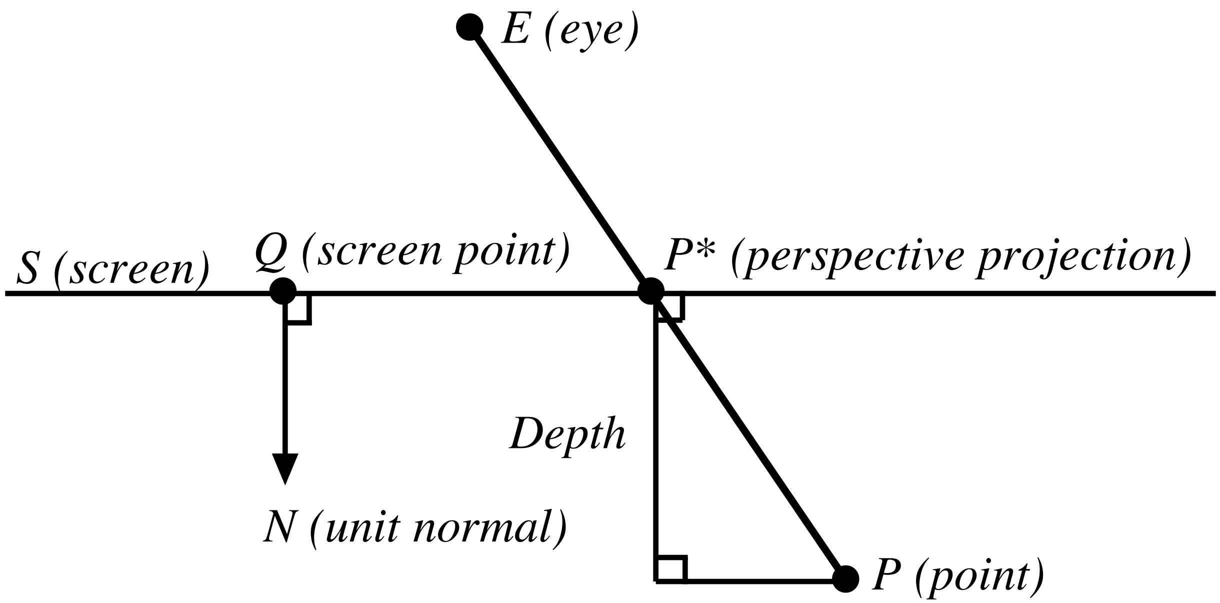
3. *Fulcrum Lies Between the Masses*

   $$(m_1P_1, m_1) \prec (m_1P_1 + m_2P_2, m_1 + m_2) \prec (m_2P_2, m_2)$$

4. *Mass Has Inertia*

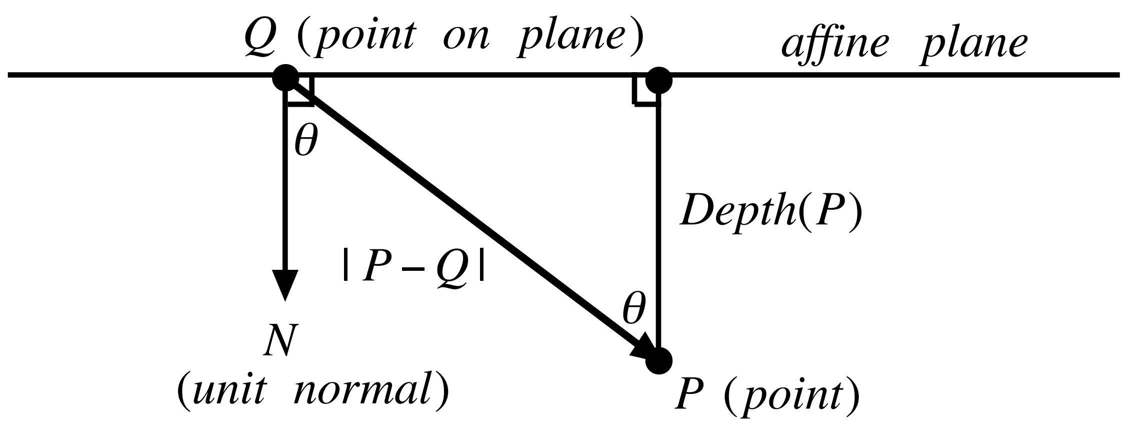
   $$(mP, m) \oplus (F, 0) = (mP + F, m)$$

<u>Perspective</u>



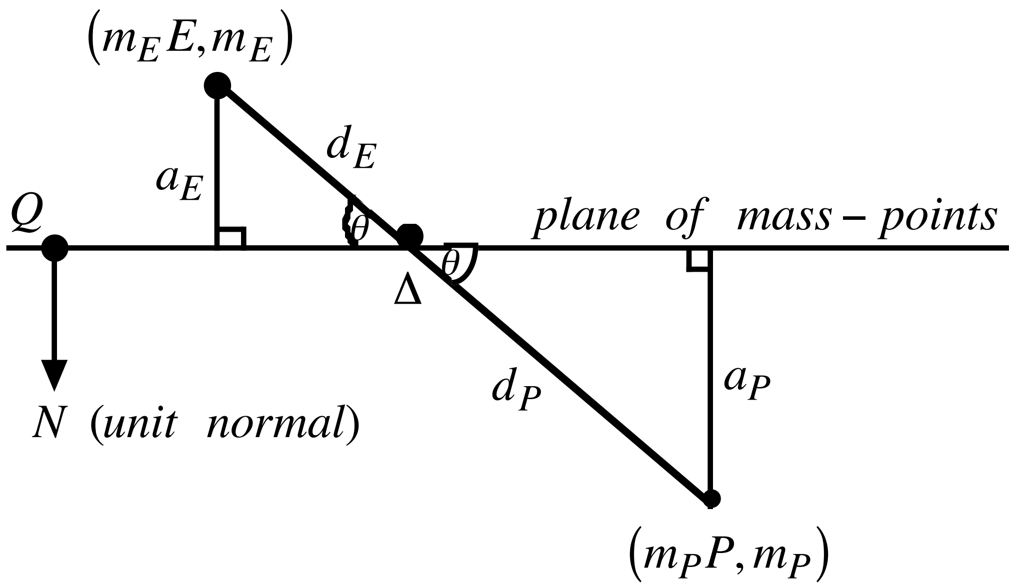
$$P^* = \frac{\{(Q - E) \bullet N\}P + \{(P - Q) \bullet N\}E}{(P - E) \bullet N}$$

Depth



$$Depth(P) = |P - Q| \cos\theta = (P - Q) \bullet N$$

<u>Perspective and the Law of the Lever</u>



$$\Delta = center\ \ of\ \ mass \ \Rightarrow\ m_p d_p = m_E d_E$$

$$a_P / d_P = \sin\theta = a_E / d_E \ \Rightarrow\ m_P a_P / m_P d_P = m_E a_E / m_E d_E$$

$$\therefore \quad m_P d_P = m_E d_E \Leftrightarrow m_P a_P = m_E a_E$$

To find the intersection Δ of the plane S with the line EP, let
$m_P = a_E = (Q - E) \bullet N \quad and \quad m_E = a_P = (P - Q) \bullet N$ and compute center of mass.

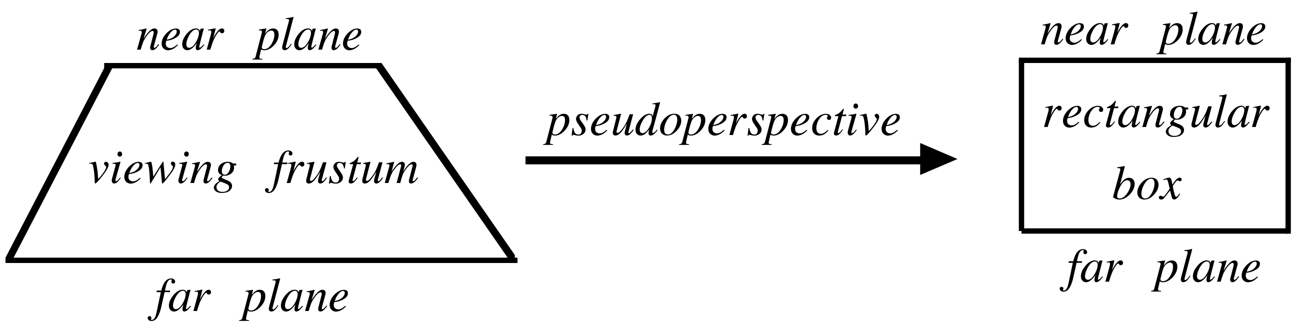
$$\therefore \quad \Delta = \frac{m_P P + m_E E}{m_P + m_E} = \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N}$$

Part III: Pseudoperspective

Baseball Arithmetic in Computer Graphics

Pseudoperspective

Mapping



Goals

i. Clipping Algorithms -- Map the viewing frustum into a rectangular box.

ii. Projections -- Replace perspective projection by orthogonal projection.

iii. Hidden Surface Algorithms -- Preserve relative depth.

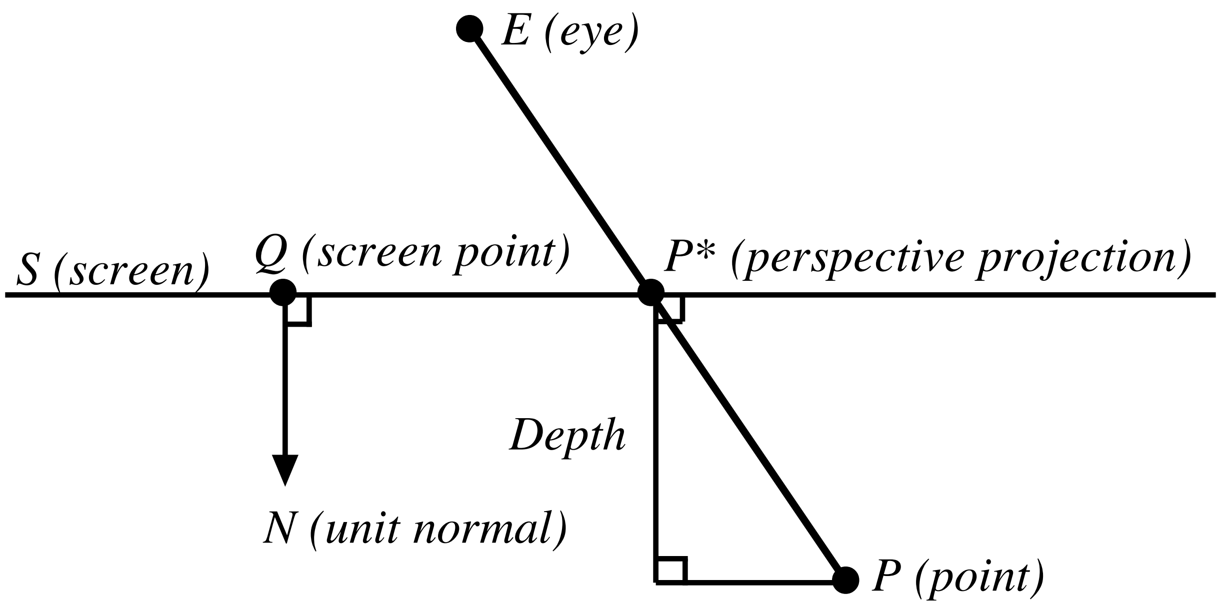
Pseudoperspective

Projective Geometry

- Every projective transformation is defined by the image of 5 generic points.
- Solve 15 homogeneous linear equations in 16 unknowns.

Grassmann Geometry

- *Pseudoperspective* = *Perspective Projection* $\oplus$ *Depth Vector*



Formulas

Perspective Projection

$$P^* = \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N}$$

Depth (Projection on Normal Vector)

$$Depth(P) = (P-Q)\bullet N$$

Perspective + Depth Vector

$$P^{**} = P^* + \{Depth(P)\}N$$

$$= \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N} + \{(P-Q)\bullet N\}N$$

$$= \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E + \{(P-Q)\bullet N\}\{(P-E)\bullet N\}N}{(P-E)\bullet N}$$

Problem

$\{(P-Q)\bullet N\}\{(P-E)\bullet N\}N$ -- not linear in P

Non-Linearity

$$\{(P-Q)\bullet N\}\{(P-E)\bullet N\}N$$

Projection into xy-plane
Eye on z-axis

$$Q=(0,0,0)$$

$$N=(0,0,-1)$$

$$E=(0,0,1)$$

$$P=(x,y,z)$$

$$(P-Q)\bullet N=(x,y,z)\bullet(0,0,-1)=-z$$

$$(P-E)\bullet N=(x,y,z-1)\bullet(0,0,-1)=-(z-1)$$

$$\{(P-Q)\bullet N\}\{(P-E)\bullet N\}N=z(z-1)(0,0,-1)$$

Formulas Revisited

Perspective Projection

$$P^* = \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N}$$

Depth (Projection on Normal Vector)

$$Depth(P) = (P-Q)\bullet N$$

Perspective ⊕ Depth Vector

$$P^{**} = P^* \oplus \{Depth(P)\}N$$

$$= \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N} \oplus \frac{\{(P-Q)\bullet N\}N}{0}$$

$$= \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E + \{(P-Q)\bullet N\}N}{(P-E)\bullet N}$$

No Problem

$\{(P-Q)\bullet N\}N$ -- is linear in P

Pseudodepth

Pseudoperspective

$$P^{**} = \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N} \oplus \frac{\{(P-Q)\bullet N\}N}{0}$$

$$P^{**} = \frac{\{(Q-E)\bullet N\}P + \{(P-Q)\bullet N\}E}{(P-E)\bullet N} + \underbrace{\frac{(P-Q)\bullet N}{(P-E)\bullet N}}_{Pseudodepth} N$$

Pseudodepth

$$P^{**} = P^{*} \oplus \{Depth(P)\}N$$

$$P^{**} = P^{*} + \{Pseudodepth(P)\}N$$

$$Depth(P) = (P-Q)\bullet N = \textit{Distance to Plane of Screen}$$

$$Pseudodepth(P) = \frac{(P-Q)\bullet N}{(P-E)\bullet N} = \frac{\textit{Distance to Plane of Screen}}{\textit{Distance to Plane of Eye}}$$

Laws of Pseudoperspective

1. *Clipping*

$$0 \le Pseudodepth(P) \le 1$$

2. *Averaging*

$$Depth\left(\frac{P_1 + P_2}{2}\right) = \frac{Depth(P_1) + Depth(P_2)}{2}$$

$$Pseudodepth\left(\frac{P_1 + P_2}{2}\right) = Pseudodepth(P_1) \oplus Pseudodepth(P_2)$$

3. *Relative Depth is Preserved*

$$Depth(P_2) \ge Depth(P_1) \Rightarrow Pseudodepth(P_2) \ge Pseudodepth(P_1)$$

$$\frac{(P_2 - Q) \bullet N}{(P_2 - E) \bullet N} = \frac{(P_1 - Q) \bullet N}{(P_1 - E) \bullet N} \oplus \frac{(P_2 - P_1) \bullet N}{(P_2 - P_1) \bullet N}$$

4. *Correction to Perspective Projection*

$$Pseudoperspective(P) = Perspective(P) \oplus \frac{\{Depth(P)\}N}{0}$$

The Laws of Averages

1. *Boundedness*

$$0 \le \frac{a}{b} \le 1$$

2. *Addition*

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$$

3. *Scalar Multiplication*

$$n \otimes \frac{a}{b} = \frac{na}{nb}$$

4. *Affect of Streaks and Slumps*

$$\frac{a}{b} \le \frac{c}{d} \quad \Rightarrow \quad \frac{a}{b} \le \frac{a}{b} \oplus \frac{c}{d} \le \frac{c}{d}$$

5. *Official Scorer's Correction Rule*

$$\frac{a}{b} \oplus \frac{h}{0} = \frac{a+h}{b}$$

Summary

Baseball	*Computer Graphics*	*Physics*
Batting Average	Pseudodepth	Point
At Bats	Distance to Plane of the Eye	Mass
Hits	Distance to the Screen	Mass × Point
Official Scorer	Depth Vector	Force
Consistency	Stability	Inertia

Table 1: The Elements

Summary (continued)

Baseball	*Computer Graphics*	*Physics*
Boundedness of Batting Average	Boundedness of Pseudodepth	Boundedness of Space (No Points at Infinity)
Addition and Scalar Multiplication	Pseudodepth is an Affine Map	Archimedes' Law of the Lever Mass does not Affect Location
Streaks and Slumps Affect Average	Pseudodepth Preserves Relative Depth	Fulcrum Lies Affect Between the Masses
Official Scorer's Correction Rule	Pseudodepth Correction to Perspective Projection	How Forces Act on Mass

Table 2: The Laws

Conclusions

1. Baseball Arithmetic has Many Applications.
   a. Classical Physics -- Laws of the Lever
   b. Computer Graphics -- Perspective and Pseudoperspective
   c. Geometric Modeling -- Physical Effects for Rational Bezier Surfaces

2. In Computer Graphics, Homogeneous Coordinates are Really Mass–Points.

3. Homogeneous Coordinates Represent a 4–Dimensional Vector Space.
   a. First Three Dimensions are Spatial
   b. Fourth Dimension is Mass

4. Reexamining the Basic Laws of Arithmetic is Often Worthwhile.

5. Do Not Automatically Accept the Basic Laws of Algebra or Geometry.
   Seek Novel, Useful Rules.