The Perfect Logicians Meet the Marquis de Sade

One day 100 people from the Island of Perfect Logicians decide to sail to the Island of Perfect Truth Tellers. During their voyage, they become shipwrecked on the Island of the Marquis de Sade. The Marquis and his henchman capture all 100 Logicians and decide to play the following sadistic game. They tell the Logicians that tomorrow all 100 Logicians will be blindfolded and lined up on a long staircase. The Sadist will then place a hat on each Logician. The hat will be either red or blue. The Sadists will now remove the blindfolds. Each Logician will be able to see only the hats in front of them, but will not be able to see their own hat or the hats of the people behind them.   Starting from the Logician at the top of the staircase, who can see everyone's hat but his own. the Sadists will ask each Logician in turn to guess the color of their hat. Each Logician will be allowed to say only one word: *red* or *blue*; if they say anything else, the Sadists will immediately throw all 100 Logicians off a cliff. If a Logician correctly guesses the color of their hat, then the Sadists will release that Logician; if a Logician guesses incorrectly, then the Sadists will throw that Logician off a cliff. The night before the game, the Logicians get together to discuss their strategy for the next day. For example, one strategy could be that the odd numbered Logicians on the staircase guess the color of the hat immediately in front of them, and the even numbered Logicians guess the color that the person behind them has guessed. This strategy is guaranteed to save at least 50 Logicians and on average will save 75 Logicians. You job is to:

a. develop a better strategy for the Logicians;

b. prove that your strategy works and is optimal.