Undecidable Problems

Undecidable Problems

- Halting Problem
- Detecting Division by Zero
- Determining if Two Arbitrary Programs Always Generate the Same Output on the Same Input
- Finding Optimal Programs

Halting Problem

Halting Program

- $H(P,I)$ -- prints YES, if P HALTS on input I
- $H(P,I)$ -- prints NO, if P LOOPS FOREVER on input I
- Note: $H(P,I)$ halts for all input P, I.

Negation of Halting Program

- $K(P)$

  -- Run $H(P,P)$

  -- If Output is YES, then LOOP FOREVER

  -- If Output is NO, then HALT

Halting Problem (continued)

Paradox

- $K(K)$

  -- Run $H(K,K)$

  -- If Output is YES, then LOOP FOREVER

  -- If Output is NO, then HALT

- $H(K,K)$

  – If Output is YES, then $K(K)$ LOOPS FOREVER

  -- If Output is NO, then $K(K)$ HALTS

  Therefore H FAILS to solve the Halting Problem!

Detecting Division by Zero

Problem

- Given a program and some input, does the program ever divide by zero?

Observation

- Detecting division by zero is an Undecidable problem.
- If we could solve Division by Zero Problem, then we could solve the Halting Problem.

Theorem: Division by Zero is an Undecidable Problem

Proof: For every program:

- Replace every HALT command by a Division by Zero.
- Replace every division by;
  - -- A Test to Determine if the Denominator is Zero
  - -- If the Test is Positive:
    - -- Perform an action equivalent to Division by Zero
    - -- Jump around the Division

    Otherwise just Perform the Division
- New Programs Divides by Zero $\Leftrightarrow$ Old Program HALTS
- If we could solve the Division by Zero Problem we could also solve the HALTING Problem.
- Therefore we cannot solve the Division by Zero Problem.

More Undecidable Problems

Theorem: Determining whether two programs are equivalent is undecidable

Proof: For two programs to be equivalent they must at least either both HALT or both Loop on the same input. But the Halting Problem is Undecidable.

Theorem: Determining whether a program is optimal is undecidable.

Proof: To determine if a program is optimal, we must first determine equivalence, but equivalence is undecidable.