Testing

Four Problems

1. Sum of Consecutive Odd Numbers -- Speed of Algorithms

2. Sum of Consecutive Cubes -- Speed of Algorithms

3. Formula for Generating Prime Numbers -- Cryptography

4. Formula for Counting Prime Numbers -- Cryptography

Sum of Consecutive Odd Numbers

Example

- $1 = 1^2$

Conclusion

- *The sum of consecutive odd numbers is equal to a square.*

Sum of Consecutive Odd Numbers

Examples

$$1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

Conclusion

The sum of consecutive odd numbers is equal to a square.

Sum of Consecutive Odd Numbers

Examples

$$1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

Conclusion

The sum of consecutive odd numbers is equal to a square.

Sum of Consecutive Odd Numbers

Examples

$$1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

Sum of Consecutive Odd Numbers

Examples

$$1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

Conclusion

The sum of the first n odd numbers is equal to n^2

$$1 + 3 + \cdots + 2n - 1 = n^2$$

Sum of Consecutive Cubes

Example

- $1^3 = 1^2$

Conclusion

- *The sum of consecutive cubes is equal to a square.*

Sum of Consecutive Cubes

Example

$$1^3 = 1^2$$

$$1^3 + 2^3 = 1 + 8 = 9 = 3^2$$

Conclusion

The sum of consecutive cubes is equal to a square.

Sum of Consecutive Cubes

Example

$$1^3 = 1^2$$

$$1^3 + 2^3 = 1 + 8 = 9 = 3^2$$

$$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2$$

Conclusion

The sum of consecutive cubes is equal to a square.

Sum of Consecutive Cubes

Example

$$1^3 = 1^2$$

$$1^3 + 2^3 = 1 + 8 = 9 = 3^2$$

$$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2$$

$$1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 = 10^2$$

Sum of Consecutive Cubes

Example

$$1^3 = 1^2$$

$$1^3 + 2^3 = 1 + 8 = 9 = 3^2$$

$$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2$$

$$1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 = 10^2$$

Conclusion

The sum of consecutive cubes is equal to a square.

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$

Formula for Generating Prime Numbers

$$F(n) = n^2 + n + 41$$

$$F(1) = 1^2 + 1 + 41 = 43$$

Conclusion

The function $F(n)$ *generates a prime number for every value of* n.

Formula for Generating Prime Numbers

$$F(n) = n^2 + n + 41$$

$$F(1) = 1^2 + 1 + 41 = 43$$

$$F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47$$

Conclusion

The function $F(n)$ *generates a prime number for every value of* n.

Formula for Generating Prime Numbers

$$F(n) = n^2 + n + 41$$

$$F(1) = 1^2 + 1 + 41 = 43$$

$$F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47$$

$$F(3) = 3^2 + 3 + 41 = 9 + 3 + 41 = 53$$

Conclusion

The function $F(n)$ *generates a prime number for every value of* n.

Formula for Generating Prime Numbers

$$F(n) = n^2 + n + 41$$

$$F(1) = 1^2 + 1 + 41 = 43$$

$$F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47$$

$$F(3) = 3^2 + 3 + 41 = 9 + 3 + 41 = 53$$

$$F(4) = 4^2 + 4 + 41 = 16 + 4 + 41 = 61$$

Conclusion

The function $F(n)$ *generates a prime number for every value of* n.

Formula for Counting Prime Numbers

$$PrimePi(n) = number\ \ of\ \ primes\ \ less\ \ than\ \ or\ \ equal\ \ to\ \ n$$

$$Li(n) = \int_0^n \frac{dt}{Ln(t)}$$

Theorem

$$\frac{n}{Li(n)} \leq \frac{n}{PrimePi(n)} \leq Ln(n) \qquad \text{for } n > 20$$

Four Problems

1. *The sum of the first n odd numbers is equal to* n^2.

   $1+3+\cdots+2n-1=n^2$

2. *The sum of consecutive cubes is equal to a square*.

   $1^3+2^3+\cdots+n^3=(1+2+\cdots+n)^2$

3. *The function* $F(n)=n^2+n+41$ *generates a prime number for every value of* n.

4. $$\frac{n}{Li(n)} \le \frac{n}{PrimePi(n)} \le Ln(n) \qquad \text{for } n>20.$$