Comp280 05.spring
Summations commentary

Rosen p.229–232 has a fine introduction to summations. The key element to remember is that $\sum$ is just a notation; whenever you see it you should mentally expand it into the sum it represents.

Here are a few problems from Rosen that introduce some standard tricks:

- Rosen 3.2, #15a:

$$\begin{aligned}\sum_{j=0}^{8} 3 \cdot 2^j &= 3 \cdot 2^0 + 3 \cdot 2^1 + 3 \cdot 2^2 + \cdots + 3 \cdot 2^8 \\ &= 3(2^0 + 2^1 + 2^2 + \cdots + 2^8) \\ &= 3 \cdot \sum_{j=0}^{8} 2^j \\ &= 3 \cdot (2^9 - 1) \text{by Rosen 3.2 Th'm 1}\end{aligned}$$

  The handy trick is that you can pull out the constant factor 3.

- Rosen 3.2, #17d: The previous trick is often useful in double sums:

$$\begin{aligned}\sum_{i=0}^{2} \sum_{j=0}^{3} ij &= \sum_{i=0}^{2} \left(\sum_{j=0}^{3} ij \right) \\ &= \sum_{i=0}^{2} \left(i \sum_{j=0}^{3} j \right) \\ &= \sum_{i=0}^{2} (i \cdot 6) \\ &= 6 \sum_{i=0}^{2} i \\ &= 6 \cdot 3 \\ &= 18\end{aligned}$$

Why was it valid, in the first line, to factor out i from the inner sum? Because (with respect to the inner sum over j) it was a constant. Again, writing it out explicitly makes this clear.

- Difference of sums: When a sum's initial index isn't a nice even 0 or 1, often we can express the sum as a difference of two others. See Rosen Section 3.2, Example 15:

$$\sum_{k=50}^{100} k^2 = \sum_{k=1}^{100} k^2 - \sum_{k=1}^{49} k^2$$

  , and now each of the two sums can be individually computed from Section 3.2, Table 2.