Algebra Laws

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John Greiner
Ian Barland

Abstract

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Algebra Laws

The following lists some propositional formula equivalences. Remember that we use the symbol $\equiv$ as a relation between two WFFs, not as a connective inside a WFF. In these, ϕ, ψ, and θ are meta-variables standing for any WFF.

Propositional Logic Equivalences

Double Complementation	$\neg\neg\phi \equiv \phi$	
Complement	$(\phi \vee \neg\phi) \equiv \text{true}$	$(\phi \wedge \neg\phi) \equiv \text{false}$
Identity	$(\phi \vee \text{false}) \equiv \phi$	$(\phi \wedge \text{true}) \equiv \phi$
Dominance	$(\phi \vee \text{true}) \equiv \text{true}$	$(\phi \wedge \text{false}) \equiv \text{false}$
Idempotency	$(\phi \vee \phi) \equiv \phi$	$(\phi \wedge \phi) \equiv \phi$
Absorption	$(\phi \wedge (\phi \vee \psi)) \equiv \phi$	$(\phi \vee (\phi \wedge \psi)) \equiv \phi$
Redundancy	$(\phi \wedge (\neg\phi \vee \psi)) \equiv (\phi \wedge \psi)$	$(\phi \vee (\neg\phi \wedge \psi)) \equiv (\phi \vee \psi)$
DeMorgan's laws	$\neg(\phi \wedge \psi) \equiv (\neg\phi \vee \neg\psi)$	$\neg(\phi \vee \psi) \equiv (\neg\phi \wedge \neg\psi)$
Associativity	$(\phi \wedge (\psi \wedge \theta)) \equiv ((\phi \wedge \psi) \wedge \theta)$	$(\phi \vee (\psi \vee \theta)) \equiv ((\phi \vee \psi) \vee \theta)$
Commutativity	$(\phi \wedge \psi) \equiv (\psi \wedge \phi)$	$(\phi \vee \psi) \equiv (\psi \vee \phi)$
Distributivity	$(\phi \wedge (\psi \vee \theta)) \equiv ((\phi \wedge \psi) \vee (\phi \wedge \theta))$	$(\phi \vee (\psi \wedge \theta)) \equiv ((\phi \vee \psi) \wedge (\phi \vee \theta))$

Equivalences for implication are omitted above for brevity and for tradition. They can be derived, using the definition $(a \rightarrow b) \equiv (\neg a \vee b)$.

Example 1:

For example, using Identity and Commutativity, we have $(\text{true} \rightarrow b) \equiv (\neg\text{true} \vee b) \equiv (\text{false} \vee b) \equiv (b \vee \text{false}) \equiv b$.