Chemistry 312: Carnot Cycle for Ideal Gas

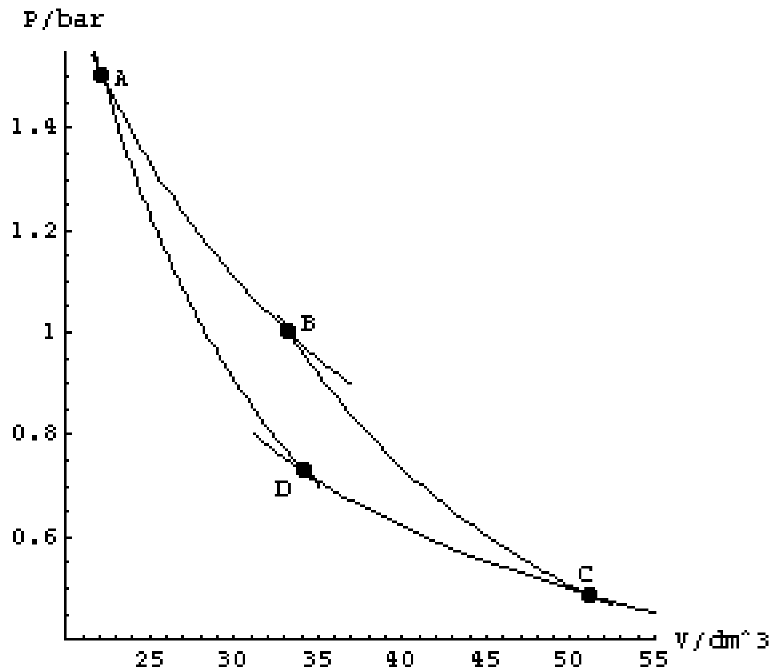

Figure 2: Carnot cycle, plotted as P vs V, for one mole of an ideal gas. The high temperature reservoir is assumed to be at 400K and the lower reservoir at 300K. The starting point (A) is at a pressure of 1.5 bar. The line AB is an isothermal expansion at 400K to a pressure of 1 bar at point B. The line BC is a reversible adiabatic expansion taking the system to T= 300K; CD is an isothermal compression at 300K; DA is a reversible adiabatic compression.

In the following, the quantities that must be specified to determine the cycle are shown in **bold** type the first time they appear. In the ideal gas equation of state, the appropriate value for R is .083145 if the volume is expressed in dm^3 and the pressure in bars.

I. Isothermal expansion from A to B:

$T_A = T_1 = 400$ K; **$P_A = 1.5$ bar**;

$V_A = RT/P = .083145 \times 400/1.5 = 22.172\ dm^3$

Reversible isothermal expansion to new pressure

$P_B = 1.0$ bar; $V_B = 33.258\ dm^3$; $T_B = T_1$

$dU_{AB} = \delta q_{AB} + \delta w_{AB} = 0$ (U for ideal gas depends only on T)
$\delta w_{AB} = -PdV = -RT_1 dV/V$
$w_{AB} = -RT_1 \ln(V_B/V_A) = -1348.5$ J $= -q_{AB}$
$\Delta U_{AB} = \Delta H_{AB} = 0$
$\Delta S_{AB} = q_{AB}/T = R \ln(V_B/V_A) = 3.371$ J K^{-1}

II. Reversible adiabatic expansion from B to C:

T_2 = 300 K; C_V = 3/2 R
$V_C = V_B (T_2/T_1)^{C_V/R} = 51.204$ dm^3; $P_C = 0.48714$ bar
$\Delta U_{BC} = C_V (T_2 - T_1) = -1247.2$ J
$\Delta H_{BC} = C_P (T_2 - T_1) = -2078.6$ J
$q_{BC} = 0$; $w_{BC} = \Delta U_{BC} = -1247.2$ J
$\Delta S_{BC} = 0$

III. Reversible isothermal compression from C to D:

Need to calculate V_D so that it lies on the same reversible adiabat as A: $V_D = V_A (T_2/T_1)^{C_V/R} = 34.136$ dm^3; $P_D = .7307$ bar
$dU_{CD} = \delta q_{CD} + \delta w_{CD} = 0$
$\delta w_{CD} = -PdV = -RT_2 dV/V$
$w_{CD} = -RT_2 \ln(V_D/V_C) = 1011.4$ J $= -q_{CD}$
$\Delta U_{CD} = \Delta H_{CD} = 0$
$\Delta S_{CD} = q_{CD}/T = R \ln(V_D/V_C) = -3.371$ J K^{-1}

IV. Reversible adiabatic compression from D to A:

$\Delta U_{DA} = C_V (T_1 - T_2) = 1247.2$ J
$\Delta H_{DA} = C_P (T_1 - T_2) = 2078.6$ J
$q_{DA} = 0$; $w_{DA} = \Delta U_{DA} = 1247.2$ J
$\Delta S_{DA} = 0$

Summary:

PATH	q	w	ΔU	ΔH	ΔS
AB	1348.5 J	-1348.5 J	0	0	3.371 J/K
BC	0	-1247.2 J	-1247.2 J	-2078.6 J	0
CD	-1011.4 J	1011.4 J	0	0	-3.371 J/K
DA	0	1247.2 J	1247.2 J	2078.6 J	0
TOTAL	337.1 J	-337.1 J	0	0	0

Efficiency: $\eta = -w/q_{AB} = 337.1/1348.5 = 0.25$

$1 - T_1/T_2 = 1 - 300/400 = 0.25$