Problem 1

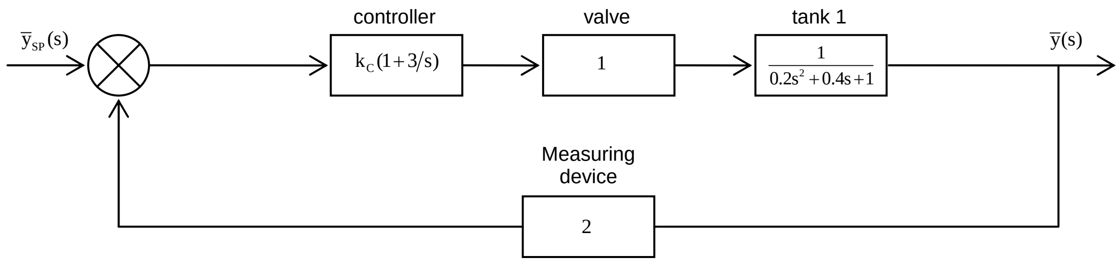

Figure 1

The characteristic equation for the closed loop in figure 1 can be written as follows:

$$1+G_{OL}(s)=1+\frac{2k_C(1+3/s)}{0.2s^2+0.4s+1} \quad \text{(S1.1)}$$

Setting (S1.1) equal to zero and substituting k_C=2 yields:

$$0.2s^3+0.4s^2+5s+12=0 \quad \text{(S1.2)}$$

Test 1: All the coefficients of (S1.2) are positive. Therefore, one can move to test 2.
Test 2: (S1.2) is a 3rd order polynomial. Thus, the Routh-Hurwitz array is composed of 4 rows.

	Column 1	**Column 2**
Row 1	0.2	5
Row 2	0.4	12
Row 3	$[(0.4*5)-(12*0.2)]/0.4=-1$	0
Row 4	$(-1*12)/-1=12$	0

As all the coefficients of the first column of the Routh-Hurwitz array are not positive, the system is unstable. Moreover, as the sign changes two times (from row 2 to row 3 and row 3 to row 4), there are two roots with positive real part.

Problem 2

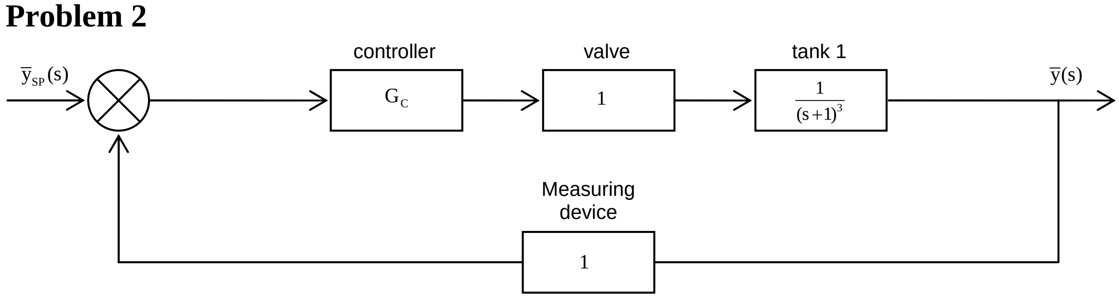

Figure 2

For the proportional controller, the characteristic equation for the closed loop in figure 2 can be written as follows:

$$1 + G_{OL}(s) = 1 + \frac{k_C}{(s+1)^3} \tag{S2.1}$$

Setting (S2.1) equal to zero yields:

$$s^3 + 3s^2 + 3s + (1 + k_C) = 0 \tag{S2.2}$$

Test 1: All the coefficients of (S2.2) are positive. Therefore, one can move to test 2.
Test 2: (S2.2) is a 3rd order polynomial. Thus, the Routh-Hurwitz array is comprised of 4 rows.

	Column 1	**Column 2**
Row 1	1	3
Row 2	3	$1 + k_C$
Row 3	$A_1 = [9 - (1 + k_C)]/3$	0
Row 4	$1 + k_C$	0

In order to ensure stability, it is sufficient to impose that A_1 is positive. Thus, if $k_C < 8$ the system is stable, while the condition $k_C = 8$ leads the system to the verge of instability.

For the proportional-derivative controller, the characteristic equation for the closed loop in figure 2 can be written as follows:

$$1 + G_{OL}(s) = 1 + \frac{k_C(1 + \tau_D s)}{(s+1)^3} \tag{S2.3}$$

Setting (S2.3) equal to zero yields:

$$s^3 + 3s^2 + (3 + 10\tau_D)s + 11 = 0 \tag{S2.4}$$

Test 1: All the coefficients of (S2.2) are positive. Therefore, one can move to test 2.
Test 2: (S2.4) is a 3rd order polynomial. Thus, the Routh-Hurwitz array is comprised of 4 rows.

	Column 1	**Column 2**
Row 1	1	$3 + 10\tau_D$
Row 2	3	11
Row 3	$(-2 + 30\tau_D)/3$	0
Row 4	11	0

In order to ensure stability, it is sufficient to impose that $-2 + 30\tau_D$ is positive, which yields $\tau_D > 1/15$.

Problem 3

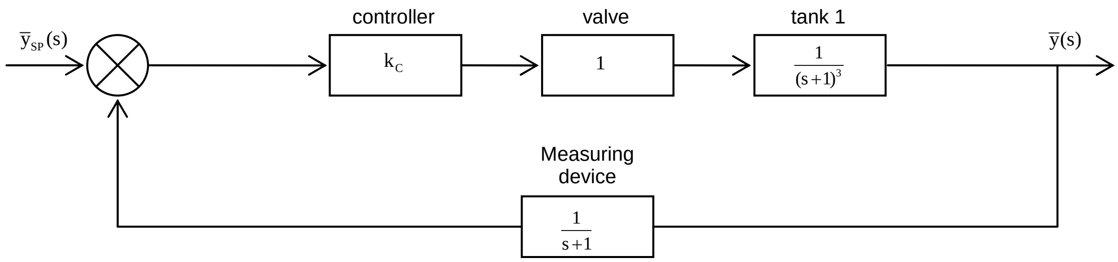

Figure 3

The characteristic equation for the closed loop in figure 3 can be written as follows:

$$1 + G_{OL}(s) = 1 + \frac{k_C}{(s+1)^4} \tag{S3.1}$$

Setting (S3.1) equal to zero yields:

$$s^4 + 4s^3 + 6s^2 + 4s + (1 + k_C) = 0 \tag{S3.2}$$

Test 1: All the coefficients of (S3.2) are positive. Therefore, one can move to test 2.
Test 2: (S3.2) is a 4^{th} order polynomial. Thus, the Routh-Hurwitz array is comprised of 5 rows.

	Column 1	**Column 2**	**Column3**
Row 1	1	6	$1+k_C$
Row 2	4	4	0
Row 3	5	$1+k_C$	0
Row 4	$B_1 = [20 - 4(1+k_C)]/5$	0	0
Row 5	$1+k_C$	0	0

In order to ensure stability, it is sufficient to impose that B_1 is positive. Thus, if $k_C < 4$ the system is stable.

In order to get two imaginary roots, the elements of the 3^{rd} and 5^{th} row have to be positive while that of the 4^{th} row is zero. This is accomplished when $k_C = 4$, which gives the following polynomial:

$$a_1 s^2 + a_2 = 0 \tag{S3.3}$$

where a_1 = 5 (3^{rd} row element) and a_2 = $1+k_C$ = 5 (5^{th} row element). Therefore, (S3.3) becomes:

$$5s^2 + 5 = 5(s - j)(s + j) = 0 \tag{S3.4}$$

which gives $s_1 = j$ and $s_2 = -j$.