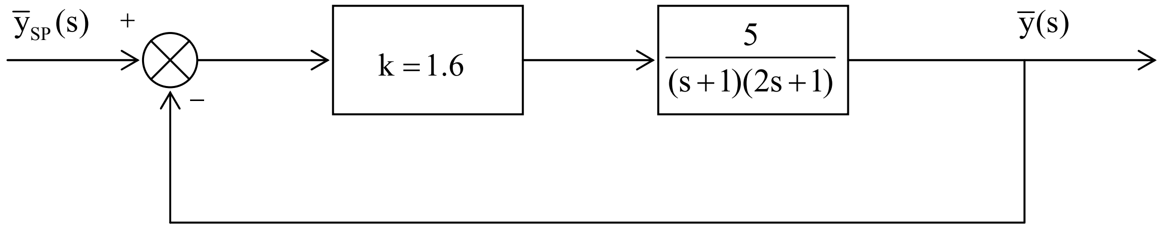
Problem 1



The closed loop transfer function can be written as follows:

$$G_{CL}(s) = \frac{\bar{y}(s)}{\bar{y}_{SP}(s)} = \frac{\frac{8}{(s+1)(2s+1)}}{1+\frac{8}{(s+1)(2s+1)}} = \frac{8}{2s^2+3s+9} = \frac{8/9}{2/9\,s^2+1/3\,s+1} \quad \text{(S1.1)}$$

Thus, comparing (S1.1) to the standard form of 2nd order systems, one obtains:

$$\tau^2 = 2/9 \quad \Rightarrow \quad \tau = \sqrt{2}/3 = 0.471 \quad \text{(S1.2)}$$

$$2\xi\tau = 1/2 \quad \Rightarrow \quad \xi = \sqrt{2}/4 = 0.354 \quad \text{(S1.3)}$$

$$k = 8/9 = 0.889 \quad \text{(S1.4)}$$

Since ξ is smaller than 1, the system is underdamped.

a) From textbook, we know that:

$$OS = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.305 = \frac{\text{max dev final value}}{\text{final value}} = \frac{y_{max} - y(\infty)}{y(\infty)} \quad \text{(S1.5)}$$

The output of the system for a step change of magnitude 0.1 in the set point is:

$$\bar{y}(s) = \frac{0.1}{s}\frac{8/9}{2/9\,s^2+1/3\,s+1} \quad \text{(S1.6)}$$

Applying the final value theorem, yields:

$$y(\infty) = \lim_{s\to 0} s\bar{y}(s) = \lim_{s\to 0}\frac{0.8/9}{2/9\,s^2+1/3\,s+1} = \frac{0.8}{9} = 0.0889 \quad \text{(S1.7)}$$

Hence, the maximum value of the response is:

$$y_{max} = y(\infty)(1+OS) = 0.116 \quad \text{(S1.8)}$$

b) For a servo problem, the offset is given by:

$$\text{offset} = \text{new set point} - y(\infty) = 0.1 - 0.0899 = 0.0111 \tag{S1.9}$$

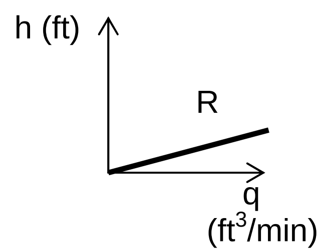
c) From the textbook, the period of the oscillation is:

$$T = 2\pi/\omega \quad \text{where} \quad \omega = \sqrt{1-\xi^2}\Big/\tau \tag{S1.10}$$

Therefore, the value of the period of the oscillation T is:

$$T = 3.17\,\text{min} \tag{S1.11}$$

Problem 2



The resistance of a liquid to a hydrostatic pressure can be defined as the rate of changing of the liquid level due to the change of the output low rate. Thus:

$$R = \frac{dh}{dq} \tag{S2.1}$$

Assuming linear resistances, R_1 and R_2 can be directly evaluated by plotting h (ft) versus q (ft^3/min) and estimating the slope of the straight line. Hence, one obtains:

$$R_1 = R_2 = 0.5 \tag{S2.2}$$

Since the cross sections are equal to 2, the time constant of the two tanks are both equal to $\tau_1 = A_1R_1 = 1\ (\text{min})$ and $\tau_2 = A_2R_2 = 1\ (\text{min})$, while the static gains of two tanks are equal to $k_1 = R_1 = 0.5\ (\text{min}/\text{ft}^2)$ and $k_2 = R_2/R_1 = 1\ (\text{min}/\text{ft}^2)$. Thus, the transfer functions of the two tanks are the following:

$$G_1(s) = \frac{0.5}{s+1} \quad \Rightarrow \quad G_2(s) = \frac{1}{s+1} \tag{S2.3}$$

Employing a proportional controller, the corresponding transfer function is:

$$G_s(s) = k_c \tag{S2.4}$$

Plotting the change in pressure to the valve versus the change in flow provides the transfer function for the final control element:

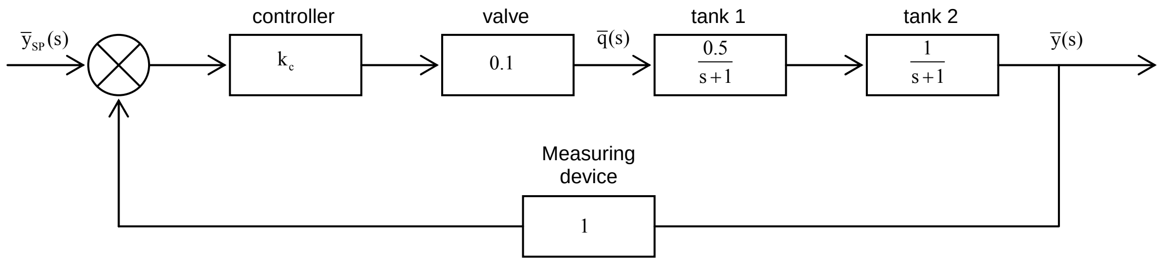
$$G_f(s) = \frac{dP}{dq} = \frac{0.1}{1} = 0.1 \tag{S2.5}$$

With no lag in the measuring device dynamics, the corresponding transfer function is:

$$G_m(s) = 1 \tag{S2.6}$$

Therefore, the block diagram is the following:

a)



b) The close loop transfer function is:

$$G_{CL}(s) = \frac{\bar{y}(s)}{\bar{y}_{SP}(s)} = \frac{\frac{0.05\ k_c}{(s+1)^2}}{1+\frac{0.05\ k_c}{(s+1)^2}} = \frac{\frac{0.05\ k_c}{1+0.05\ k_c}}{\frac{1}{1+0.05\ k_c}s^2 + \frac{2}{1+0.05\ k_c}s+1} \tag{S2.7}$$

Comparing the closed loop transfer function to the standard form of a 2nd order system transfer function one obtains:

$$\tau = 1\Big/\sqrt{1+0.05\ k_c} \tag{S2.8}$$

$$\xi = 1\Big/\sqrt{1+0.05\ k_c} \tag{S2.9}$$

$$k = 0.05\ k_c/(1+0.05\ k_c) \tag{S2.10}$$

For a critically damped 2nd order system, ξ=1 and therefore:

$$k_{c(CD)} = 0 \tag{S2.11}$$

Therefore, the critical damping cannot occur.

c) For interacting tanks, the transfer function between the output of the second tank and the input of the first tank is given by:

$$G(s) = \frac{\bar{y}(s)}{\bar{q}(s)} = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2+A_1R_2)s+1} = \frac{0.5}{s^2+3s+1} \tag{S2.12}$$

Thus, the close loop transfer function becomes:

$$G_{CL}(s) = \frac{\bar{y}(s)}{\bar{y}_{SP}(s)} = \frac{\dfrac{0.05\ k_c}{s^2+3s+1}}{1+\dfrac{0.05\ k_c}{s^2+3s+1}} = \frac{\dfrac{0.05\ k_c}{1+0.05\ k_c}}{\dfrac{1}{1+0.05\ k_c}s^2 + \dfrac{3}{1+0.05\ k_c}s+1} \quad \text{(S2.13)}$$

Comparing the closed loop transfer function to the standard form of a 2nd order system transfer function one obtains:

$$\tau = 1\Big/\sqrt{1+0.05\ k_c} \quad \text{(S2.14)}$$

$$\xi = \frac{3}{2\sqrt{1+0.05\ k_c}} \quad \text{(S2.15)}$$

$$k = 0.05\ k_c/(1+0.05\ k_c) \quad \text{(S2.16)}$$

For a critically damped 2nd order system, ξ=1 and therefore:

$$k_{c(CD)} = 25 \text{ psi/ft} \quad \text{(S2.17)}$$

Hence, for interacting capacities the critical damping does occur.

d) Assuming k_c=1.5$k_{c(CD)}$, one obtains:

$$k_c = 1.5k_{c(CD)} = 37.5 \text{ psi/ft} \quad \text{(S2.18)}$$

Thus, the natural period of the oscillations, damping factor and gain becomes:

$$\tau = 0.590 \text{ (min)} \quad \text{(S2.19)}$$

$$\xi = 0.885 \quad \text{(S2.20)}$$

$$k = 0.652 \text{ (ft)} \quad \text{(S2.21)}$$

For a step change of 1/12 ft in the se point, $y_{sp}(s)$ is equal to 1/(s12). Therefore, one obtains:

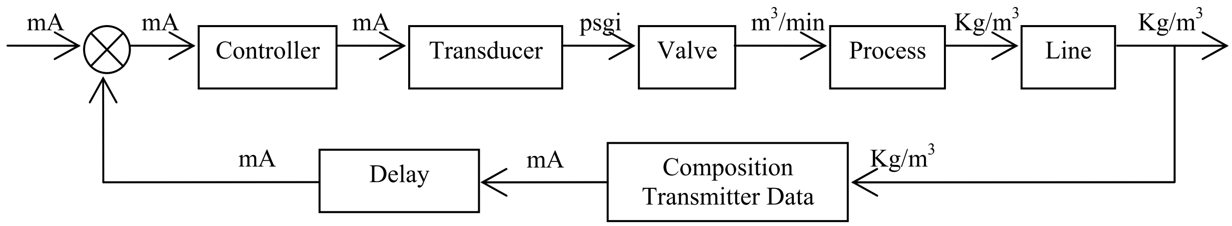
$$\bar{y}(s) = \frac{0.652}{0.348s^2+1.043s+1}\bar{y}_{SP}(s) = \frac{0.054}{(0.348s^2+1.043s+1)s} \quad \text{(S2.22)}$$

Since ξ is smaller than 1, from textbook we have:

$$y(t) = 0.054\left\{1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi t/\tau_1}\sin\left[\sqrt{1-\xi^2}\,\frac{t}{\tau_1}+tg^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]\right\} = $$
$$= 0.054-0.116e^{-1.5t}\sin(0.8t+28^o) \quad \text{(S2.23)}$$

Problem 3:

a) The block diagram is the following:



b) Controller (pure PID)

Transfer function is: $G_c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)$

Transducer

Transfer Function is: $G_T(s) = \frac{\tilde{p}_v(s)}{\tilde{p}(s)} = \frac{p_v(s) - 3}{p(s) - 4} = \frac{15 - 3}{20 - 4} = \frac{12}{16} = \frac{3}{4}$

Control Valve

Linearizing q_A around an operative point:

$q_A = q_{A0} + 0.03\ln(20)20^{(p_{v0}-3)/12}(p_v - p_{v0})$

Choosing q_{A0}=0.17 and p_{V0}=3, gives: $q_A = q_{A0} + 0.0025\ln(20)(p_v - p_{v0})$

Transfer Function is: $G_v(s) = \frac{\tilde{q}_A(s)}{\tilde{p}_v(s)} = \frac{q_A(s) - 0.17}{p_v(s) - 3} = 0.0025\ln(20)$

Also, considering delay: $G_v(s) = 0.0025\ln(20)e^{-s}$

Process

$V\frac{dc}{dt} = q_A c_A + q_F c_F - (q_A + q_F)c$. Note $q_A + q_F \approx q_F$.

Hence $V\frac{dc}{dt} + q_F c = q_A c_A + q_F c_F$.

At steady state: $q_F c_s = q_{As} c_A + q_F c_{Fs}$

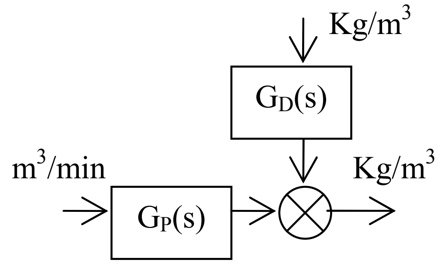
Defining $\tilde{c} = c - c_s$, $\tilde{q}_A = q_A - q_{As}$, $\tilde{c}_F = c_F - c_{Fs}$ yields:

$V\frac{d\tilde{c}}{dt} + q_F\tilde{c} = c_A\tilde{q}_A + q_F\tilde{c}_F$ or $\tau_p\frac{d\tilde{c}}{dt} + \tilde{c} = k_p\tilde{q}_A + k_d\tilde{c}_F$ where $\tau_p = V/q_F$, $K_P = c_A/q_F$ and $K_d = 1$.

Taking Laplace transform gives:

$c(s) = \frac{k_P}{\tau_P s + 1}q_A(s) + \frac{k_d}{\tau_P s + 1}c_F(s) = G_P(s)q_A(s) + G_D(s)c_F(s)$

Therefore, process can be expanded as illustrated in figure.



Trasmission Line

Transfer function is: $G_L(s) = e^{-t_L s}$,

where $t_L = \left[20\pi(0.5)^2\right]/4q_F$

Composition Transmitter Data

Transfer Function is: $G_{CTA}(s) = \dfrac{\tilde{c}_m(s)}{\tilde{c}(s)} = \dfrac{c_m(s)-4}{c(s)} = \dfrac{20-4}{200} = \dfrac{16}{200}$