Problem 1

From table 7.1 in the textbook, the Laplace transform of $y(t)=te^{-t}$ is:

$$L\left[te^{-t}\right]=\bar{y}(s)=\frac{1}{(s+1)^2} \qquad (S1.1)$$

Moreover, the Laplace transform of the unit-impulse $\delta(t)$ is:

$$L\left[\delta(t)\right]=\bar{u}(s)=1 \qquad (S1.2)$$

Since $u(t)=0$ all the time but at $t=t_o$, assuming that $y(t)$ is in deviation form (i.e. $y_s=0$), the transfer function of the system in exam can be written as follows:

$$G(s)=\frac{\bar{y}(s)}{\bar{u}(s)}=\frac{1}{(s+1)^2} \qquad (S1.3)$$

Problem 2

Assuming constant density and reactor volume, the overall material balance can be written as follows:

$$A\frac{dh(t)}{dt}=F_i(t)-F=F_i(t)-8h(t)^{1/2} \qquad (S2.1)$$

As initial condition for equation (S2.1), we chose the steady state value of the hydrostatic pressure h_s given by:

$$h(0)=h_s=F_{is}^2/64 \qquad (S2.2)$$

Linearizing the right hand side of equation (S2.1) about the steady state value (S2.2), one obtains:

$$F_i(t)-8h(t)^{1/2}\cong\left[F_i(t)-F_s\right]-\left[4h_s^{-1/2}\right]\left[h(t)-h_s\right] \qquad (S2.3)$$

Defining the deviation variables $H(t)=h(t)-h_s$ and $Q(t)=F_i(t)-F_s$, and considering (S2.3), equation (S2.2) becomes

$$\frac{Ah_s^{1/2}}{4}\frac{dH(t)}{dt}+H(t)=\frac{h_s^{1/2}}{4}Q(t) \qquad (S2.4)$$

subject to the following initial condition

$$H(0)=0 \qquad (S2.5)$$

Therefore, by comparison to the standard form of 1st order systems, the time constant τ and the gain k are the following:

$$\tau = \frac{Ah_s^{1/2}}{4} \qquad k = \frac{h_s^{1/2}}{4} \tag{S2.6}$$

Hence:

$$\begin{aligned} &\text{a)}\quad h_s = 3\text{ft} \quad \Rightarrow \quad \tau = 3\sqrt{3}/4 \\ &\text{b)}\quad h_s = 9\text{ft} \quad \Rightarrow \quad \tau = 9/4 \end{aligned} \tag{S2.7}$$

Problem 3

The material balance on the component A is the following:

$$V\frac{dc_A(t)}{dt} = F\left[c_{Ai}(t) - c_A(t)\right] - kc_A(t)V \tag{S3.1}$$

subject to the initial condition $c_A(0)=c_{Ass}=F/(F+kV)c_{Ai,ss.}$ As the differential equation describing the dynamics of the system is linear, it is already in deviation form when one chooses the deviation variables $x(t)=c_A(t)-c_{ASS}$ and $y(t)=c_{Ai}(t)-c_{Ai,SS}$. Thus, (S3.1) can be rewritten as follows:

$$\frac{V}{F+kV}\frac{dx(t)}{dt} + x(t) = \frac{F}{F+kV}y(t) \tag{S3.2}$$

subject to the initial condition x(0)=0. Therefore, by comparison to the standard form of 1st order systems, the time constant τ and the gain k of the system are given by:

$$\tau = \frac{V}{F+kV} \qquad k = \frac{F}{F+kV} \tag{S3.3}$$

and the transfer function G(s) can be expressed as follows:

$$G(s) = \frac{\bar{x}(s)}{\bar{y}(s)} = \frac{\dfrac{F}{F+kV}}{\dfrac{V}{F+kV}s+1} = \frac{k/\tau}{s+1/\tau} \tag{S3.4}$$

Unit-step change $\bar{y}(s) = 1/s$. Therefore

$$\bar{x}(s) = \frac{k/\tau}{\left[s+1/\tau\right]s} = k\left(\frac{1}{s} - \frac{1}{s+1/\tau}\right) \tag{S3.5}$$

which yields:

$$x(t) = k(1-e^{-t/\tau}) \tag{S3.6}$$

Thus, the sketch to a unit-step change is shown in figure S3.a

Unit-impulse change $\overline{y}(s) = 1$. Therefore

$$\overline{x}(s) = \frac{k/\tau}{s + 1/\tau} \tag{S3.7}$$

which yields:

$$x(t) = -\frac{k}{\tau} e^{-t/\tau} \tag{S3.8}$$

Thus, the sketch to a unit-impulse change is shown in figure S3.b

Sinusoidal input $\overline{y}(s) = A\omega / (s^2 + \omega^2)$. Therefore

$$\overline{x}(s) = \frac{k/\tau}{s + 1/\tau} \frac{A\omega}{s^2 + \omega^2} \tag{S3.9}$$

Repeating what we did in class (see also page 318 of the textbook), one gets:

$$x(t) = \frac{kA}{\sqrt{\tau^2\omega^2 + 1}} \sin(\omega t + \phi) \qquad \phi = \tan^{-1}(-\omega t) \tag{S3.10}$$

Thus, the sketch to a sinusoidal input is shown in figure S3.c

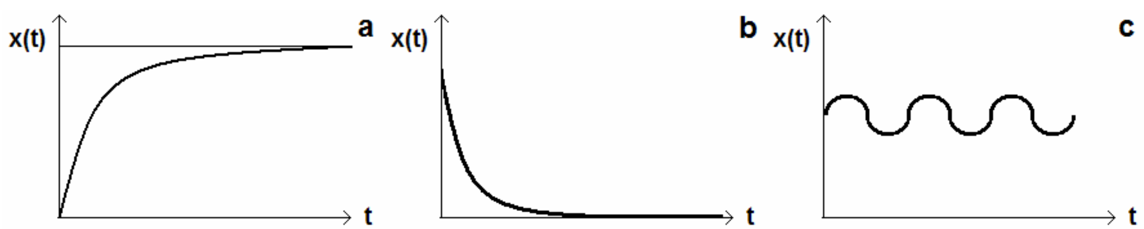

Figure S3

Problem 4

Assuming constant density and reactor volume, the overall material balance can be written as follows:

$$AR \frac{dh(t)}{dt} + h(t) = RF_i(t) \tag{S4.1}$$

As initial condition for equation (S4.1), we chose the steady state value of the hydrostatic pressure h_s given by:

$$h(0) = h_s = RF_{is} \tag{S4.2}$$

Since (S4.1) is linear, defining the deviation variables $H(t)=h(t)-h_s$ and $Q(t)=F_i(t)-F_{is}$ (S4.1) can be immediately rewritten in deviation form as follows:

$$AR\frac{dH(t)}{dt} + H(t) = RQ(t) = Ra\sin(\omega t) \tag{S4.3}$$

subject to the initial condition

$$H(0) = 0 \tag{S4.4}$$

The first order system given by (S4.3)-(S4.4) has time constant and gain respectively given by:

$$\tau = AR \qquad k = aR \tag{S4.5}$$

We know from what we did in class that the output of a first order system subject to a sinusoidal input is a sinusoidal wave with the same frequency and with an amplitude ratio given by:

$$ARatio = b = \frac{k}{\sqrt{\tau^2\omega^2 + 1}} = \frac{aR}{\sqrt{A^2R^2\omega^2 + 1}} \tag{S4.6}$$

Solving (S4.6) for A gives:

$$A = \frac{1}{R\omega b}\sqrt{(aR)^2 - b^2} \tag{S4.7}$$