CHBE 470 – Process Dynamics and Control – Fall 2007

Homework Set 4

Assigned: Wednesday, September 26 **Due:** Wednesday, October 3

Note: Please staple your papers and include your name in the first page

Use table 7.1 from textbook wherever you find appropriate

Problem 1: Consider the 2-tank liquid level system where the outlet from the first tank is the inlet to the second tank. Assume linear resistances and that the system operates at steady state when a unit-step change is made in the flow rate entering tank 1. The transient response is critically damped and it takes 1 min for the change in level of the second tank to reach 50 percent of the total change. If the ratio of the cross-sectional areas of the tanks is $A_1/A_2=2$,

a) Calculate the ratio R_1/R_2, where R_1, R_2 are the resistances in the 2 tanks.
b) Calculate the time constant for each tank.
c) How long does it take for the change in the level of the first tank to reach 90 percent of the total change?

Problem 2: A step change of magnitude A is introduced into the transfer function

$$G(s) = \frac{10}{2s^2 + 0.3s + 0.5} \tag{2.1}$$

Determine the overshoot and the frequency of oscillation $\nu=\omega/(2\pi)$.

Problem 3: Consider a 2-tank liquid level system with linear resistances to flow. There are 2 outputs from the first tank: the first is the inlet to the second tank while the second is a simple output flow rate with resistance $R_a=2$. The resistance of the outlet flow rate F_1 which enters the second tank is $R_1=1$ and the resistance of the outlet flow rate F_2 from the second tank is also $R_2=1$. The cross-sectional areas of the two tanks are $A_1=2ft^2$ and $A_2=1ft^2$, respectively. Starting from first principles (mass balances), derive the transfer functions $H_1(s)/Q(s)$ and $H_2(s)/Q(s)$, where $H_1(s)$, $H_2(s)$ are the Laplace transforms of the liquid levels in the first and second tanks, respectively, while Q(s) is the Laplace transform of the input to the 1st tank (*all in deviation form*). You're expected to give numerical values of the parameters in the transfer functions and show clearly how you derived the transfer functions.

Problem 4: A reaction $A\rightarrow R$ takes place in a constant-volume isothermal CSTR. The component mass balances are:

$$\tau\frac{dC_A}{dt} = C_{Ai} - C_A - kC_AC_R \tag{4.1a}$$

$$\tau\frac{dC_R}{dt} = C_{Ri} - C_R + kC_AC_R \tag{4.1b}$$

In equations (4.1) C_{Ai} is the feed concentration of A (input), C_{Ri} is the feed concentration of R (assumed constant) C_A and C_R are respectively the concentrations of A and R in the reactor and outlet while τ and k are positive constants.

a) Linearize the equations around the design steady-state (C_{Ais}, C_{As}, C_{Rs}) and put them in deviation variable form
b) Calculate the transfer function between C_R and C_{Ai}.
c) Under what conditions on the process parameters and steady state values is the above transfer function stable?
d) Does the above transfer function correspond to an overdamped or underdamped system? Provide a detailed explanation.