CHBE 470 – Process Dynamics and Control – Fall 2007

Homework Set 1

Assigned: Wednesday, September 5 **Due:** Wednesday, September 12

Note: Please staple your papers and include your name in the first page

Problem 1: Consider an isothermal chemical reactor, with the following set of reactions:

$$A \xrightarrow{k_1} B \tag{1.1}$$

$$B \xrightarrow{k_2} C \tag{1.2}$$

$$2A \xrightarrow{k_3} D \tag{1.3}$$

where k_1, k_2, and k_3 are the reaction rates constants of the given set of reactions in appropriate units. The feed stream (flow rate F_i) contains only component A (concentration C_{Af}). Assume 1) perfect mixing, 2) the input and output density are equal and 3) the volume of the reactor is kept constant.

a) Derive a process model for this system
b) Identify the input, output and state variables as well as the parameters of this process

Problem 2: Consider a biochemical reactor where biomass is generated by the consumption of a substrate. Let x and s denote the biomass and substrate concentrations, in units of mass of cells/ unit reactor volume and mass of substrate/unit reactor volume, respectively. Assume that the rate of biomass production r_1 (in units of mass of cells generated per unit reactor volume per unit time) is proportional to the biomass concentration and the proportionality coefficient is the specific growth rate μ. Further assume that the specific growth rate depends on the substrate concentration according to the expression (Michaelis-Menten kinetics):

$$\mu = \frac{\mu_{max} S}{K_s + S} \tag{2.1}$$

Define Y as the ratio of the rate of biomass production r_1 (in units of mass of cells generated per unit reactor volume per unit time) divided by the rate of substrate consumption r_2 (in units of mass of substrate consumed per unit reactor volume per unit time). Assume constant reactor volume, constant input and output densities, constant temperature, perfect mixing and also that the feed stream (volumetric flow rate F_i) contains only substrate (concentration s_f).

a) Derive a process model for this system
b) Identify the input, output and state variables as well as the parameters of this process

Problem 3: Consider a continuous stirred tank reactor, with the following exothermic, 1st order irreversible reaction:

$$A \xrightarrow{k,-\Delta H} B \tag{3.1}$$

where k is the reaction rate constant (1/h) and -ΔH is the heat produced per mole of A consumed during the reaction (kcal/mol). The feed stream contains only component A. Let F_o, c_{Ao} and T_o respectively denote the volumetric flow rate (lt/h), concentration (mol/lt) and temperature (k) of the feed stream, while F, c_A and T respectively represent the volumetric flow rate (lt/h), concentration (mol/lt) and temperature (k) of the output stream.

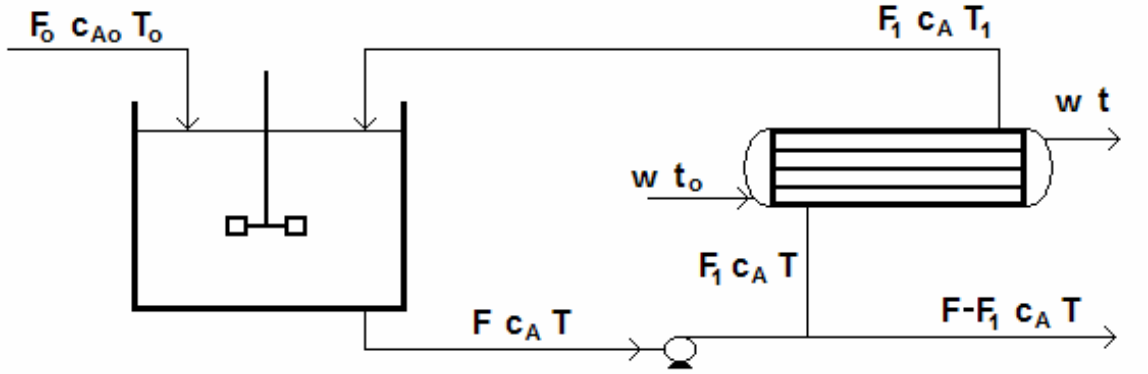

Figure 1: schematic representation of the process

In order to remove the heat generated during the reaction, a fraction F_1 (lt/h) of the output volumetric flow rate is recycled to the reactor after being cooled down at temperature T_1 (K) through a heat exchanger, as shown in figure 1. The refrigerant fluid employed in the heat exchanger is water, which is fed at constant molar flow rate w (moles/h) with input and output temperatures respectively equal to t_o and t. Assume that the pressure and the reactor volume are kept constant. Assume, also, that the density (moles/lt) and the heat (kcal/moles/k) capacity of both the liquid fluid and water are independent on the temperature and thus constant. Finally, assume that the dependence of the rate constant k on the temperature follows the Arrhenius law:

$$k = k_o \exp(-E/RT) \tag{3.2}$$

where E is the activation energy (kcal/mole) and R is the ideal gas constant (kcal/mol/K).

a) Derive a process model for this system
b) Identify the input, output and state variables as well as the parameters of this process