Chapter 8 Well Models

Reading assignment: Chapter 7 in Mattax and Dalton, <u>Reservoir Simulation</u>

      Up to now we have treated wells are point or line sources or sinks with specified rates. In practice the well rates may be the quantity desired to be predicted. The control variable that an operator has in producing a reservoir may be the surface or well head pressure in the case of a flowing well or the bottom hole pressure in the case of a pumping well. There may be a hierarchy of controls governing the production or injection between the following: (1) grid blocks, (2) sand members, (3) well, (4) group of wells, (5) reservoir, and (6) field. Here we will treat only the most fundamental level, i.e., the relation between the reservoir variables and well variables, i.e., bottom hole or surface pressure.

8.1 Relation with Grid Block

      Rather than considering a well as a line source or sink, the well is considered as an interior boundary condition with a radius, r_W, flow rate, q, and pressure, p_W. See Fig. 8.1a. The following assumptions are made.

1) The flow from (to) the grid block to (from) the well is treated as axisymmetric in an annular region with external radius, r_e, where $r_e = \sqrt{\Delta x \Delta y / \pi}$, and internal radius, r_W, where $r_W << r_e$.

2) Uniform thickness and homogeneous properties and saturations in this region.

3) Semi-steady state, i.e., neglect transients except for change in average pressure.
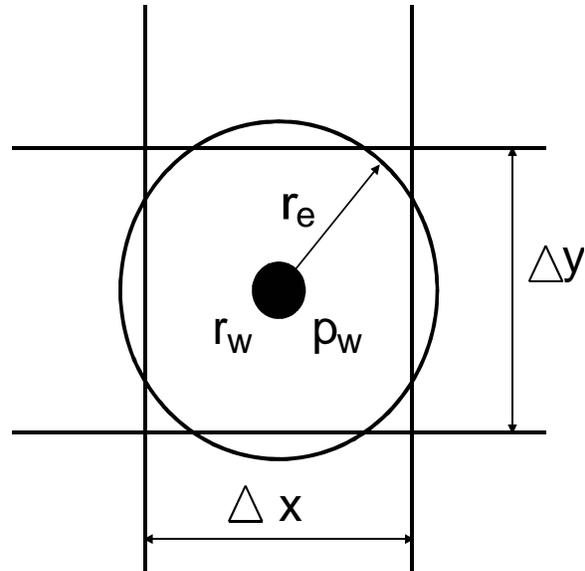
Fig. 8.1 a Well located in center of grid block

$$\frac{\partial(\phi b S)}{\partial t} = -\nabla \cdot b\vec{u} = 0$$

$$\nabla \cdot b\vec{u} = \frac{d(r b u_r)}{dr} = -\frac{d}{dr}\left(r \frac{k k_r b}{\mu} \frac{dp}{dr}\right) = 0$$

(8.1a)

$$r \frac{k \, k_r \, b}{\mu} \frac{dp}{dr} = constant \quad (\text{i.e.,not function of } r)$$

$$\text{at } r = r_w, \quad 2\pi \, r_w \, h \, b \, u_r = q$$

$$\text{implies}: \quad const = \frac{-q}{2\pi \, h} \tag{8.1b}$$

$$r \frac{dp}{dr} = -\frac{q \, \mu}{2\pi \, h \, k \, k_r \, b}$$

Let:

$$p_D = \frac{2\pi \, h \, k \, k_r \, b}{q \, \mu} (p - p_w) \tag{8.1c}$$

$$r_D = r / r_w$$

$$r_D \frac{dp_D}{dr_D} = -1$$

$$dp_D = -\frac{dr_D}{r_D} = -d \ln r_D \tag{8.1d}$$

$$p_D = -\ln r_D + const2$$

$$\text{at } r_D = 1, \quad p_D = 0, \quad \Rightarrow \quad const2 = 0$$

This equation gives the pressure profile. The computed grid block pressure corresponds to the average pressure in the annular region.

$$\overline{p}_D = \frac{\int_1^{r_e/r_w} 2\pi \, r_D \, p_D \, dr_D}{\int_1^{r_e/r_w} 2\pi \, r_D \, dr_D}$$

$$\approx \frac{\int_1^{r_e/r_w} r_D \, p_D \, dr_D}{\frac{1}{2}(r_e/r_w)^2} \tag{8.1e}$$

$$\int r \ln r \, dr = \frac{r^2}{4}(2 \ln r - 1) \tag{8.1f}$$

$$\bar{p}_D \approx \frac{-\dfrac{1}{4}\left(r_e / r_w\right)^2 \left(2\ln\left(r_e / r_w\right)-1\right)}{\dfrac{1}{2}\left(r_e / r_w\right)^2}$$

$$= -\left[\ln\left(r_e / r_w\right)-1/2\right] \tag{8.1g}$$

$$= \frac{2\pi\, h\, k\, k_r\, b}{q\,\mu}\left(\bar{p}-p_w\right)$$

$$q = -\frac{2\pi\, h\, k\, k_r\, b}{\mu}\frac{\left(\bar{p}-p_w\right)}{\left(\ln\left(r_e / r_w\right)-1/2\right)}$$

or in oil field units $\tag{8.1h}$

$$q = -\frac{2\pi\, h\, k\, k_r\, b}{887.2\mu}\frac{\left(\bar{p}-p_w\right)}{\left(\ln\left(r_e / r_w\right)-1/2\right)}$$

where

q(STB/d)	b(STB/RB)
h(ft)	μ(cp)
k(md)	p(psi)

The proceeding calculations were for a well in the center of a grid block. If a model is an element of symmetry, then the element may contain only a half or quarter well as illustrated in Fig. 8.1b.
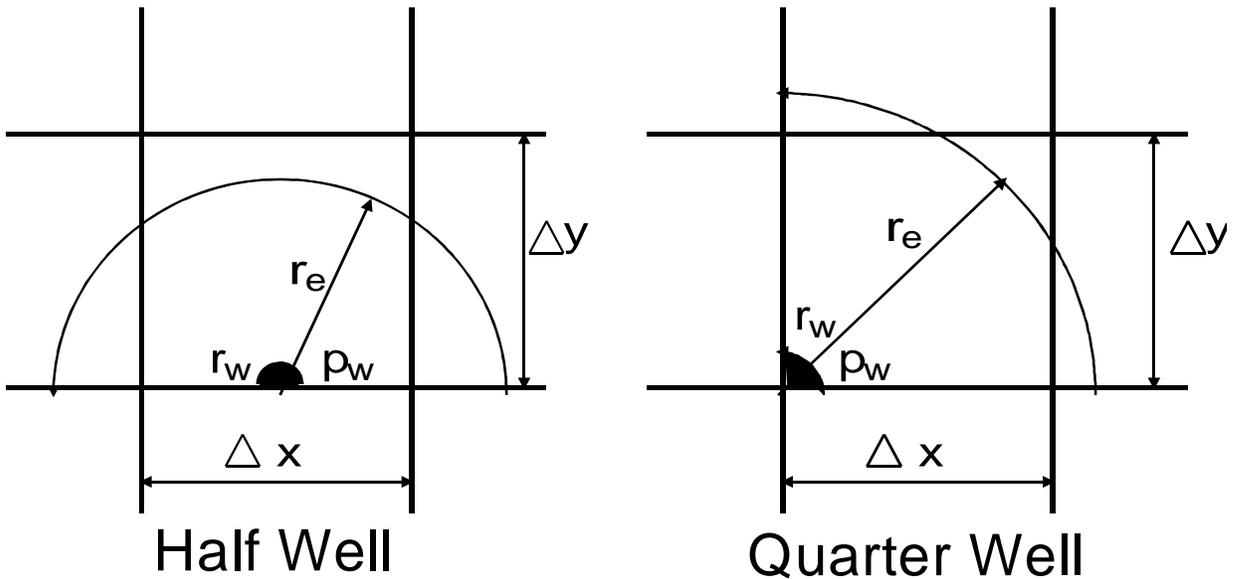
Fig. 8.1 b Half and quarter well in a grid block

In the cases of the **half well** the relation between the equivalent radius and grid block dimensions are as follows.

$$\pi\, r_e^2 = 2\Delta x\, \Delta y$$
$$r_e = \sqrt{\frac{2\Delta x\, \Delta y}{\pi}}$$

(8.1i)

In the cases of the **quarter well** the relation between the equivalent radius and grid block dimensions are as follows.

$$\pi\, r_e^2 = 4\Delta x\, \Delta y$$
$$r_e = \sqrt{\frac{4\Delta x\, \Delta y}{\pi}}$$

(8.1j)

In the case of the half well and quarter well, the factor 2π in equation 8.1h is replaced by π and $\pi/2$, respectively and the rates are one half or one quarter of the whole well. All these modifications for whole, half, or quarter well can be encompassed by defining a well fraction parameter, W_{FRAC}.

$$r_e = \sqrt{\Delta x\, \Delta y\, /\, \pi\, W_{FRAC}}$$
$$q = -\frac{2\pi\, h\, k\, k_r\, b\, W_{FRAC}}{\mu}\, \frac{\left(\bar{p} - p_w\right)}{\left(\ln\left(r_e\, /\, r_w\right) - 1/2\right)}$$

The expression for a well with cylindrical coordinates was given in equation 7.2g. The sign is changed to denote production as negative and injection as positive rates.

$$p\left(r_w, t\right) = p\left(r_1, t\right) + \frac{q\,\mu}{2\pi\, k\, h}\ln\left(r_w\, /\, r_1\right)$$

$$q = \frac{-2\pi\, k\, h}{\mu\, \ln\left(r_w\, /\, r_1\right)}\left[p\left(r_1, t\right) - p\left(r_w, t\right)\right]$$

Mattax and Dalton (1990) references several alternative well inflow coefficients. Peaceman (1978) found that expression that best described steady flow in a square domain with Cartesian coordinates is different from Eq. (8.1h). A skin factor is introduced to find the equivalence between the two models.

$$q = -\frac{2\pi \, h k \, k_r \, b}{\mu} \frac{\left(p_o - p_w\right)}{\left(\ln\left(r_e / r_w\right) - 1/2 + s\right)} \qquad , \text{Eq(8.1h)}$$

$$r_e = \sqrt{\Delta x \, \Delta y / \pi} \qquad , \text{whole well}$$

$$q = -\frac{2\pi \, h k \, k_r \, b}{\mu} \frac{\left(p_o - p_w\right)}{\ln\left(0.2\Delta x / r_w\right)} \qquad , \text{Peaceman}, \quad \Delta y = \Delta x$$

$$s = \log\left(0.2\sqrt{\pi}\right) + 1/2$$

$$= -0.54$$

The use of this pseudo-skin factor will match the analytical solution when the well is in the center of a square domain. However, the original expression (with WFRAC) matches the analytical solution for a quarter-well at the corner of the one-quarter element of symmetry.

Equation 8.1h is abbreviated by calling the coefficient the productivity index, PI, (or injectivity index) and identifying the average pressure as the grid block pressure, p_{ij}. The well rate is made semi-implicit in the grid block pressure and saturation.

$$q_{ij} = -PI\left(p_{ij} - p_w\right)$$

$$= -PI\left(p_{ij}^n - p_w\right) - PI\left(p_{ij}^{n+1} - p_{ij}^n\right) - \frac{dPI}{dS}\left(p_{ij}^n - p_w\right)\left(S_{ij}^{n+1} - S_{ij}^n\right) \qquad (8.1k)$$

$$= -PI\left(p_{ij}^n - p_w\right) - PI\,\Delta_t p_{ij} - \frac{dPI}{dS}\left(p_{ij}^n - p_w\right)\Delta_t S_{ij}$$

The latter two terms called the "implicit coefficients" and are very important for stability. The importance of this semi-implicit formulation is illustrated by a simple stability analysis for the pressure equation. The well rate and accumulation terms in equation 6.6ll are all that are of interest for the stability analysis.

$$C\left(p_{n+1} - p_n\right) = q$$

$$= -PI\left(p_n - p_w\right) - PI^{(IMP)}\left(p_{n+1} - p_n\right) \qquad (8.1l)$$

The implicit coefficient, $PI^{(IMP)}$, is zero for the explicit rate formulation and is equal to the *PI* for the semi-implicit well formulation. The equation is rewritten as follows.

$$\left(C + PI^{(IMP)}\right) p_{n+1} = \left(C + PI^{(IMP)} - PI\right) p_n + PI \, p_w \qquad (8.1m)$$

Suppose e_n is the difference between two solutions of the difference equation that differ by a small amount at one time step. The difference equation for e_n after this time step is as follows.

$$\left(C + PI^{(IMP)}\right)e_{n+1} = \left(C + PI^{(IMP)} - PI\right)e_n$$

$$e_{n+1} = \frac{\left(C + PI^{(IMP)} - PI\right)}{\left(C + PI^{(IMP)}\right)}e_n \qquad (8.1n)$$

The coefficient of the second equation is an amplification factor. For the semi-implicit formulation, the amplification factor is always less than unity. With the explicit formulation, the factor becomes negative for $PI > C$ and is negative with a magnitude greater than unity if $PI > 2C$. In the latter case, the system is unstable.

Assignment 8.1 Well model with rate and pressure constraints a) Simulate the problem of assignment 7.1 with NX=10 and following well model. The productivity index for this cylindrical system is given by equation 7.2g. Let the well radius and kh be as in assignment 7.1. Specify the minimum bottom hole pressure to be 14.7 psi. b) Use the same pressure constraint for the one-quarter well in the corner of a square as in assignment 6.4. c) Use the same pressure constraint for a whole well in the center of a 2000' by 2000' square. Find the pseudo-skin factor that matches the analytical solution. In all cases compare the well pressure with the analytical solution for constant rate.

8.2 Well Impairment

The previous section assumed that the formation properties were homogeneous throughout the grid block. In practice, the formation usually does not have uniform properties next to the well. The formation may have been invaded by the drilling mud solids, precipitation of pressure sensitive minerals may have resulted is scaling, the formation may have compacted due to pressure depletion, or the well may be cased and cemented with perforations providing the communication with the formation. The combined effect of the departure from a homogeneous formation is represented by a "skin factor" that can be determined by transient pressure analysis. It is assumed that the flow is axisymmetric and the permeability distribution is described as in Figure 8.2a.

The same assumptions are made as before except that the formation is
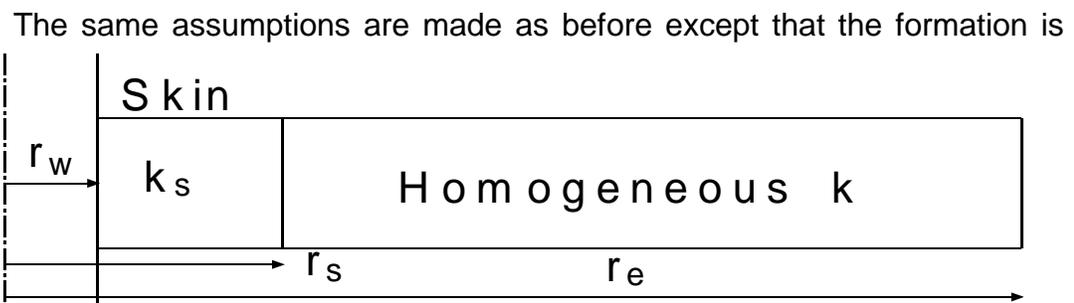
Fig. 8.2a Schematic of well with skin of modified permeability, k_S

homogeneous except for a region of radius, r_s, in which the permeability is k_s.

$$r_D \frac{k(r_D)}{k} \frac{dp_D}{dr_D} = -1 \tag{8.2a}$$

$$r_D \frac{dp_D}{dr_D} = \begin{cases} -k/k_s, & 1 < r_D < r_s/r_w \\ -1, & r_s/r_w < r_D < r_e/r_w \end{cases} \tag{8.2b}$$

$$dp_D = \begin{cases} -\dfrac{k}{k_s} d \ln r_D, & 1 < r_D < r_s/r_w \\ -d \ln r_D, & r_s/r_w < r_D < r_e/r_w \end{cases} \tag{8.2c}$$

$$p_D(r_s) - p_D(r_w) = -\frac{k}{k_s} \ln(r_s/r_w)$$
$$p_D(r_e) - p_D(r_s) = -\ln(r_e/r_s) \tag{8.2d}$$

$$p_D(r_e) - p_D(r_w) = -\frac{k}{k_s} \ln(r_s/r_w) - \ln(r_e/r_s)$$
$$= -\left(\frac{k}{k_s} - 1\right) \ln(r_s/r_w) - \ln(r_e/r_w) \tag{8.2e}$$

Define the skin factor, s, as follows.

$$s = \left(\frac{k}{k_s} - 1\right) \ln(r_s/r_w)$$
$$= \ln\left[(r_s/r_w)^{\left(\frac{k}{k_s} - 1\right)}\right] \tag{8.2f}$$

Substituting the skin factor into equation 8.2e results in a much simpler expression.

$$p_D(r_e) - p_D(r_w) = -\left[\ln(r_e/r_s) + s\right] \tag{8.2g}$$

The effect of the inhomogeneity is to add the "skin factor" to the $\ln(r_e/r_w)$ term of the inflow equation, (8.1h). It is called a skin because usually r_S is not very large, i.e., just a "skin" of reduced permeability around the well. Notice that the inhomogeneity was initially described by two parameters, r_S and k_S. These two parameters can not be determined uniquely from the well performance. Thus they are now combined into a single parameter, s.

The effect of the skin factor is be examined by defining an "effective well bore radius", r_w'.

$$\ln\left(r_e / r_w'\right) = \ln\left(r_e / r_w\right) + s \qquad\qquad (8.2h)$$

$$\begin{aligned}\ln r_w' &= \ln r_w - s \\ &= \ln\left(r_w\, e^{-s}\right)\end{aligned} \qquad\qquad (8.2i)$$

$$\begin{aligned}r_w' &= r_w\, e^{-s} \\ &= r_w\left(r_s / r_w\right)^{-\left(k/k_s - 1\right)}\end{aligned} \qquad\qquad (8.2j)$$

$$\begin{aligned}&\text{if} \quad k_s < k, \quad r_w' < r_w \\ &\text{if} \quad k_s = k, \quad r_w' = r_w \\ &\text{if} \quad k_s > k, \quad r_w < r_w' < r_s \\ &\text{if} \quad k_s \to \infty, \quad r_w' \to r_s\end{aligned} \qquad\qquad (8.2k)$$

These relations express the effective well radius as less than the actual radius if the permeability in the skin region is less than the formation permeability and greater than the actual radius if it is greater. Thus a well "stimulation" such as a hydraulic fracture results in an enlarged effective well radius.

8.3 Saturation Dependence

The coefficient for the well model included a relative permeability. The concept of upstream mobility weighting should be used for the well coefficient just as in calculating the transmissibility coefficients. If the flow is from the grid block to the well as in a production well, the upstream mobility is calculated from the relative permeability and viscosity of the fluids in the grid block. If more than one phase has a nonzero relative permeability, then more that one phase will be produced and the resulting fractional flow is in proportion to the mobility of the respective phase. If the flow is from the well to the grid block as in an injection well or an interval of a production well that is backflowing, then the only phase(s) that is flowing from the well is the phase(s) that is present in the well. The mobility of the injected phase(s) can be calculated either as the end point mobility of that phase or the total mobility of the fluids in the grid block. The former choice assumes that the resistance is dominated by the saturation next to the well bore and the latter choice assumes that the resistance is in series and is dominated by the fluids in the grid block. The need for an inexact choice arises because the saturation profile around the well is not uniform as assumed in section 8.1. After the displacement has progressed and only the injected fluid(s) is present in the grid block, either choice will give the same result.

The oil and water fraction entering a production well is a function of the saturation in the grid block from which the fluids enter. The IMPES procedure is explicit in saturation and this fractional flow may be expected to be calculated based on the saturation at the old time level (t_n). Explicit calculation of the well fraction flow has a time step size limitation beyond which oscillations will occur. This is more severe in cases of extreme mobility ratio, e.g. gas/oil flow. This time step size limitation can be overcome by making the production well fractional flow semi-implicit in saturation.

$$q_w = q \, f_w \left(S_n \right) + q \, \frac{df_w}{dS} \left(S_{n+1} - S_n \right)$$

When computing mass balance for the change in water saturation over the time step, the above equation should be substituted for the production rate. Time change of saturation appears in both the accumulation term and production term. The coefficients of the time change of saturation should be combined and the saturation computed.

$$z_{11} \, \Delta_t S_w = q_w - z_{10} \, \Delta_t p + \Delta \left[T_w \left(\Delta p_w - \rho_w \, g \, \Delta D \right) \right]$$

$$= q \, f_w \left(S_n \right) + q \, \frac{df_w}{dS} \left(S_{n+1} - S_n \right) - z_{10} \, \Delta_t p + \Delta \left[T_w \left(\Delta p_w - \rho_w \, g \, \Delta D \right) \right]$$

$$\left(z_{11} - q \, \frac{df_w}{dS} \right) \Delta_t S_w = q \, f_w \left(S_n \right) + - z_{10} \, \Delta_t p + \Delta \left[T_w \left(\Delta p_w - \rho_w \, g \, \Delta D \right) \right]$$

$$\Delta_t S_w = \frac{q \, f_w \left(S_n \right) + - z_{10} \, \Delta_t p + \Delta \left[T_w \left(\Delta p_w - \rho_w \, g \, \Delta D \right) \right]}{z_{11} - q \, \dfrac{df_w}{dS}}$$

$$q_w = q \, f_w \left(S_n \right) + q \, \frac{df_w}{dS} \, \Delta_t S_w$$

$$q_o = q \left[1 - f_w \left(S_n \right) \right] - q \, \frac{df_w}{dS} \, \Delta_t S_w$$

8.4 Well Common to Several Grid Blocks

A well may be completed in several grid blocks as in Figure 8.4a. If the bottom hole pressure, P_W, is specified, then the production from each grid block is given by equation 8.1h and the total rate from the well is the total from the grid blocks in which it is completed. However, if the total flow rate of the well is specified, then a model is needed to distribute the flow between the grid blocks. First, the grid blocks in which the wells are completed must be specified. The data required may be as follows.
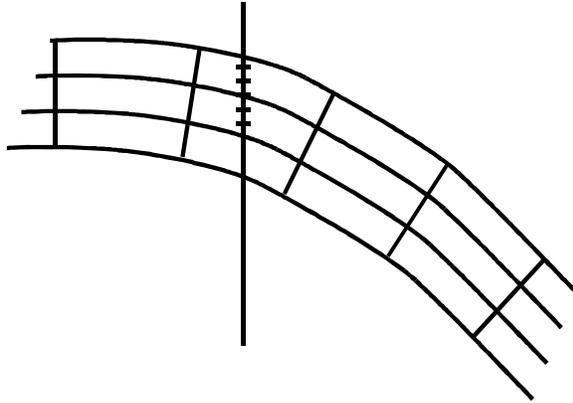
Fig. 8.4a Well with limited completion

Table 8.4a Well completion variables

Variable	Name	Range
well number	NW	1,2,...,NWT
completion interval	IC	1,2,...,ICT
I index of grid block	IW(NW,IC)	$1 \leq IW \leq NX$
J index of grid block	JW(NW,IC)	$1 \leq JW \leq NY$
K index of grid block	KW(NW,IC)	$1 \leq KW \leq NZ$
kh of completion in grid block	KH(NW,IC)	

The simplest way to distribute the flow between several grid blocks is to assume that the pressure drop between the well and the grid block pressure (pressure draw down) is the same for each grid block. In this case the total flow is distributed between the grid blocks in proportion to the mobility×thickness product. However, the assumption of equal pressure draw down may not be valid for a multi zone reservoir that is responding at different times to a water flood. In this case some zones may be producing while other zones may be back flowing. If the well pressure is specified, then the rates from (to) each grid block is calculated from equation 8.1h with the mobilities calculated as described in section 8.3. If the rate is specified, then the well pressure is an unknown and must be determined. The following describes how the well pressure and the rate into each grid block can be calculated if the well rate of one phase is specified. If the total liquid or total fluid (water, oil, and gas) rate is specified, then a summation over the phases will be required.

$$q = \sum_{IC=1}^{ICT} q_{IC}$$

$$= -\sum_{IC=1}^{ICT} PI_{IC} \left(P_{IW,JW} - P_w + \bar{\rho} g D_{IC} \right) \qquad (8.4a)$$

$$= -\sum_{IC=1}^{ICT} PI_{IC} P_{IW,JW} + P_w \sum_{IC=1}^{ICT} PI_{IC} - \sum_{IC=1}^{ICT} PI_{IC} \bar{\rho} g D_{IC}$$

$$P_w = \frac{q + \sum_{IC'=1}^{ICT} PI_{IC'} P_{IW,JW} + \sum_{IC=1}^{ICT} PI_{IC'} \bar{\rho} g D_{IC'}}{\sum_{IC'=1}^{ICT} PI_{IC'}} \qquad (8.4b)$$

$$q_{IC} = -PI_{IC} \left[P_{IW,JW} - \frac{q + \sum_{IC'=1}^{ICT} PI_{IC'} P_{IW,JW} + \sum_{IC'=1}^{ICT} PI_{IC'} \bar{\rho} g D_{IC'}}{\sum_{IC'=1}^{ICT} PI_{IC'}} + \bar{\rho} g D_{IC} \right] \qquad (8.4c)$$

where

D_{IC} is the depth of grid block relative to a reference depth

g is the gravitational constant

$P_{IW,JW}$ is the grid block pressure located at grid location, *IW, JW*

P_w is the well pressure relative to a reference depth

PI_{IC} is the well productivity (injectivity) index in grid block *IC*

q is the total rate from (to) the well

q_{IC} is the rate from (to) grid block, *IC*

$\bar{\rho}$ is the average density of the fluids in the well

Equation 8.4c gives an expression for calculating the distribution of the production (injection) between grid blocks assuming that the fluids in the well bore are in hydrostatic equilibrium with a average density, $\bar{\rho}$. It was mentioned earlier that if the pressure draw down is identical in each grid block, the production could be distributed in proportion to the productivity index, PI_{IC}, or in proportion to the mobility×thickness product. Equation 8.4c is needed when the draw down is not identical for all grid blocks in which the well is completed.

It was mentioned earlier that if the production (injection) rate is a function of the grid block pressure, the rate expression should be semi-implicit in pressure to preserve stability. Equation 8.4c has the production rate a function of the pressure of every grid block in which it is completed.   If the numerical calculations are made semi-implicit for each of these pressures, additional terms

may have to be added to the coefficient matrix. The rate expression made semi-implicit in pressure is as follows.

$$q_{IC} = -PI_{IC} \left[P_{IW,JW} - \frac{q + \sum_{IC'=1}^{ICT} PI_{IC'} P_{IW,JW}^n + \sum_{IC'=1}^{ICT} PI_{IC'} \bar{\rho} g D_{IC'}}{\sum_{IC'=1}^{ICT} PI_{IC'}} + \bar{\rho} g D_{IC} \right]$$

$$-PI_{IC} DP_{IW,JW} + PI_{IC} \frac{\sum_{IC'=1}^{ICT} PI_{IC'} DP_{IW,JW}}{\sum_{IC'=1}^{ICT} PI_{IC'}}$$

(8.4d)

If the well is completed in only <u>one</u> grid block, making the rate expression semi-implicit is identical to specifying the total rate to be the rate from that grid block (notice that everything cancels except the total rate). If the well is completed in more than the adjoining two grid blocks, additional terms will be needed for the coefficient matrix. An approximation with minimal recoding is to make the rate semi-implicit only with respect to the pressure of the current grid block. This will require adding a term to the diagonal of the coefficient matrix.

8.5 Vertical Lift Performance (VLP)

The previous section treated the bottom hole flowing pressure,(BHFP or p_{wf}) as if it was a specified parameter for calculating the inflow performance (IFP) of a well. However, the fluids are not produced until it reaches the surface facilities. The pressure drop in bringing the fluids from the bottom of the well to the surface is often an integral part of the simulation of reservoir performance. This pressure drop determines the well bottom hole pressure for a specified well head pressure.

The pressure drop in the well is the sum of two components: (1) the change in potential energy due to the earth's gravitational field and (2) the drag as a result of the flow of the fluids. The pressure drop in the well is usually illustrated as curves of depth versus pressure called "pressure traverse or gradient curves" with flow rate and/or gas/liquid ratio as parameters. Well configuration and pressure profiles are illustrated in figure 8.5a.

This example illustrates the pressure profiles for a case when all the produced fluids are flowing up the tubing compared with the case when the gas flows up the annulus and the liquid flows up the tubing after the pressure is increased by ΔP_{pump} with a pump. Pumping wells can usually be described in a reservoir simulation with a maximum liquid rate and minimum bottom hole well pressure constraints. Wells that are flowing unassisted or with gas lift usually require the vertical lift performance (VLP) to be calculated simultaneously with the well inflow performance (IFP).
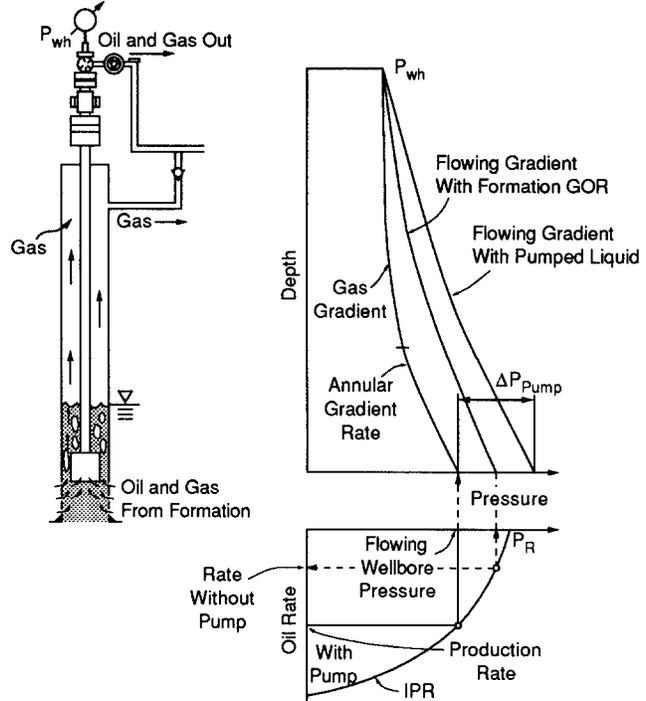
Fig. 8.5a   Well configuration and pressure profiles for an oil well [Economides, Hill, and Ehlig-Economides, 1994 (Golan and Whitson, 1991)]

The most important variable in determining pressure gradient due to both potential energy and drag is the local gas/liquid volumetric ratio in the tubing.   This ratio needs to be distinguished from the produced gas/liquid ratio (GLR) which is the ratio of the produced separator gas at standard conditions (MCF) and the produced stock tank barrels (STB).   Furthermore, the liquid is producing at a given water cut or water/oil ratio (WOR).   The local volumetric ratio will change with depth because of evolution of gas from the oil, expansion of the gas and the relative velocity of one fluid to the other.   The different gas-liquid flow regimes as a function of the gas superficial velocity on the abscissa and the superficial liquid velocity on the ordinate are illustrated in figure 8.5b.
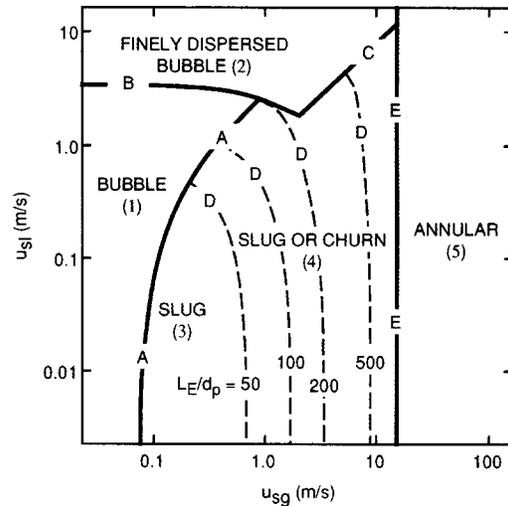
Fig. 8.5b   Flow regimes for gas-liquid flow in a vertical tube [Economides, Hill, and Ehlig-Economides, 1994 (Taitel, et al, 1976)]

Correlations are used to calculate the liquid or gas hold-up and drag coefficient in a given flow regime. The hold-up and drag coefficient determine the potential energy and drag contributions to the vertical pressure gradient. Calculated pressure traverses for a given tubing size, tubing head pressure, WOR, and liquid rate but different GLR may appear as in figure 8.5c. The curve

Fig. 8.5c Pressure traverses with different GLR (Economides, Hill, and Ehlig-Economides, 1994)

for zero GLR has the highest BHFP. It has a gradient that is close to the hydrostatic gradient of 0.433 psi/ft for a fluid with a density of 1.0 g/cm^3. Increasing the GLR reduces the average density of the fluids and the BHFP decreases. There is diminishing reduction in BHFP with increase in GLR because of increasing drag with additional gas flow.

The beneficial effect of gas in reducing the BHFP is the principle of **gas lift** design. Gas injected down the annulus of the well enters the tubing through valves near the bottom of the well. The resulting reduced average density reduces the BHFP.

Calculation of pressure traverses for different liquid flow rates at a given GLR results in a curve of BHFP versus the flow rate. This curve called the vertical lift performance curve is illustrated in figure 8.5d. It passes through a minimum and increases at low flow rates. At low rates the gas and liquid separate and the average density approach the liquid density. If the flow rate drops below the minimum in the curve, the production may jump to a lower rate or cease flowing.

IFP curves and the intersection are as follows.

Fig.   8.5d   Combination   of   inflow performance   (IFP)   and   vertical   lift performance (VLP) curves (Economides, Hill, and Ehlig-Economides, 1994

Th

$$P_{wf} = a + b\,q, \qquad \text{VLP curve}$$

$$P_{wf} = \frac{q + \sum_{IC'=1}^{ICT} PI_{IC'}\, P_{IW,JW} + \sum_{IC=1}^{ICT} PI_{IC'}\, \bar{\rho}\, g\, D_{IC'}}{\sum_{IC'=1}^{ICT} PI_{IC'}}$$

$$= a' + b'q, \qquad \text{IFP curve} \tag{8.5a}$$

$$q = -\frac{a - a'}{b - b'}$$

$$P_{wf} = a' - b'\left(\frac{a - a'}{b - b'}\right)$$

The flow from each grid block is as follows.

$$q_{IC} = -PI_{IC}\left(P_{i,j} - a' - b'\left(\frac{a - a'}{b - b'}\right)\right) \tag{8.5b}$$

8.6 **Oil-Water Coning Model**

We saw earlier that the detailed pressure profile near a well in a coarse Cartesian grid simulation model can be modeled by an analytical solution. We will see here how to model the detailed saturation profile near a well for use in a coarse grid, two dimensional simulation. If a reservoir satisfies the condition of vertical equilibrium (i.e., the vertical saturation profile at a point in the reservoir is determined by the conditions of hydrostatic equilibrium), then the reservoir can be simulated in two dimensions by using averaged or "pseudo" relative permeability and "pseudo" capillary pressure curves that are a function of the average saturation in the grid block. A well model is needed to represent the effect of a limited perforated interval and the water cone around the well. Figure 8.6a illustrates the saturation profile near a well that has a limited perforated interval in a reservoir with an original oil-water contact.
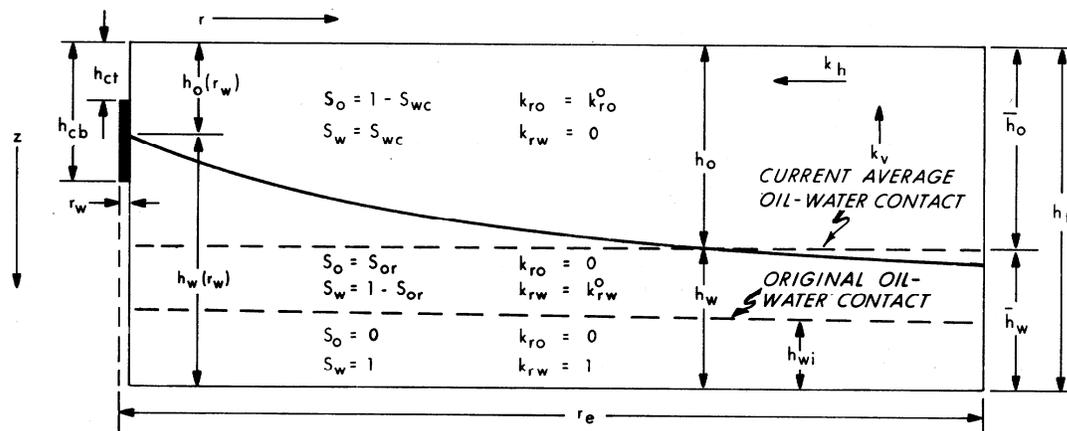
Fig. 8.6a Schematic of saturation profile near a well (Chappelear and Hirasaki, 1974, 1976)

Assumptions:

The geometric configuration for the coning model is a radially symmetric, homogeneous, anisotropic system with inflow at the outer boundary and with a partially perforated well. The fluid distribution is shown in Fig. 8.6a. The presence of initial water at 100% water saturation is considered. The perforated interval is assumed to be within the original oil column.

The fluids are assumed to be incompressible. The model is developed for steady-state flow. The transient time for the start of flow is short for most practical problems and, thus, the rise of the cone is represented as a succession of steady states.

The fluids are assumed to flow in segregated regions as shown in Fig. 8.6a. The fractional flow into the perforated interval is assumed to be only a

function of the fraction of the interval covered by each fluid and of the mobility ratio.

The fluids are assumed to be in vertical equilibrium everywhere except near the well bore. The departure from vertical equilibrium near the well caused by the vertical flow resistance is represented by an "effective radius." The expression for the effective radius represents the anisotropy through the vertical-to-horizontal permeability ratio.

The fluid flow equations are linearized by assuming that the average oil-column thickness over the drainage area can be used to compute the vertically averaged relative permeability function for the entire drainage area.

<u>The model</u>

The coning model is a relationship between the water cut and the average thickness of the oil column, given the total liquid rate and reservoir parameters.

$$
\begin{aligned}
&f_w^2 (1-M) + f_w \left\{ M - \frac{N_{mt}}{1+N_{mt}} \left[(1-M) \left(1 - \frac{q_c}{q_t} \frac{\bar{h}_o}{(\bar{h}_o - h_{cb})} \right) + \frac{q_c}{q_t} \frac{(h_{ct} - M\, h_{cb})}{(\bar{h}_o - h_{cb})} \right] \right\} \\
&- \frac{M\, N_{mt}}{1+N_{mt}} \left(1 - \frac{q_c}{q_t} \right) = 0
\end{aligned}
\tag{8.6a}
$$

where

$$M = \frac{k_{rw}^o \, \mu_o \, B_o}{k_{ro}^o \, \mu_w \, B_w},$$ end point mobility and formation volume factor ratio

$$N_{mt} = \frac{\bar{k}_{rw}^o \, \mu_o \, B_o}{\bar{k}_{ro}^o \, \mu_w \, B_w},$$ mobility thickness ratio

$$\bar{k}_{ro} = \frac{\bar{h}_o}{h_t} k_{ro}^o,$$ depth averaged oil relative permeability

$$\bar{k}_{rw} = \frac{1}{h_t} \left[h_{wi} + \left(h_t - \bar{h}_o - h_{wi} \right) k_{rw}^o \right],$$ depth averaged water relative permeability

$$q_c = \frac{2\pi\, h_t\, k_h\, \bar{k}_{ro}\, \Delta\rho\, g \left(\bar{h}_o - h_{cb} \right)}{887.2\, \mu_o\, \overline{\ln r}\,' B_o},$$ critical oil production rate

$$\overline{\ln r'} = \frac{\ln\left[\dfrac{r_e}{(r_w + r')}\right]}{\left[1 - \dfrac{(r_w + r')^2}{r_e^2}\right]} - \frac{1}{2},$$ average of ln r with effective radius correction

$$\approx \ln\left[\frac{r_e}{(r_w + r')}\right] - \frac{1}{2}$$

$$r' = 4\overline{h}_o \sqrt{k_h/k_v}\left(\left|\frac{\overline{h}_o - h_{cb}}{\overline{h}_o - h_{ct}}\right| \cdot \frac{\overline{h}_o + h_{ct}}{\overline{h}_o + h_{cb}}\right)^{\frac{\overline{h}_o}{2(h_{cb} - h_{ct})}}$$ effective well bore radius

Limiting Cases

Water cut is zero below the critical rate.

$$f_w = 0 \quad \Leftrightarrow \quad q_t \leq q_c,$$

At limit of high rates, the WOR is equal to the mobility thickness ratio.

$$q_t \gg q_c \quad \Rightarrow$$
$$f_w = \frac{N_{mt}}{1 + N_{mt}}, \quad \text{or}$$
$$WOR = N_{mt}$$

A simple expression results for small interval and unit mobility ratio.

$$h_{cb} \approx h_{ct} \quad and \quad M = 1 \quad \Rightarrow$$
$$f_w = \frac{N_{mt}}{1 + N_{mt}}\left(1 - \frac{q_c}{q_t}\right)$$

8-18

Comparison of model with numerical simulation

The coning model is compared with a semi-implicit R-Z numerical simulator to test the validity of the model. Comparison for the base case is shown in Fig. 8.6b. The base case has an original oil column of 40 feet and underlying water of 40 feet. The water cut is shown as a function of the average water saturation. The simulation and coning model are shown as solid and dashed curves, respectively. The curve of "without gravity" is the limiting water cut that would result if production rate is very high compared to the critical rate. The flat interface case is the water cut that would result if there was a flat interface (no cone) and water is produced only when interface is up to the perforations.
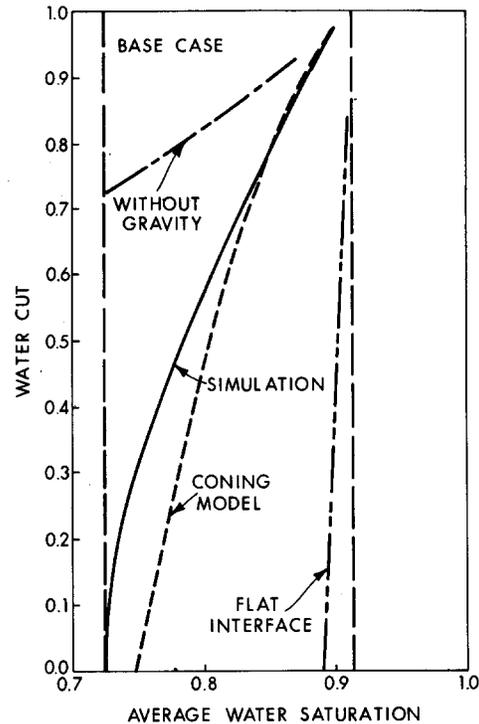
Fig. 8.6b Comparison of coning model with simulation (Chappelear and Hirasaki, 1974, 1976)

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