Chapter 7 Reservoir Geometry and Properties

Reading assignment: Chapter 3 in Reservoir Simulation

If reservoirs were rectangular parallelpipeds then they could be modeled with Cartesian coordinates. However, nature is not as orderly as the laboratory machine shop and thus the reservoir model must conform with the natural boundaries of the reservoir. Also, when modeling the details of a single well, the well is a cylindrical surface that is a boundary for the reservoir. Coordinates that conform to the curved reservoir boundaries are curvilinear coordinates.

7.1 Curvilinear Coordinates

The coordinates in a curvilinear system may not have units of distance. e.g., coordinates in cylindrical polar and spherical polar coordinates may be an angle. The metric tensor relates distance to the infinitestimal coordinate increments. Denote y^i as a Cartesian system of coordinates and x^i as a curvilinear system of coordinates. The distance between two points with coordinates y^i and $y^i + dy^i$ is ds, where

$$ds^2 = \sum_{k=1}^{3} dy^k\, dy^k \tag{7.1a}$$

However,

$$dy^k = \frac{\partial y^k}{\partial x^i} dx^i \tag{7.1b}$$

,where summation is understood between a pair of upper and lower indicies, hence

$$\begin{aligned} ds^2 &= \sum_{k=1}^{3}\left(\frac{\partial y^k}{\partial x^i} dx^i\right)\left(\frac{\partial y^k}{\partial x^j} dx^j\right) \\ &= g_{ij}\, dx^i\, dx^j \end{aligned} \tag{7.1c}$$

where

$$g_{ij} = \sum_{k=1}^{3} \frac{\partial y^k}{\partial x^i} \frac{\partial y^k}{\partial x^j} \tag{7.1d}$$

g_{ij} is called the *(covariant) metric tensor* since it relates distance to the infinitestimal coordinate increments (Aris, 1962).

For example for the ***cylindrical polar*** coordinates

$$
\begin{aligned}
x^1 &= \sqrt{\left(y^1\right)^2 + \left(y^2\right)^2} && = r \\
x^2 &= \tan^{-1}\frac{y^2}{y^1} && = \theta \\
x^3 &= y^3 && = z
\end{aligned}
\tag{7.1e}
$$

we have

$$
\begin{aligned}
&g_{11} = 1, \quad g_{22} = \left(x^1\right)^2, \quad g_{33} = 1 \\
&\text{and all} \quad g_{ij} = 0, \quad i \neq j
\end{aligned}
\quad \text{, cylindrical polar coordinates} \tag{7.1f}
$$

The divergence of the gradient or the Laplacian of a scaler is given by the following expression.

$$
\nabla \cdot \nabla \Phi = \nabla^2 \Phi = \frac{1}{g^{1/2}} \frac{\partial}{\partial x^j} \left\{ g^{1/2} \, g^{ij} \, \frac{\partial \Phi}{\partial x^i} \right\} \tag{7.1g}
$$

where g^{ij} is *contravariant metric tensor* which is equal to the inverse of the matrix of g_{ij} and g is the determinate of g_{ij}. g is also equal to the square of the Jacobian of the transformation from Cartesians y to the coordinates x. If the coordinate system is orthogonal, the metric tensor will have nonzero term only along the diagonal. If the coordinate system is not orthogonal, the equation 7.1g will have cross partial terms. Thus an orthogonal curvilinear coordinate system should be used unless the algorithm includes cross partial terms.

A differential element of volume transforms between Cartesian and curvilinear coordinates as follows.

$$
\begin{aligned}
dV &= dy^1 \, dy^2 \, dy^3 \\
&= J \, dx^1 \, dx^2 \, dx^3 \\
&= g^{1/2} \, dx^1 \, dx^2 \, dx^3
\end{aligned}
\tag{7.1h}
$$

To illustrate the form of the finite difference expression, the x^1 term of the Laplacian will be expressed in finite differences.

$$
\frac{1}{g^{1/2}} \frac{\partial}{\partial x^1} \left\{ g^{1/2} g^{11} \frac{\partial \Phi}{\partial x^1} \right\}
$$

$$
\approx \frac{1}{g^{1/2} \Delta x^1 \Delta x^2 \Delta x^3} \left\{ \begin{array}{l} \left(g^{1/2} g^{11} \right)_{i+1/2} \dfrac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i+1/2}} \Phi_{i+1} \\ -\left[\left(g^{1/2} g^{11} \right)_{i+1/2} \dfrac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i+1/2}} + \left(g^{1/2} g^{11} \right)_{i-1/2} \dfrac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i-1/2}} \right] \Phi_i \\ + \left(g^{1/2} g^{11} \right)_{i-1/2} \dfrac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i-1/2}} \Phi_{i-1} \end{array} \right\} \tag{7.1i}
$$

These coefficients are the grid block bulk volume and transmissibility coefficients (without permeability).

$$
\begin{aligned}
V &= g^{1/2} \Delta x^1 \Delta x^2 \Delta x^3 \\
T^1_{i+1/2} &= \left(g^{1/2} g^{11} \right)_{i+1/2} \frac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i+1/2}} \\
T^1_{i-1/2} &= \left(g^{1/2} g^{11} \right)_{i-1/2} \frac{\Delta x^2 \Delta x^3}{\Delta x^1{}_{i-1/2}}
\end{aligned} \tag{7.1j}
$$

These coefficients are analogous to those with Cartesian coordinates except for the metric tensor. The finite difference expression with these latter coefficients appear the same as with Cartesian coordinates.

$$
\frac{1}{g^{1/2}} \frac{\partial}{\partial x^1} \left\{ g^{1/2} g^{11} \frac{\partial \Phi}{\partial x^1} \right\} = \frac{1}{V} \left[T^1_{i+1/2} \Phi_{i+1} - \left(T^1_{i+1/2} + T^1_{i-1/2} \right) \Phi_i + T^1_{i-1/2} \Phi_{i-1} \right] \tag{7.1k}
$$

7.2 **Cylindrical r-z Coordinates**

Problems that are axisymmetric can be modeled with cylindrical $(x^1, x^2, x^3) = (r, \theta, z)$ coordinates. Since there is no dependence on the θ coordinate, Δx^2 is replaced with 2π. The components of the metric tensor and its determinant are as follows.

$$
\begin{aligned}
&g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = 1, \quad g = r^2 \\
&g^{11} = 1, \quad g^{22} = r^{-2}, \quad g^{33} = 1
\end{aligned} \tag{7.2a}
$$

The grid block volume and transmissibility coefficients (without permeability) are as follows.

$$
\begin{aligned}
V &= 2\pi\,\overline{r_i}\,\Delta r\,\Delta z \\
T^r_{i+1/2} &= \frac{2\pi\, r_{i+1/2}\,\Delta z}{\Delta r_{i+1/2}} \\
T^z_{k+1/2} &= \frac{2\pi\,\overline{r_i}\,\Delta r}{\Delta z_{k+1/2}}
\end{aligned}
\tag{7.2b}
$$

The mean radius, $\overline{r_i}$, must be defined such that it will yeild the correct grid volume and area for T^z.

$$
\begin{aligned}
2\pi\,\overline{r_i}\,\Delta r &= 2\pi\,\overline{r_i}\left(r_{i+1/2} - r_{i-1/2}\right) \\
&= \pi\, r^2_{i+1/2} - \pi\, r^2_{i-1/2} \\
&\Rightarrow \\
\overline{r_i} &= \frac{r_{i+1/2} + r_{i-1/2}}{2}
\end{aligned}
\tag{7.2c}
$$

The evaluation of $\Delta r_{i+1/2}$ depends on where $\Phi_i \text{ and } \Phi_{i+1}$ are evaluated. If they are evaluated at the arithemetic mean radius, then $\Delta r_{i+1/2}$ is evaluated as in a block centered grid. Other possible locations for evaluating the potentials is at the controid of the grid block or or the log mean radius. Numerical experiments showed that the transmissibility calculated based on the potential evaluated at the log mean (geometric mean) radius gives the best comparison with the analytical solution. Settari and Aziz (1974) suggests using a grid point centered grid and locating the grid block boundaries at the logarithmic mean radius in r^2.

The expression for $\Delta r_{i+1/2}$ assuming that the pressure is evaluated at the log mean radius is given below.

$$
\begin{aligned}
r_i &= \sqrt{r_{i-1/2}\left(r_{i-1/2} + \Delta r_i\right)} = r_{i-1/2}\sqrt{1 + \Delta r_i / r_{i-1/2}} \\
r_{i-1} &= \sqrt{r_{i-1/2}\left(r_{i-1/2} - \Delta r_{i-1}\right)} = r_{i-1/2}\sqrt{1 - \Delta r_{i-1} / r_{i-1/2}} \\
r_i - r_{i-1} &= r_{i-1/2}\left[\sqrt{1 + \Delta r_i / r_{i-1/2}} - \sqrt{1 - \Delta r_{i-1} / r_{i-1/2}}\right]
\end{aligned}
\tag{7.2d}
$$

The expression for the radial grid dependent terms of the transmissibility is as follows.

$$\bar{k}_{i-1/2} = \frac{r_i - r_{i-1}}{\frac{r_i - r_{i-1/2}}{k_i} + \frac{r_{i-1/2} - r_{i-1}}{k_{i-1}}}$$

$$\frac{r_{i-1/2}}{r_i - r_{i-1}} \bar{k}_{i-1/2} = \frac{k_i \, k_{i-1}}{k_{i-1}\left(\sqrt{1+\Delta r_i / r_{i-1/2}} - 1\right) + k_i\left(1 - \sqrt{1 - \Delta r_{i-1} / r_{i-1/2}}\right)} \tag{7.2e}$$

The use of Equation 7.2d or 7.2e for the transmissibility coefficient gave a much better comparison with an analytical solution compared to assuming that the pressure was evaluated at the midpoint of the grid block. When plotting the pressure profile it did not make much difference whether it was assumed that the pressure was evaluated at the log mean radius, arithmetic mean radius, or the centroid.

The choice of radial grid spacing depends on the error that is dominating. Near the well the pressure profile is a linear function of the logarithm of the radial distance. Here the discretization error in the profile is reduced by using a grid that is evenly spaced in the logarithm of the radial distance. Near the external boundary the approach to semi-steady state is affected by the difference between the location of where in the grid block the pressure is evaluated and the controid of the grid block. In coning problems the grid spacing is determined to obtain resolution of the shape of the cone.

The following is an algorithm for calculating the grid spacing for equal spacing in the logarithm of the radial distance.

$$\begin{aligned}
\Delta \ln r &= \ln\left(r_e / r_w\right) / NX = \ln\left[\left(r_e / r_w\right)^{1/NX}\right] \\
&= \ln\left(r_{i+1/2} / r_{i-1/2}\right) \\
r_{i+1/2} &= r_{i-1/2} \exp\left(\Delta \ln r\right) = r_w \exp\left(i \, \Delta \ln r\right) = r_w \left(r_e / r_w\right)^{i/NX} \\
\Delta r_i &= r_{i+1/2} - r_{i-1/2} \\
&= r_w \left(r_e / r_w\right)^{\left(\frac{i-1}{NX}\right)} \left[\left(r_e / r_w\right)^{\left(\frac{1}{NX}\right)} - 1\right]
\end{aligned} \tag{7.2f}$$

Assignment 7.1 Pressure response with cylindrical coordinates. Simulate the system of assignments 6.4 and 6.5 using cylindrical coordinates. We now have all four quadrants so adjust the rates accordingly. Choose the external radius as to have the same area as assignment 6.4 and 6.5. Let the well radius be 0.4 feet. Use equal grid spacing in the logarithm of radial distance. Let NX=10 and 20. Compare the profiles (at 1, 3, 10, 30, and 50 days) and history of the pressure at the well with the analytical solution. Use *gjh/class/asg7_1.m* and *alf.dat* for plotting.

The pressure at the well is calculated from the pressure in the first grid block by the following equations for a well in an infinite domain.

$$
\begin{aligned}
p(r_1,t) &= p_i - \frac{q\,\mu}{2\pi\,k\,h}\ln(r_1) + \frac{q\,\mu}{4\pi\,k\,h}\ln\left(\frac{4k\,t}{1.78\phi\,\mu\,c}\right) \\
p(r_w,t) &= p_i - \frac{q\,\mu}{2\pi\,k\,h}\ln(r_w) + \frac{q\,\mu}{4\pi\,k\,h}\ln\left(\frac{4k\,t}{1.78\phi\,\mu\,c}\right) \qquad (7.2g) \\
p(r_w,t) &= p(r_1,t) - \frac{q\,\mu}{2\pi\,k\,h}\ln(r_w/r_1)
\end{aligned}
$$

The analytical solution for the bounded circular reservoir is given on page 11 of Matthews and Russell, <u>Pressure Buildup and Flow Tests in Wells</u>.

$$
p(r,t) = p_i + \frac{q\,\mu}{2\pi\,k\,h}\left\{
\begin{aligned}
&\frac{2}{r_{eD}^2-1}\left(\frac{r_D^2}{4}+t_{Dw}\right) - \frac{r_{eD}^2\ln r_D}{r_{eD}^2-1} \\
&-\frac{3r_{eD}^4-4r_{eD}^4\ln r_{eD}-2r_{eD}^2-1}{4\left(r_{eD}^2-1\right)^2} \\
&+\pi\sum_{n=1}^{\infty}\frac{\exp\left(-\alpha_n^2 t_{Dw}\right)J_1^2(\alpha_n r_{eD})\left[J_1(\alpha_n)Y_0(\alpha_n r_D)-Y_1(\alpha_n)J_0(\alpha_n r_D)\right]}{\alpha_n\left[J_1^2(\alpha_n r_{eD})-J_1^2(\alpha_n)\right]}
\end{aligned}
\right\}
$$

(7.2h)

where

$$
r_D = \frac{r}{r_w}, \quad r_{eD} = \frac{r_e}{r_w}, \quad t_{Dw} = \frac{k\,t}{\phi\,\mu\,c\,r_w^2} \qquad (7.2i)
$$

and α_n values are roots of

$$
J_1(\alpha_n r_{eD})Y_1(\alpha_n) - J_1(\alpha_n)Y_1(\alpha_n r_{eD}) = 0. \qquad (7.2j)
$$

7.3 Reservoir Geometry

Aquifers and petroleum reservoirs are natural features that have boundaries that do not conform easily to a Cartesian coordinate system. A reservoir prototype may be a representative element of a reservoir modeled as a rectangular parallelpipid with Cartesian coordinates that are rotated with respect to the sea level. More often reservoirs can not be modeled with planer boundaries and the shape of the reservoir must be taken into account. Figure 7.3a is an example of cross sectional and three dimensional grids. Here the reservoir structure map (contours of the subsea depth) is projected on to the top surface of the reservoir. The grid extending through the thickness of the reservoir is projected vertically downward. It is clear that this grid is not orthogonal and cross derivatives will be required to properly describe the flux. Usually, these cross derivatives are neglected.

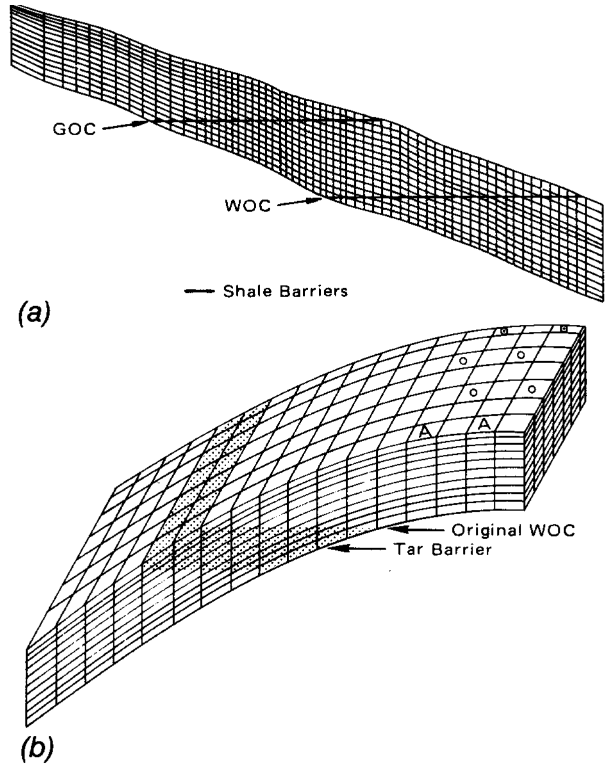

Fig. 7.3a Example grid system used in reservoir simulation studies: (a) cross-sectional model and (b) 3D model (Mattax and Dalton, 1990)

Fig. 7.3b is a curvilinear coordinate system in which the Cartesian coordinates of the reference plane are projected on to the reservoir and the top and bottom reservoir surfaces are coordinate surfaces of the third coordinate. The coordinate axis of the third coordinate is thus normal or orthogonal to the reservoir surfaces (Hirasaki and O'Dell, 1970). This system is much closer to being orthogonal than the system illustrated in Fig. 7.3a.

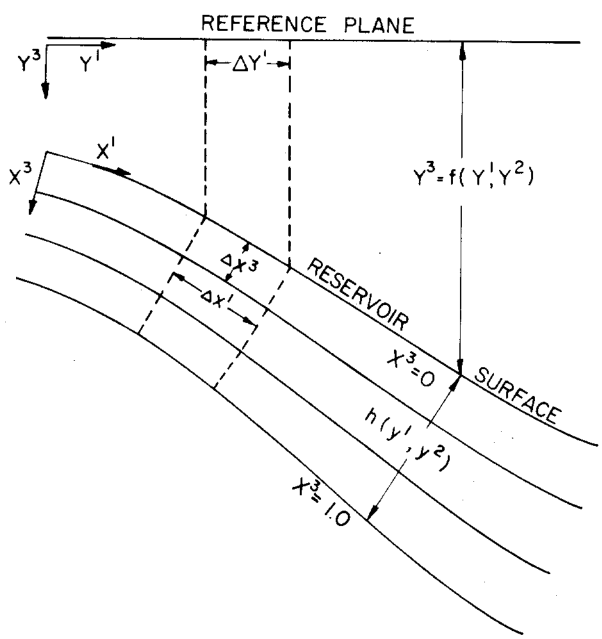

Fig. 7.3b Curvilinear coordinate system with x^3 coordinate normal to reservoir surface (Hirasaki and O'Dell, 1970)

The reservoir geometry data that is usually available are contour maps of reservoir thickness and the depth of the top reservoir surface trom sea level (Fig. 7.3c). Usually these contour maps have either sea level as a reference plane or a plane that is approximately parallel with the reservoir if the reservoir is steeply dipping. A system of Cartesian coordinates, (y^1, y^2, y^3), may be defined such that the coordinate surface, $y^3 = 0$, coincides with the reference plane. Position on the reference plane may be determined by the coordinates, ($y^1, y^2, 0$). If the reference plane is sea level, then y^3 is the depth from sea level; however, if the reference plane is other than sea level, then the relation between y^3 and depth is slightly more complex. In either case the reservoir thickness and the distance of the top reservoir surface from the reference plane is a function of the coordinates, (y^1, y^2), on the contour map.

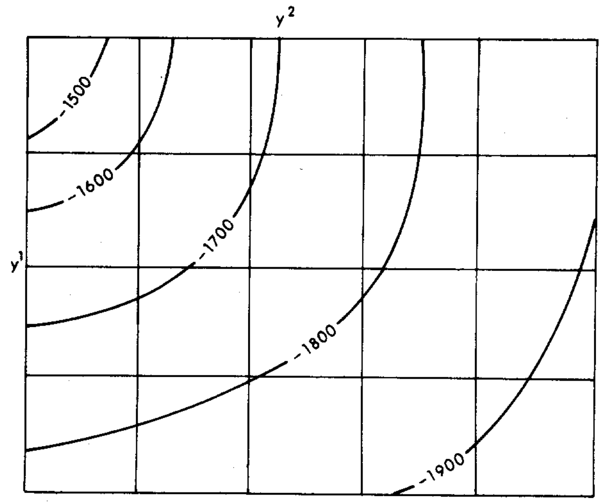

Fig. 7.3c Contour map of reservoir structure with superimposed Cartesian grid (Hirasaki and O'Dell, 1970)

Denote the system of curvilinear coordinates for the reservoir as (x^1, x^2, x^3). We may specify the coordinate system to have surfaces that coincide with the reservoir surface by defining the surfaces $x^3 = 0$ and $x^3 = 1.0$ to coincide with the top and bottom reservoir surfaces, respectively (Fig. 7.3b). Since the coordinates (y^1, y^2) have already been defined for the reference plane, it provides a convenient means for specifying the coordinates, (x^1, x^2), on the top reservoir surface. On the top reservoir surface let

$$x^1 = y^1, \quad x^2 = y^2, \quad x^3 = 0 \qquad (7.3a)$$

Eq. 7.3a simply define (x^1, x^2) on the top reservoir surface to be the projection of (y^1, y^2) from the reference plane on to the top of the reservoir. This definition is convenient, as a coordinate grid on the reference plane will project on to the same grid on the top reservoir surface (Fig. 7.3d). Another restriction that we may place on our coordinate system is for x^3 coordinate lines to be orthogonal to the coordinate lines of x^1 and x^2. This can be accomplished by making the x^3 coordinate lines normal to the top and bottom reservoir surfaces. However, the x^1 and x^2 coordinate lines may not be orthogonal. The coordinate lines of x^1 and x^2 will be orthogonal if and only if the dip is zero in one of the coordinate directions. The effect of x^1 and x^2 not being orthogonal is discussed in Hirasaki and O'Dell, 1970.

The coordinate grid needs to be computed for the interior of the reservoir. A coordinate grid specified for the (y^1, y^2) coordinate on the reference plane may be used to specify the (x^1, x^2) coordinate grid on the top surface of the reservoir. The coordinate intervals in the direction normal to the top reservoir surface, Δx^3, needs to be specified. The coordinate grid for (x^1, x^2, x^3) define a set of "grid cells" or "grid blocks" for the reservoir. We will refer to the center of the grid blocks as "grid points ".

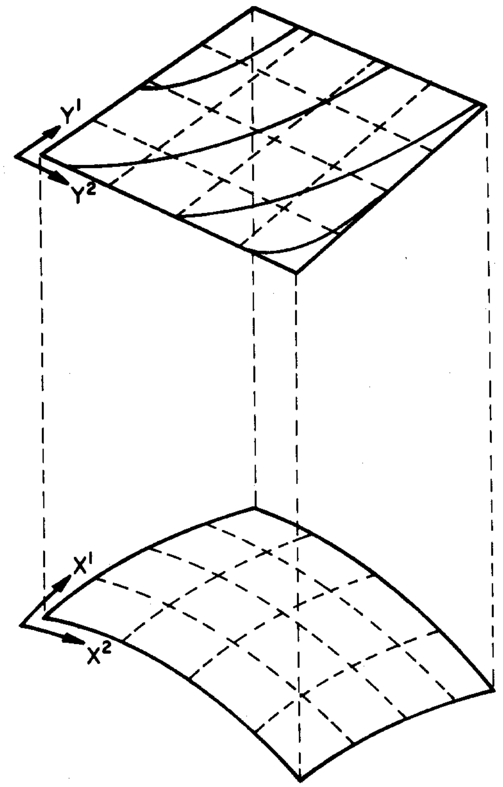

Fig. 7.3d A Cartesian coordinate grid (y^1, y^2) on the reference plane is projected on to the topmost reservoir surface to define a curvilinear coordinate grid, (x^1,x^2). (Hirasaki and O'Dell, 1970)

A point on the top reservoir surface has its position in space determined in terms of the Cartesian coordinate system of the reference plane. For example, a point (x^1, x^2, 0) on the top reservoir surface has the Cartesian coordinate position, (y^1, y^2, $D(y^1, y^2)$], where $D(y^1, y^2)$ is the depth of the top reservoir surface if the sea level is the reference plane. The local dip and the direction normal to the reservoir surface may be determined by computing the spatial derivatives of depth on the surface. The direction normal to the reservoir surface is in the direction of the x^3 coordinate line. The Cartesian coordinate positions of grid points in the interior of the reservoir may be computed by integrating along the x^3 coordinate lines (Fig. 7.3b).

The calculations are illustrated for integration from the top surface of a reservoir that is thin compared to the minimum radius of curvature. Let D be the depth and h be the thickness measured in the direction normal to the reservoir surfaces.

$$\frac{\partial y^1}{\partial x^3} = \frac{-h\dfrac{\partial D}{\partial y^1}}{\left[\left(\dfrac{\partial D}{\partial y^1}\right)^2 + \left(\dfrac{\partial D}{\partial y^2}\right)^2 + 1\right]^{1/2}}$$

$$\frac{\partial y^2}{\partial x^3} = \frac{-h\dfrac{\partial D}{\partial y^2}}{\left[\left(\dfrac{\partial D}{\partial y^1}\right)^2 + \left(\dfrac{\partial D}{\partial y^2}\right)^2 + 1\right]^{1/2}} \tag{7.3b}$$

$$\frac{\partial y^3}{\partial x^3} = \frac{h}{\left[\left(\dfrac{\partial D}{\partial y^1}\right)^2 + \left(\dfrac{\partial D}{\partial y^2}\right)^2 + 1\right]^{1/2}}$$

The determinant of the metric tensor is

$$g = \left\{\left[1+\left(\frac{\partial D}{\partial y^1}\right)^2\right]\left[1+\left(\frac{\partial D}{\partial y^2}\right)^2\right] - \left(\frac{\partial D}{\partial y^1}\right)^2\left(\frac{\partial D}{\partial y^2}\right)^2\right\}h^2 \tag{7.3c}$$

and the conjugate metric tensor is

$$\left(g^{ij}\right) = \begin{bmatrix} g^{-1}\left[1+\left(\dfrac{\partial D}{\partial y^2}\right)^2\right]h^2 & -g^{-1}\dfrac{\partial D}{\partial y^1}\dfrac{\partial D}{\partial y^2}h^2 & 0 \\ -g^{-1}\dfrac{\partial D}{\partial y^1}\dfrac{\partial D}{\partial y^2}h^2 & g^{-1}\left[1+\left(\dfrac{\partial D}{\partial y^1}\right)^2\right]h^2 & 0 \\ 0 & 0 & h^{-2} \end{bmatrix}. \tag{7.3d}$$

A coordinate system is orthorgonal if and only if

$$g^{ij} = 0, \quad i \neq j \tag{7.3e}$$

Thus, we see that the coordinate system on the top surface will be orthogonal if and only if the reservoir has zero dip in one of the coordinate directions. From Eq. 7.3d we see that the off-diagonal terms of the conjugate metric tensor on the surface will all be zero if and only if the reservoir has zero dip in one coordinate direction (i.e., if the coordinate system is orthogonal). If there are non-zero off-diagonal terms of the conjugate metric tensor, then the conservation equations will have non-zero cross partial derivative terms.

Once the Cartesian coordinate positions are known, the metric tensor may be computed from the derivative of (y^1, y^2 y^3) with respect to (x^1, x^2, x^3). The metric tensor is a matrix which relates increments of distance, area and volume to products of coordinate increments. It may be interpreted as a "shape operator" which relates a curved grid block in a curvilinear coordinate system to a rectangular grid block in a Cartesian coordinate system. The metric tensor is needed for the computation of the pore volume and transmissibility coefficient of a grid block in a curvilinear coordinate system.

Sharp and Anderson (1990, 1993) have developed a method to compute a grid that conforms with external and internal boundaries and is as orthorgonal as possible. Figures 7.3e and 7.3f illustrates two of their grids. Aziz (1993) reviewed the state of art in reservoir gridding. Figures 7.3g and 7.3h illustrates some grids that have been introduced.

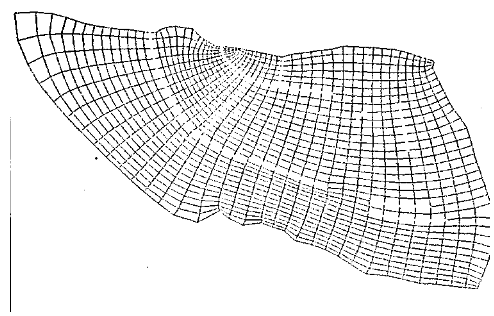

Fig. 7.3e Boundary-conforming full-field grid with faults (Sharp and Anderson, 1990, 1993)

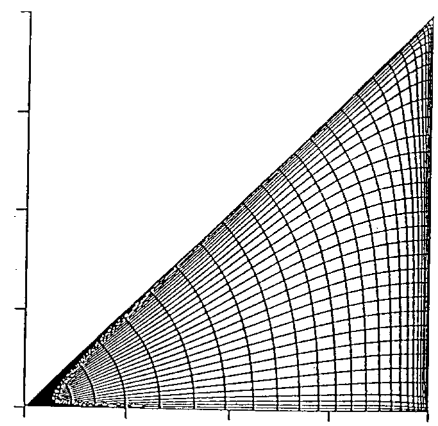

Fig 7.3f Boundary-conforming grid for 1/8 9-spot (Sharp, 1993)

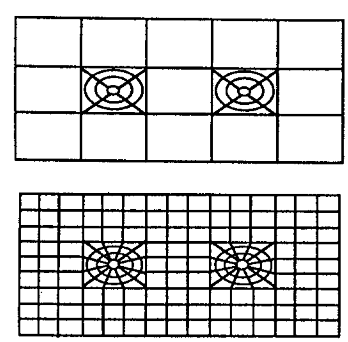

Fig. 7.3h Examples of hybrid grid (Aziz, 1993)

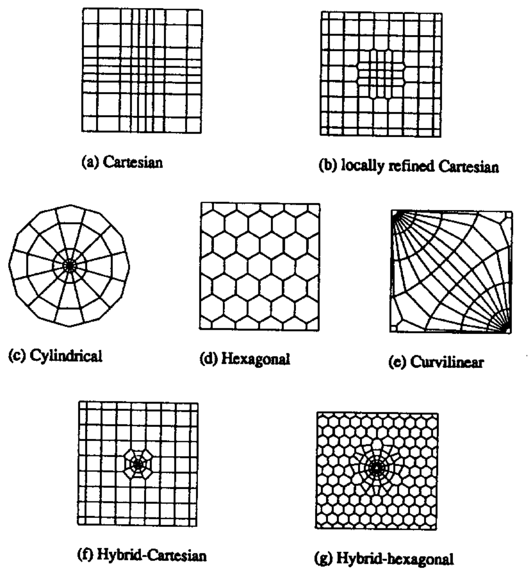

Fig. 7.3g Examples of Voronoi grid (Aziz, 1993)

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