

Chapter 5 Generalized Material Conservation Equations and Special Cases

The basic concepts discussed earlier will be generalized to the multiphase and multicomponent flow systems that occur in reservoir recovery processes. The mass conservation equations will be derived as a more general material conservation equations. The form of the equations will be illustrated for the various material quantities being conserved.

A Generalized Material Conservation Equation

The fluid flow equations in mathematical reservoir simulators consists of two basic types of equations: the equations for the conservation of momentum which is just a generalization of Darcy's law and the equations for the conservation of mass or material. The material quantity being conserved is not necessarily in mass units. For example, the quantity being conserved may be stock tank barrels (STB) of liquid hydrocarbons at stock tank conditions (14.7 psia and 60°F) produced from the separator. A barrel is a unit of volume equal to 5.615 ft³, 42 U.S. gallons, or 0.15899 m³. The mass of one STB of oil will not change unless the specific gravity or API gravity changes during the depletion of the reservoir. Other conserved quantities may include STB of water, MCF of gas, or moles of individual components or groups of components. These quantities will be equivalent to an unit of mass if the specific gravity or average molecular weight does not change. [The "gravity" of an oil is generally expressed in "API degrees," computed from the specific gravity at 60°F, ρ , by the relation $^{\circ}\text{API} = (141.5/\rho) - 131.5$.]

A general concentration will be denoted as C . The concentration, C , is defined as the amount of the conserved quantity per unit volume (bulk) of the system. For example, the concentration of stock tank oil per unit of bulk volume of the reservoir is

$$C_{oil} = b_o S_o \phi \left(\frac{\text{STB}}{\text{bbl of bulk reservoir volume}} \right)$$

where

- S_o oil saturation
- ϕ formation porosity
- b_o reciprocal oil formation volume factor, STB/RB

A general source term will be denoted as \tilde{q} . It is defined as the injection rate or the negative of the production rate per unit of bulk reservoir volume. It may be expressed as the amount of the quantity injected or produced per unit volume per unit time. The source term will be used to represent wells in reservoirs in the differential form of the conservation equations.

A general flux term will be denoted as the vector, \vec{f} . The flux can be interpreted as the rate of transport or flow rate of a quantity per unit area. Since the rate of transport per unit area depends on the orientation of the surface, the value of \vec{f} in any direction may be interpreted as the rate of transport across a unit of area with its normal in that direction. Thus, if we have a Cartesian coordinate system, the components of \vec{f} , f_x , f_y , and f_z can be interpreted as the rate of transport per unit of area in the x, y, and z directions respectively. An example of a flux is the mass flux vector given by

$$\vec{f}_{mass} = \rho \vec{u}$$

where

ρ	fluid density, mass/volume
\vec{u}	superficial fluid velocity, length/time or volume/(area-time)
\vec{f}	mass flux, mass/(area-time).

To illustrate a derivation of the generalized conservation equation, consider a small element of volume in Cartesian coordinates. The element is a parallelepiped with a volume equal to $\Delta x \Delta y \Delta z$. A material balance over this element of volume over a time interval Δt can be expressed as

$$\text{ACCUMULATION} = \text{IN} - \text{OUT} + \text{SOURCE}$$

The ACCUMULATION term represents the accumulation or gain of the conserved quantity in the element of volume over the time interval from t to $t + \Delta t$.

$$\text{ACCUMULATION} = (C|_{t+\Delta t} - C|_t) \Delta x \Delta y \Delta z$$

The IN-OUT terms represent the net influx of the material through the surfaces bounding the element of volume during the time interval, Δt . This may be expressed as

$$\begin{aligned} \text{IN} - \text{OUT} = & \left(f_x|_x - f_x|_{x+\Delta x} \right) \Delta y \Delta z \Delta t \\ & + \left(f_y|_y - f_y|_{y+\Delta y} \right) \Delta x \Delta z \Delta t \quad . \\ & + \left(f_z|_z - f_z|_{z+\Delta z} \right) \Delta x \Delta y \Delta t \end{aligned}$$

If f_x is positive, the first term of the sum may be interpreted as the transport of material into the volume across the face at x minus the transport of the material out of the volume at the face at $x + \Delta x$.

The SOURCE term represents the addition of material into the volume over the time interval, Δt , due to the sources. This may be expressed as

$$\text{SOURCE} = \tilde{q} \Delta x \Delta y \Delta z \Delta t \quad .$$

Substitution of the above equations into the conservation equation and division by $\Delta x \Delta y \Delta z \Delta t$ results in

$$\frac{C|_{t+\Delta t} - C|_t}{\Delta t} = -\frac{f_x|_{x+\Delta x} - f_x|_x}{\Delta x} - \frac{f_y|_{y+\Delta y} - f_y|_y}{\Delta y} - \frac{f_z|_{z+\Delta z} - f_z|_z}{\Delta z} + \tilde{q} \quad .$$

Taking the limit as $\Delta x, \Delta y, \Delta z,$ and Δt goes to zero results in

$$\frac{\partial C}{\partial t} = -\frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} - \frac{\partial f_z}{\partial z} + \tilde{q} \quad .$$

The sum of derivatives of the components of f can be represented as the divergence of f .

$$\frac{\partial C}{\partial t} = -\nabla \cdot \vec{f} + \tilde{q} \quad .$$

Since the divergence operator is independent of the coordinate system, the equation in vector notation is applicable to any other coordinate system.

For the case of two dimensional (x - y) flow in a reservoir of variable thickness, $h(x,y)$, a similar derivation will result in

$$h \frac{\partial \bar{C}}{\partial t} = -\frac{\partial (h \bar{f}_x)}{\partial x} - \frac{\partial (h \bar{f}_y)}{\partial y} + h \bar{q} \quad .$$

where

$$\bar{C} = \frac{\int_0^h C dz}{h}$$
$$\bar{f}_x = \frac{\int_0^h f_x dz}{h}$$
$$\bar{f}_y = \frac{\int_0^h f_y dz}{h}$$
$$\bar{\tilde{q}} = \frac{\int_0^h \tilde{q} dz}{h}$$

Hydrocarbon Phase Behavior

Crude oils are generally a complex mixture of hundreds of components. However, the material that is produced at the surface facilities and marketed is a "stock tank" oil and a separator gas. It will simplify matters greatly if the crude oil could be treated as if it consisted of these two components and the material conservation equations be expressed in terms of these two components. Since both fluids are compressible and the components distribute between the liquid and vapor phases at the reservoir temperature and pressure, some means are needed to describe the volumetric behavior of these fluids.

The volumetric behavior of a typical mixture of hydrocarbons with decrease in pressure is illustrated in the following figure. Suppose the pressure, p_1 represents the initial reservoir pressure where this mixture is under-saturated, one phase liquid (oil) system. As the pressure drops to p_2 , the volume increases as the liquid expands. This expansion is characterized by the **oil compressibility**.

$$c_o = -\frac{1}{V} \left(\frac{\Delta V}{\Delta p} \right)_T$$

At pressure p_3 the first bubble of a vapor phase appears. This pressure is called the **bubble point pressure**, p_b . As the pressure is decreased further, additional gas is evolved and expands. Even though the pressure is decreasing, the oil volume may decrease if the oil volume decrease due to the loss of gas is greater than the oil volume increase due to compressibility.

The change in volume with pressure depletion needs to be quantified for the purpose of simulation. We do this by expressing the volumes per unit of "stock tank oil" and "separator gas" at standard temperature (60°F) and pressure (14.7 psia). The separator gas will generally be mostly methane with small amounts of ethane and carbon dioxide.

The following terms are employed:

- (a) **Formation Volume Factor (FVF) B_o** : the volume of reservoir liquid phase that would yield a unit volume of oil at stock tank conditions.
- (b) **Solution Gas Oil Ratio (GOR) R_s** : the standard volume of gas that the reservoir liquid phase could yield with a unit volume of stock tank oil.
- (c) **Gas Formation Volume Factor B_g** : the volume of reservoir vapor phase that would yield a unit volume gas at standard conditions.

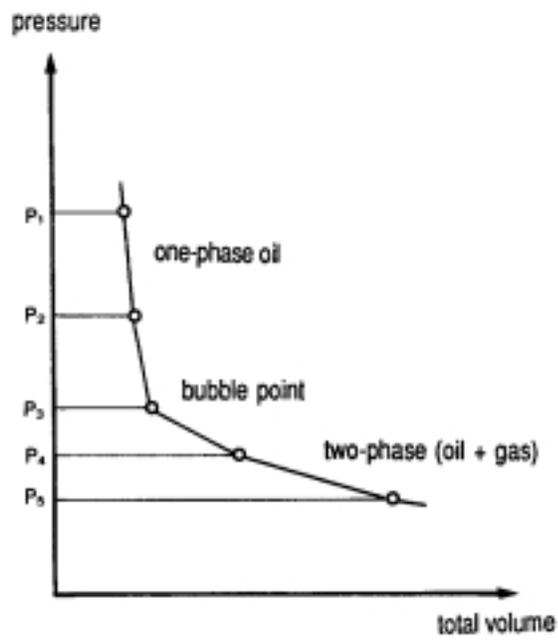
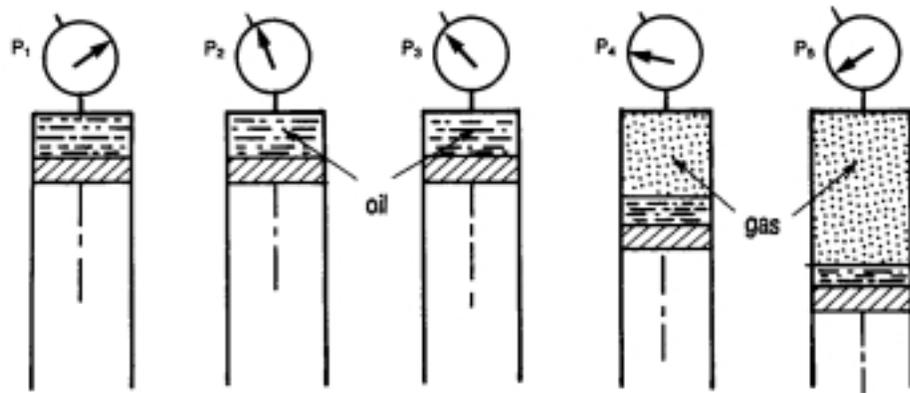


Fig. 5.1 Variation in hydrocarbon phases and volumes with decreasing pressure. (Cosse,1993)

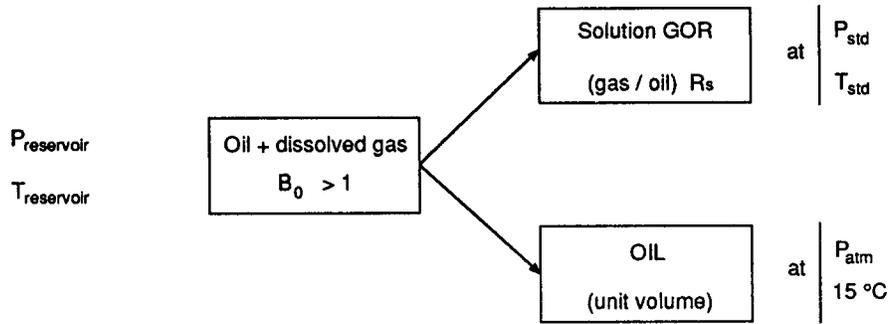


Fig. 5.2 Conditions for measuring the formation volume factor and solution gas oil ratio (Cosse, 1993).

The oil formation volume factor is expressed in reservoir barrels per stock tank barrel (in US) or cubic meter per cubic meter. The conditions for the formation volume factors are illustrated in figure 5.2.

The dependence of the properties on the pressure is illustrated in figure 3 for a light oil and a moderate gravity oil. There is a break in the slope of the curves at the bubble point pressure. Above p_b the oil expands on reduction of pressure and the solution GOR remains constant. Below p_b there is shrinkage of the oil on reduction of pressure due to loss of solution gas.

Suppose you had an option to first produce by primary depletion, i.e., oil expansion and solution gas drive, before water flooding. Also, suppose that the same average oil saturation would remain after water flooding. What would be the effect on recovery of depleting to a low pressure before water flooding compared to starting the water flood at the bubble point pressure?

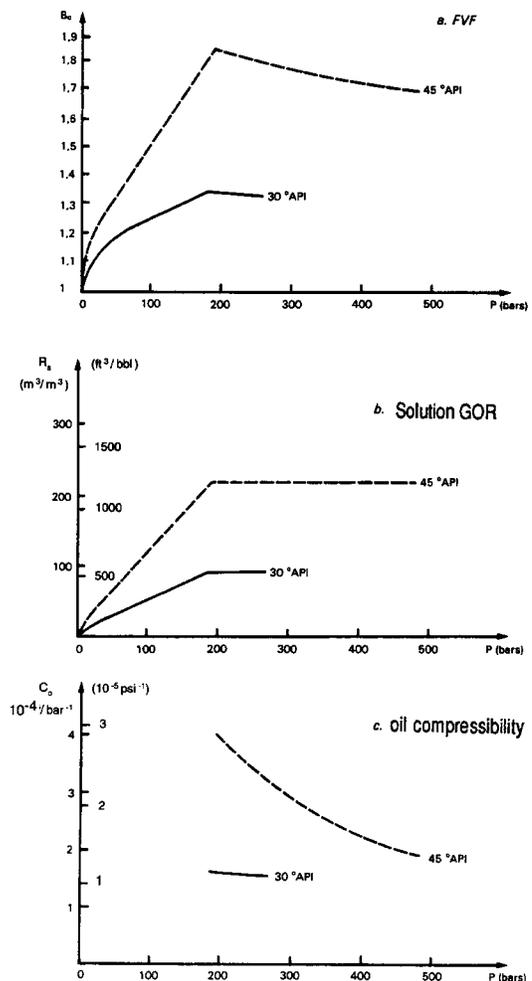


Fig. 5.3 Dependence of oil properties on pressure (Cosse, 1993)

Another factor that could have a significant dependence on pressure is the oil viscosity. The light oil has a low viscosity compared to water and has little dependence on pressure. The medium gravity oil increases in viscosity as the pressure is depleted because of the loss of the low molecular weight gas components and the remaining components have a much higher average molecular weight. The increase in viscosity will be much more significant with lower gravity (heavy) oils.

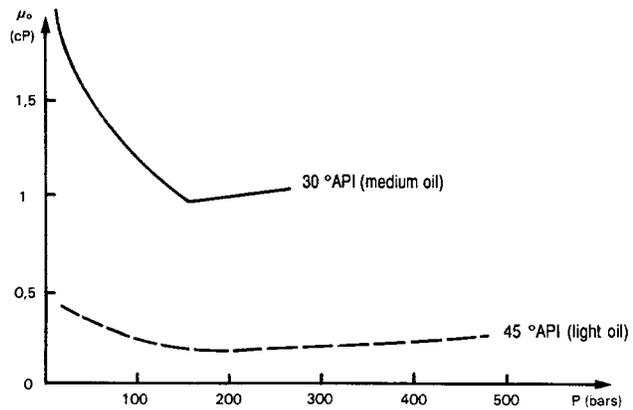


Fig. 5.4 Dependence of the oil viscosity on pressure depletion (Cosse, 1993)

If repressurization by water injection occurs after pressure depletion, the curve for the solution GOR may appear as in figure 5.5. Suppose the original conditions are at point A. A break in the curve occurs at p_b . Suppose that pressure depletion and gas evolution occurred until at point a water injection reversed the pressure trend and increased the pressure. As the pressure increases, the free gas will dissolve back into solution along the same curve. As a result of gas migration and production, a given location may not have a much gas as initially present and the gas saturation may go to zero at some pressure which is called the **saturation pressure**. (The bubble point pressure is the saturation pressure of the original oil.) As pressure is increased further R_S will not change since there is no more gas to go into solution. Also, the oil formation volume factor will change only by the compressibility effect above this pressure since there is no gas going into solution. This saturation pressure upon repressurization can be different for each grid block.

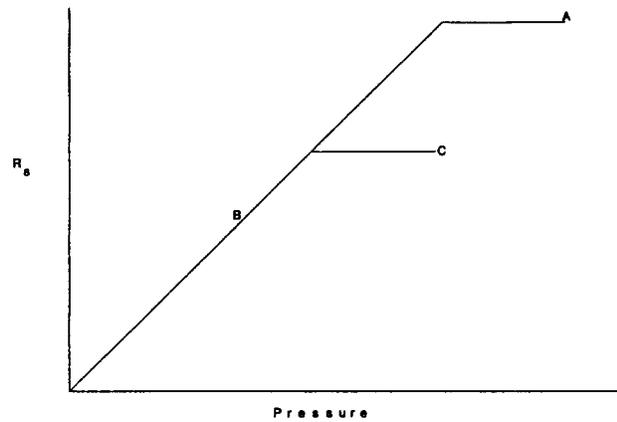


Fig. 5.5 R_S as a function of pressure, pressure depletion from production followed by repressurization from water injection (Koederitz, Harvey, Honarpour, 1989).

Conservation Equations for Particular Fluid Systems

Reading Assignment: Chapter 4 and 11 of Reservoir Simulation

The general form of the material conservation was shown earlier. The equations will now be expressed for particular reservoir fluid systems. Unless there is a significant amount of dissolved hydrocarbons, the water conservation equations will be the same for the various hydrocarbon fluid models. However, the hydrocarbon fluids can be modeled as black oil and gas, incompressible fluids, dry gas, wet gas, condensate gas, volatile oil, compositional, and miscible.

In all of the fluid systems there will be at most three fluid phases, the aqueous phase, hydrocarbon liquid phase, and hydrocarbon vapor phase. These phases will generally be denoted with the subscripts w , o , and g or aq , ℓ , and v . The phase saturations S_w , S_o , and S_g are defined as the volume fraction of the pore space occupied by each phase. Thus, we have

$$S_w + S_o + S_g = 1 .$$

The relative permeability of each phase is a function of the saturations. In a water-wet formation the water and gas relative permeabilities can be approximated as functions of the respective saturations. However, the oil relative permeability is a function of two of the saturations in a three-phase system.

$$\begin{aligned}k_{rw} &= k_{rw}(S_w) \\k_{rg} &= k_{rg}(S_g) = k_{rg}[1 - (S_w + S_o)] \\k_{ro} &= k_{ro}(S_w, S_o)\end{aligned}$$

At a given point in the reservoir, the phase pressures will be different due to the interfacial tension and the curvature of the interfaces. This phase pressure difference can be expressed as a capillary pressure.

$$\begin{aligned}p_o - p_w &= P_{cwo}(S_w) \\p_g - p_o &= P_{cgo}(S_g) = P_{cgo}[1 - (S_w + S_o)]\end{aligned}$$

where P_{cwo} is a function of S_w in a water-wet formation and P_{cgo} is always a function of the gas saturation.

(a) **Water.** The concentration and flux of water can be expressed as

$$C_w = \phi S_w b_w$$

$$\vec{f}_w = b_w \vec{u}_w$$

where

C_w water concentration, STB/(bulk reservoir volume)
 \vec{f}_w water flux, STB/(area-time)
 ϕ formation porosity
 b_w reciprocal water formation volume factor, STB/RB
 \vec{u}_w water superficial velocity, volume/(area-time)

The superficial fluid velocity for water in multiphase flow is

$$\vec{u}_w = \frac{-\mathbf{k} k_{rw}}{\mu_w} (\nabla p_w - \rho_w g \nabla D)$$

where

k_{rw} water relative permeability, function of water saturation
 ρ_w water phase pressure.

The conservation equation for water can be expressed as

$$\frac{\partial (\phi b_w S_w)}{\partial t} = \nabla \cdot \left[\frac{\mathbf{k} k_{rw} b_w}{\mu_w} (\nabla p_w - \rho_w g \nabla D) \right] + \tilde{q}_w .$$

(b) **Black oil.** Crude oil with a gravity less than 40° API and a reservoir temperature less than 150°F can be represented with a black oil model. The black oil model assumes that all of the stock tank oil is produced from the reservoir liquid phase, i.e., the reservoir vapor phase yields no stock tank oil. In the black oil fluid systems there is very little difference between the laboratory equilibrium flash vaporization and the differential vaporization data.

Lighter crude (40° - 45° API) with higher reservoir temperature (150°-250°F) will show a more significant difference between the flash and differential data. Possible approaches to modeling these crudes with a black oil model is to either conduct the laboratory experiment such as to differentially vaporize the oil sample and flash to surface conditions or to use the differential data for the reservoir fluid flow and expansion and use the flash data to convert the fluids to surface conditions.

Crudes with higher gravity (45°-55°) and higher reservoir temperature (above 250°F) cannot be modeled as a black oil as much of the stock tank oil is produced from the reservoir vapor phase. These type of fluid systems must be modeled as a volatile oil with one of the other models discussed later.

The conserved quantities in the black oil model are stock tank barrels (STB) of oil and thousands of cubic feet (MCF) of separator gas at standard conditions of 14.7 psia and 60°F. The gas is modeled to be soluble in the reservoir liquid hydrocarbon phase. The concentration of the stock tank oil in the liquid phase and separator gas in the vapor phase are represented by the reciprocal oil formation volume factor b_o (STB/RB) and the reciprocal gas formation volume factor b_g (MCF/RB) respectively. The concentration of the separator gas in the liquid phase is represented by $b_o R_s$ (MCF/RB) where the solution gas/oil ratio R_s , has the units of MCF/STB.

The concentration of stock tank oil and separator gas in the black oil model is

$$C_{STB} = \phi b_o S_o$$

$$C_{Mcf} = \phi (b_g S_g + b_o R_s S_o)$$

where

S_o saturation of hydrocarbon liquid (oil) phase, fraction of pore volume

S_g saturation of hydrocarbon vapor (gas) phase, fraction of pore volume.

The flux of stock tank oil in the black oil model is

$$\vec{f}_{STB} = b_o \vec{u}_o$$

$$= \frac{-\mathbf{k} k_{ro} b_o}{\mu_o} (\nabla p_o - \rho_o g \nabla D) \cdot$$

and the flux of separator gas is

$$\vec{f}_{Mcf} = b_g \vec{u}_g + b_o R_s \vec{u}_o$$

$$= -\frac{\mathbf{k} k_{rg} b_g}{\mu_g} (\nabla p_g - \rho_g g \nabla D) - \frac{\mathbf{k} k_{ro} b_o R_s}{\mu_o} (\nabla p_o - \rho_o g \nabla D) \cdot$$

The source terms for the black oil model are

$$\begin{aligned}\tilde{q}_{STB} &= b_o \tilde{q}_{ov} \\ \tilde{q}_{Mcf} &= b_g \tilde{q}_{gv} + b_o R_s^* \tilde{q}_{ov}\end{aligned}$$

where R_s^* is the solution gas/oil ratio existing in the reservoir for a producing well and is a specified solution gas/oil ratio if oil is being injected. \tilde{q}_{ov} and \tilde{q}_{gv} are the volumetric source terms for the hydrocarbon liquid and hydrocarbon vapor phases. To simplify the notation, \tilde{q}_{STB} and \tilde{q}_{Mcf} will just be denoted as \tilde{q}_o and \tilde{q}_g .

The stock tank oil and separator gas conservation equations for the black oil model are

$$\begin{aligned}\frac{\partial(\phi b_o S_o)}{\partial t} &= \nabla \cdot \left[\frac{\mathbf{k} k_{ro} b_o}{\mu_o} (\nabla p_o - \rho_o g \nabla D) \right] + \tilde{q}_o \\ \frac{\partial(\phi b_g S_g + \phi b_o S_o R_s)}{\partial t} &= \\ \nabla \cdot \left[\frac{\mathbf{k} k_{rg} b_g}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \right] &+ \nabla \cdot \left[\frac{\mathbf{k} k_{ro} b_o R_s}{\mu_o} (\nabla p_o - \rho_o g \nabla D) \right] + \tilde{q}_g.\end{aligned}$$

(c) **Incompressible fluid flow.** In water or gas injection pressure maintenance projects with negligible change in average pressure, the fluids and formation can be modeled to be incompressible. To model the reservoir fluids as being incompressible does not require that the fluid have negligible compressibility, but rather that the change in the pressure dependent properties with time and position in the reservoir has negligible effect on the reservoir performance. For example, the compressibility of gas may be large and is important in recovery by primary depletion, but in pressure maintenance by crestal gas injection, the gas can be modeled as incompressible. In incompressible fluid flow, the solution gas/oil ratio is constant. Since the flow and production of solution gas will always be proportional to that of oil, it is not necessary to model the solution gas. Thus, the conservation equation for gas will only need to include the free gas.

As was discussed earlier, the expression for the Darcy's law can be simplified in the case of incompressible fluids by expressing the pressure and buoyancy terms as a flow potential.

$$\bar{u}_w = \frac{-k k_{rw}}{\mu_w} \nabla \Phi_w$$

$$\bar{u}_o = \frac{-k k_{ro}}{\mu_o} \nabla \Phi_o$$

$$\bar{u}_g = \frac{-k k_{rg}}{\mu_g} \nabla \Phi_g$$

where

$$\Phi_w = p_w - \rho_w g D + C_1$$

$$\Phi_o = p_o - \rho_o g D + C_2$$

$$\Phi_g = p_g - \rho_g g D + C_3$$

where C_1 , C_2 , C_3 are constants determined by the pressure and capillary pressure at specified depths, e.g. zero capillary pressures at the free water level and free liquid level.

With the incompressible fluid assumption, the conservation equation can be expressed as follows.

$$\begin{aligned} \phi \frac{\partial S_w}{\partial t} &= \nabla \cdot \left(\frac{\mathbf{k} k_{rw}}{\mu_w} \nabla \Phi_w \right) + \frac{\tilde{q}_w}{b_w} \\ \phi \frac{\partial S_o}{\partial t} &= \nabla \cdot \left(\frac{\mathbf{k} k_{ro}}{\mu_o} \nabla \Phi_o \right) + \frac{\tilde{q}_o}{b_o} \\ \phi \frac{\partial S_g}{\partial t} &= \nabla \cdot \left(\frac{\mathbf{k} k_{rg}}{\mu_g} \nabla \Phi_g \right) + \frac{\tilde{q}_g}{b_g} \end{aligned} .$$

A restriction on the incompressible model is that if the reservoir has no-flow boundaries, then the net sum of $\int_V \left(\frac{\tilde{q}_w}{b_w} + \frac{\tilde{q}_o}{b_o} + \frac{\tilde{q}_g}{b_g} \right) dV$ must equal to zero.

That is to say that the net volumetric injection and production must be zero.

(d) **Dry gas.** The conservation equations for a water drive, dry gas reservoir can be represented by substituting gas properties for oil properties, and excluding the solution gas terms. If the solution gas in water is important, such as in extremely high pressure gas reservoirs, then the water conservation equation can be represented substituting water properties for oil and using R_S , as the solution gas/water ratio.

Single-phase dry gas flow can be more accurately computed by transforming the gas conservation equation in terms of the real gas pseudo-pressure discussed earlier.

(e) **Wet gas.** If the pressure in the reservoir stays above the dew point pressure but condensate is recovered from the separator, the reservoir is denoted as a wet gas reservoir. Since the condensate/gas ratio remains constant, a wet gas reservoir can be modeled as a dry gas reservoir and the condensate production be computed as the product of the condensate/gas ratio and the gas production.

(f) **Condensate gas.** A gas reservoir in which liquid hydrocarbon condenses out of the vapor phase in the reservoir as the pressure declines below the dew point pressure is denoted as a condensate gas reservoir. Normally the liquid hydrocarbon phase is immobile but if the liquid saturation becomes sufficiently large, it can become mobile. If the liquid hydrocarbon phase is swept by a dry gas, it can be revaporized into the vapor phase.

In general, a condensate gas system can be modeled as a multicomponent compositional system. However, the model that will be discussed is a much simpler formulation presented by A. Spivak. The formulation is analogous to the black oil model.

The hydrocarbon system is modeled as a pseudo two-component system of the STB of hydrocarbon liquid and MCF of gas produced from the separator as in the black oil model. The basic assumption of the model is that the equilibrium condensate content of the vapor is a single valued function of pressure during both the depletion and revaporization. The model also assumes that the separator gas contained in the liquid hydrocarbon phase can be neglected.

The STB of condensate and MCF of gas recovered from the separator from one reservoir barrel of the liquid hydrocarbon and vapor phase, respectively are represented by the reciprocal formation volume factors, b_O and b_G . The STB of condensate recovered from one reservoir barrel of the vapor phase is represented by $r_S b_G$, where r_S has units of STB/MCF.

The reciprocal condensate formation volume factor, b_O , is a function only of the pressure since the liquid hydrocarbon phase is assumed to always be in equilibrium with the vapor phase. The values of b_G and r_S are single valued functions of pressure when the vapor and liquid are in equilibrium. However, when no liquid phase is present (e.g., when above the initial dew point or when all of the liquid has been vaporized by the cycled dry gas), the value of b_G is a function of both pressure and r_S and r_S is computed from the conservation equations.

The concentrations, fluxes, and source terms for the STB of condensate and MCF of separator gas can be expressed as

$$\begin{aligned}
C_{STB} &= \phi (S_o b_o + S_g b_g r_s) \\
C_{MCF} &= \phi S_g b_g \\
\vec{f}_{STB} &= b_o \vec{u}_o + b_g r_s \vec{u}_g \\
&= \frac{\mathbf{k} k_{ro} b_o}{\mu_o} (\nabla p_o - \rho_o g \nabla D) + \frac{\mathbf{k} k_{rg} b_g r_s}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \\
\vec{f}_{MCF} &= b_g \vec{u}_g \\
&= \frac{\mathbf{k} k_{rg} b_g}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \\
\tilde{q}_{STB} &= b_o \tilde{q}_{ov} + b_g r_s^* \tilde{q}_{gv} \\
\tilde{q}_{MCF} &= b_g \tilde{q}_{gv}
\end{aligned}$$

where r_s^* is the condensate/gas ratio existing in the reservoir if the well is a producer and is a specified value if the well is an injector. The terms \tilde{q}_{ov} are volumetric source terms. To simplify the notation, \tilde{q}_{STB} and \tilde{q}_{MCF} will just be denoted as \tilde{q}_o and \tilde{q}_g . The conservation equations for the condensate and separator gas may be expressed as

$$\begin{aligned}
\frac{\partial (\phi S_o b_o + \phi S_g b_g r_s)}{\partial t} &= \nabla \cdot \left[\frac{\mathbf{k} k_{ro} b_o}{\mu_o} (\nabla p_o - \rho_o g \nabla D) \right] \\
&\quad + \nabla \cdot \left[\frac{\mathbf{k} k_{rg} b_g r_s}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \right] + \tilde{q}_o \\
\frac{\partial (\phi S_g b_g)}{\partial t} &= \nabla \cdot \left[\frac{\mathbf{k} k_{rg} b_g}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \right] + \tilde{q}_g
\end{aligned}$$

When no liquid hydrocarbon phase exists in the reservoir, then r_s is not a function of pressure and must be computed. In the case of zero liquid hydrocarbon phase, the equation for STB liquids reduces to

$$\frac{\partial (\phi S_g b_s r_s)}{\partial t} = \nabla \cdot \left[\frac{\mathbf{k} k_{rg} b_g r_s}{\mu_g} (\nabla p_g - \rho_g g \nabla D) \right] + \tilde{q}_o$$

Now r_s , is a dependent variable which can be computed from above equation.

(h) **Volatile oil.** The concepts of the black oil model and the condensate gas model previously discussed can be combined to represent either a volatile oil or a condensate gas system. The separator gas content in the reservoir liquid hydrocarbon phase can be represented by $b_o R_s$ and the stock tank hydrocarbon liquid in the reservoir vapor phase can be represented by $b_g r_s$. The concentration of the stock tank liquid hydrocarbon and separator gas can then be represented as

$$C_{STB} = \phi (S_o b_o + S_g b_g r_s)$$

$$C_{MCF} = \phi (S_o b_o R_s + S_g b_g)$$

The term partial density or partial specific mass has been used to denote the concentrations of the separator products in the reservoir liquid and vapor phases. For example, the concentration of the surface products could have been expressed as

$$C_{STB} = \phi (S_o \rho_{ol} + S_g \rho_{ov})$$

$$C_{MCF} = \phi (S_o \rho_{g\ell} + S_g \rho_{gv})$$

where

$$\rho_{ol} = b_o$$

$$\rho_{ov} = b_g r_s$$

$$\rho_{g\ell} = b_o R_s$$

$$\rho_{gv} = b_g$$

The partial densities are determined as a function of pressure from experimental depletion data. In one case the compositional changes were approximated by using the average molecular weight as a correlating parameter.

(i) **Compositional model.** All of the previously mentioned hydrocarbon system could have been represented with a compositional model. The compositional model represents the hydrocarbon system as a mixture of two or more hydrocarbon components or pseudocomponents that have their composition in the vapor and liquid phases determined by an equilibrium ratio, $K_i = y_i / x_i$, which may be modeled as constant, function of pressure, or a function of pressure and composition. The number of conservation equations is equal to the number of components.

The compositional model generally takes more computing time than the black oil or condensate gas model because of the additional conservation equations and the vapor-liquid equilibrium calculations. Thus, the simpler models should be used when possible. However, in recovery process with injected fluids with a composition very different from the reservoir fluids, it may

be necessary to use a compositional model. Examples are the injection of enriched gas, lean gas, carbon dioxide, or flue gas at conditions where miscibility is not attained.

(j) **Miscible displacement.** The permeability, relative permeability, and viscosity determine the mobility of a phase in immiscible displacement. However, in miscible displacement of two fluids, both fluids have the same phase relative permeability and have a viscosity that may be a blend with the other component. For example, the mobility of two miscible fluids in multiphase flow may be expressed as

$$\lambda_1 = \frac{k k_{rp}}{\mu_{1e}} \left(\frac{S_1}{S_1 + S_2} \right)$$

$$\lambda_2 = \frac{k k_{rp}}{\mu_{2e}} \left(\frac{S_2}{S_1 + S_2} \right)$$

where

λ_1, λ_2	mobility of fluid components 1 and 2
k	absolute permeability
k_{rp}	relative permeability of phase
S_1, S_2	volume fraction of pore space occupied by fluid component 1 and 2
μ_{1e}, μ_{2e}	effective viscosity of fluid component 1 and 2

It was assumed that the phase under consideration contained only the two components. A method of modeling the effective viscosity will be discussed with the miscible displacement processes.

With the mobilities described as the above equations and the concentrations described with the volume fractions, S_1 and S_2 , and formation volume factors, the conservation equations can be expressed and solved similar to the black oil model.

References

1. Jacoby, R. H., and Berry, V. J., Jr. (1957), A Method for Predicting Depletion Performance of a Reservoir Producing Volatile Crude Oil, Trans. AIME, v. 210, 27.
2. Standing, M. B. (1952), Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems, Rienhold Publishing Corp., New York.
3. Dodson, C. R., Goodwill, P., and Mayer, E. H. (1953), Application of Laboratory PVT Data to Reservoir Engineering Problems, Trans. AIME, v. 198, 287.
4. Reudelhuber, F. O., and Hinds, R. F. (1957), A Compositional Material Balance Method for Predictions of Recovery from Volatile Oil Depletion Drive Reservoirs, Trans. AIME, v. 210, 19.
5. Spivak, A. (1971), Mathematical Simulation of Condensate-Gas Reservoirs, Ph.D. Thesis, University of Texas at Austin.
6. Kniazeff, V. J., and Naville, S. A. (1965), Two-Phase Flow of Volatile Hydrocarbons, Soc. Petr. Engr. J., p. 37, March.
7. McCord, D. R., and Associates (1968), Users Reference Manual for 200-600 Point, Two-Dimensional, Two-Phase Gas, Two-Phase Liquid Unsteady-State Reservoir Simulator with Dynamic Fluid Correlation Parameters.
8. Price, H. S , and Donohue, D. A. T. (1967), Isothermal Displacement Process with Interphase Mass Transfer, Soc. Petr. Engr. J., p 205, June.
9. Culham, W. E., Farouq Ali, S. M., and Stahl, C. D. (1969), Experimental and Numerical Simulation of Two-Phase Flow with Interphase Mass Transfer in One and Two-Dimensions, Soc. Petr. Engr. J., p. 323, September.
10. Roebuck, I. F., Jr., Henderson, G. E., Douglas, J., Jr., and Ford, W. T. (1969), The Composition Reservoir Simulator: Case I - The Linear Model, Soc. Petr. Engr. J., p. 115, March.
11. Van-Quy, N., Simandoux, P., and Corteville, J. (1970), A Numerical Study of Diphasic Multicomponent Flow, SPE Preprint no. 3006, presented at the 45th Annual Fall Meeting of SPE in Houston, October 4-7, 1970.