Chapter 7 Two Phase, One Dimensional, Displacement

The case of one dimensional, two immiscible, incompressible phase displacement with zero capillary pressure will be studied by specializing the fractional flow equations derived earlier to just two phases. In addition we assume the system has a finite length, uniform initial conditions, and constant boundary conditions. We will be calculating the following: (1) trajectories of constant saturation in the distance-time domain, (2) saturation profile at a given time, (3) fractional flow history at the outflow end, and (4) recovery efficiency as a function of time.

A two phase system can be described by only one saturation and one fractional flow expression because the saturations and fractional flows must add to unity. Thus the only independent equation for this system can be given by the following.

$$\frac{\partial S_1}{\partial t} + \frac{u}{\phi}\frac{\partial f_1}{\partial x} = 0, \qquad 0 \le x \le L, \quad 0 < t$$

$$\begin{aligned} f_1 &= \frac{\lambda_{r1}}{\lambda_{r1} + \lambda_{r2}}\left[1 - \frac{kg\sin\alpha}{u}\lambda_{r2}\left(\rho_1 - \rho_2\right)\right] \\ &= f_1\left(S_1, u\right) \end{aligned}$$

α is positive for the x axis rotated in the counter clockwise direction from the horizon.

$$u_1 = u\ f_1$$

$$\frac{\partial u}{\partial x} = 0, \qquad 0 \le x \le L$$

We may identify phase 1 with water and phase 2 with oil. The initial condition will be taken to be at the residual (immobile) water saturation. Initial condition of mobile water saturation will cause no difficulty. The boundary conditions are a specified total flux and fractional flow of water equal to unity. The total flux can be any arbitrary function of time if the gravity term in the fractional flow equation is zero. If the gravity term is nonzero we assume that the total flux is constant. In the following we will be assuming that the fractional flow depends only on the saturation(s) and is independent of position and time; thus we do not want the fractional flow to be a function of time except through the saturation. Also, we will later relax the assumption of constant boundary conditions and show how a step change in the boundary condition will set up a new set of waves which may overtake slower waves from the original boundary conditions and cause "interference". A step change in the initial conditions could be treated in a similar way. No downstream boundary condition can be specified for this first order differential equation. However, we will assume that the system has a length *L* so that we can define a characteristic volume.

Dimensionless and Normalized Variables

The **dimensionless distance** will be normalized with respect to the system length.

$$x_D = x/L, \qquad 0 \le x_D \le 1$$

The saturations will be limited by the residual value of each phase. A **normalized saturation** *S* (without subscript) is defined.

$$S = \frac{S_1 - S_{r1}}{1 - S_{r2} - S_{r1}}, \qquad 0 \le S \le 1$$

The relative permeabilities will also be normalized with respect to their end point values.

A **dimensionless time** is defined as a number of movable pore volumes of throughput.

$$\begin{aligned} t_D &= \frac{uAt}{(1 - S_{r2} - S_{r1})\phi AL} \\ &= \frac{qt}{(1 - S_{r2} - S_{r1})V_p} \\ &= \frac{Q}{(1 - S_{r2} - S_{r1})V_p}, \qquad 0 \le t_D \end{aligned}$$

The **end point mobility ratio**, M (invading/displaced) is defined as follows.

$$M = \frac{k_{r1}^o \mu_2}{k_{r2}^o \mu_1}$$

A mobility ratio greater than unity is called *unfavorable* because the invading fluid will tend to bypass the displaced fluid. It is called f*avorable* if less than unity and called *unit mobility ratio* when equal to unity.

A dimensionless **gravity number**, N_G is defined.

$$N_G = \frac{k\, k_{r2}^o\, g\, \left(\rho_1 - \rho_2\right)}{\mu_2\, u}$$

The differential equation and fractional flow expression with these dimensionless or normalized variables are as follows.

$$\frac{\partial S}{\partial t_D} + \frac{\partial f_1}{\partial x_D} = 0, \qquad 0 \le x_D \le 1, \quad 0 < t_D$$

$$f_1 = \frac{[1 - \left(k_{r2} / k_{r2}^o\right) N_G \sin\alpha]}{\left[1 + \frac{1}{M} \frac{\left(k_{r2}/k_{r2}^o\right)}{\left(k_{r1}/k_{r1}^o\right)}\right]}$$

$$= f_1\left(S, N_G \sin\alpha, M\right)$$

In the following, we will work with dimensionless and normalized variables. The subscript *D* in t_D and x_D and the subscript 1 in f_1 will be dropped. The relative permeabilities will be normalized with respect to the end point values.

Assignment 7.1 Fractional Flow Curves

$$k_{r1} = k_{r1}^o\, S^{n_1}$$
$$k_{r2} = k_{r2}^o \left(1 - S\right)^{n_2}$$

$$n_1 = n_2 = 1$$
$$\alpha = 0$$
$$M = 0.5, \quad 1.0, \quad 2.0$$

Plot: (1) *f* versus *S*, and (2) *df/dS* versus *S*

Trajectories in Distance - Time (x,t) Space

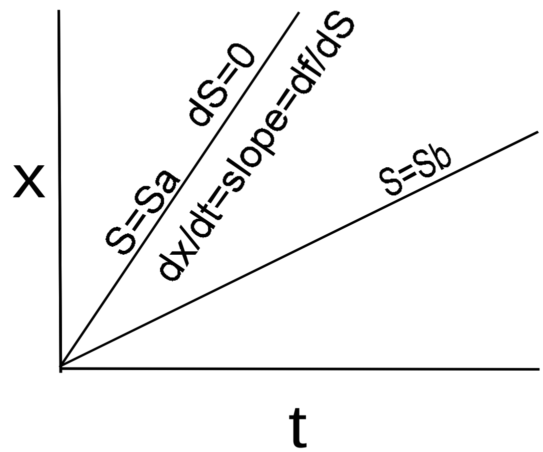
The Buckley-Leverett theory calculates the velocity that different saturation values propagate through the permeable medium. Since the fractional flow is a function of saturation, the conservation equation will be expressed in terms of derivatives of saturation.

$$\frac{\partial S}{\partial t} + \frac{df}{dS}\frac{\partial S}{\partial x} = 0$$

The differential, *df/dS* is easily calculated since there is only one independent saturation. If there were three or more phases this differential would be a Jacobian matrix. The locus of constant saturation will be sought by taking the total differential of *S(x,t)*.

$$dS = \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial x}dx = 0$$

$$\begin{aligned}\left(\frac{dx}{dt}\right)_{dS=0} &= -\frac{\partial S/\partial t}{\partial S/\partial x} \\ &= \frac{df}{dS}(S) \\ &= v_S\end{aligned}$$



This equation expresses the velocity that a particular value of saturation propagates through the system, i.e., the *saturation velocity,* v_S is equal to the slope of the fractional flow curve. It is also the slope of a trajectory of constant saturation (i.e., d*S*=0) in the (*x,t*) space. Since we are assuming constant initial and boundary conditions, changes in saturation originate at (*x,t*)=(0,0). From there the changes in saturation, called waves, propagate in trajectories of constant saturation. We assume that df/dS is a function of saturation and independent of time or distance. This assumption will result in the trajectories from the origin being straight lines if the initial and boundary conditions are constant. The trajectories can easily be calculated from the equation of a straight line.

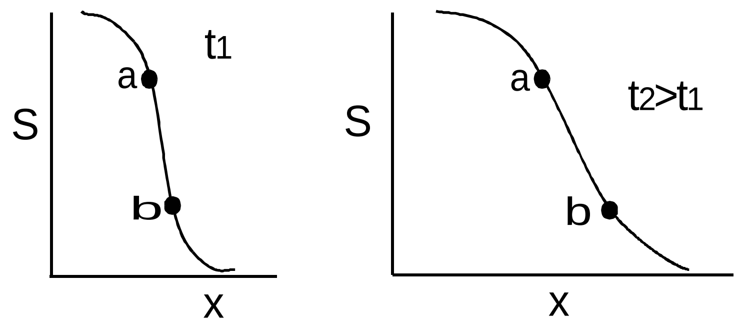
$$x(S) = \frac{df(S)}{dS}t$$

Definition of Waves

Wave: A composition change that propagates through the system.

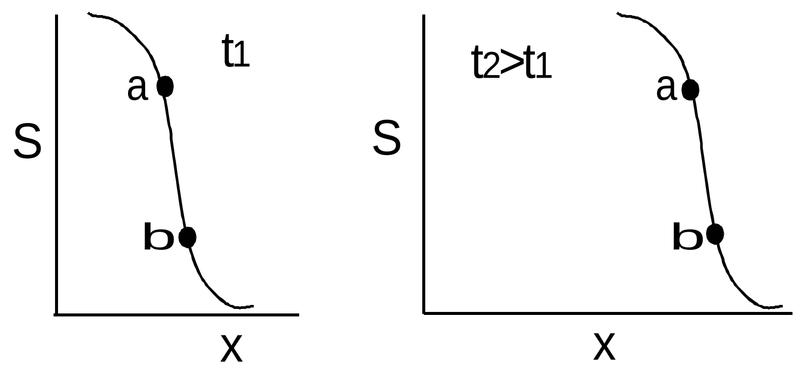
Spreading wave: A wave in which neighboring composition (or saturation) values become more distant upon propagation.

$$\left(\frac{dx}{dt}\right)_{S_a} < \left(\frac{dx}{dt}\right)_{S_b}$$

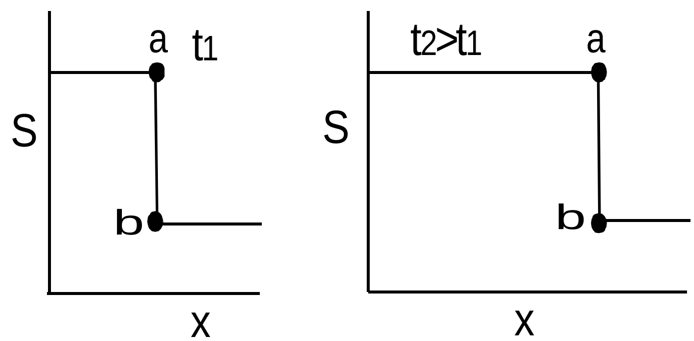


Indifferent waves: A wave in which neighboring composition (or saturation) values maintain the same relative position upon propagation.

$$\left(\frac{dx}{dt}\right)_{S_a} = \left(\frac{dx}{dt}\right)_{S_b}$$

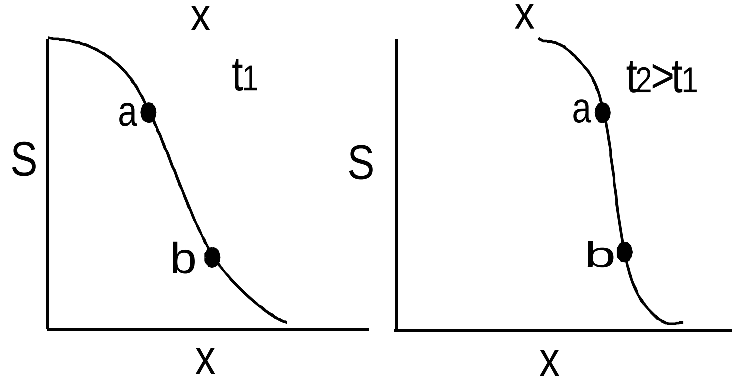


Step Wave: An indifferent wave in which the compositions change discontinuously.

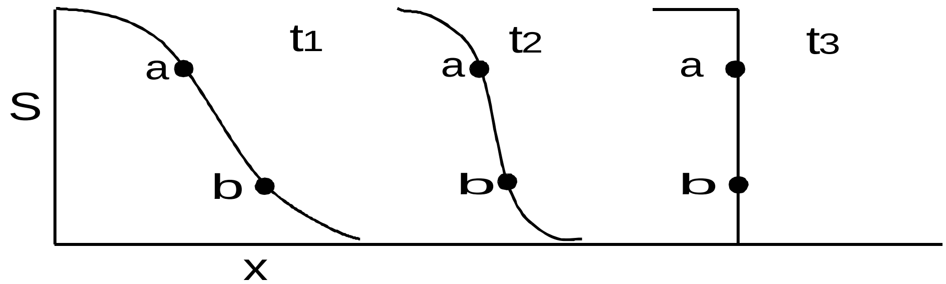


Self Sharpening Waves: A wave in which neighboring compositions (saturations) become closer together upon propagation.

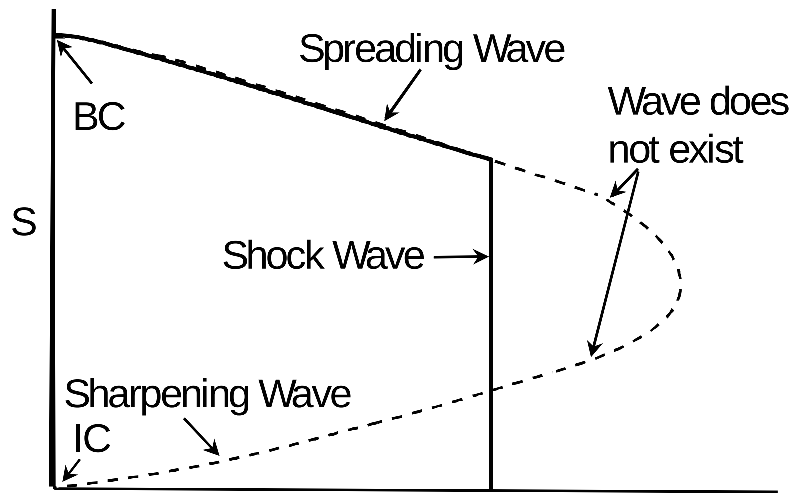
$$\left(\frac{dx}{dt}\right)_{S_a} > \left(\frac{dx}{dt}\right)_{S_b}$$



Shock Wave: A wave of composition (saturation) discontinuity that results from a self sharpening wave.



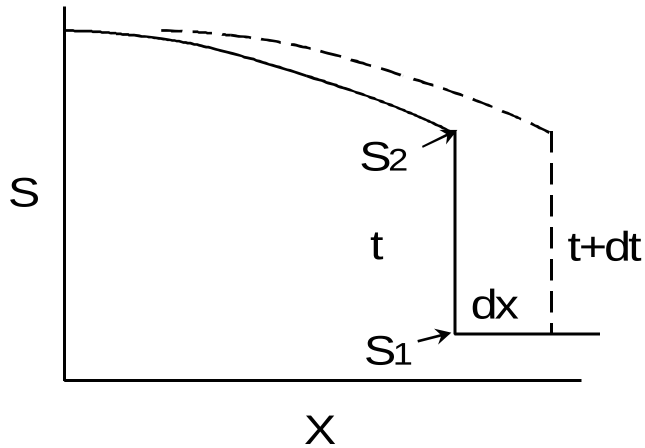
Rule: Waves originating from the same point (e.g., constant initial and boundary conditions) must have nondecreasing velocities in the direction of flow. This is another way of saying that when several waves originate at the same time, the slower waves can not be ahead of the faster waves. If slower waves from compositions close to the initial conditions originate ahead of faster waves, a shock will form as the faster waves overtake the slower waves. This is equivalent to the statement that a sharpening wave can not originate from a point; it will immediately form a shock.



$$\frac{x}{t} = \left(\frac{dx}{dt}\right)_{dS=0} = \frac{df}{ds}(S) = v(S)$$

Mass Balance Across Shock

We saw that sharpening wave must result in a shock but that does not tell us the velocity of a shock nor the composition (saturation) change across the shock. To determine these we must consider a mass balance across a shock. This is sometimes called an integral mass balance as opposed to the differential mass balance derived earlier for continuous composition (saturation) changes.

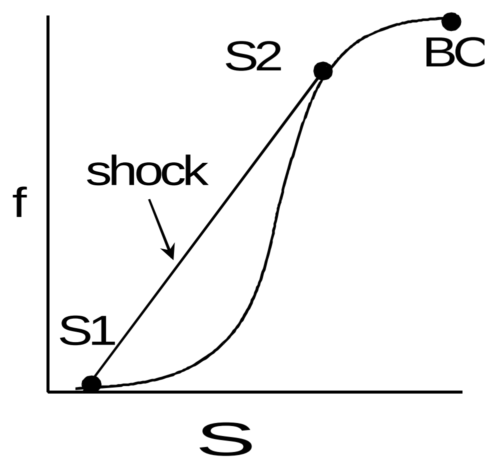


$$\text{Accumulation:} \qquad \phi A \Delta x \left(S_2 - S_1 \right)$$

$$\text{input} - \text{output:} \qquad u A \Delta t \left(f_2 - f_1 \right)$$

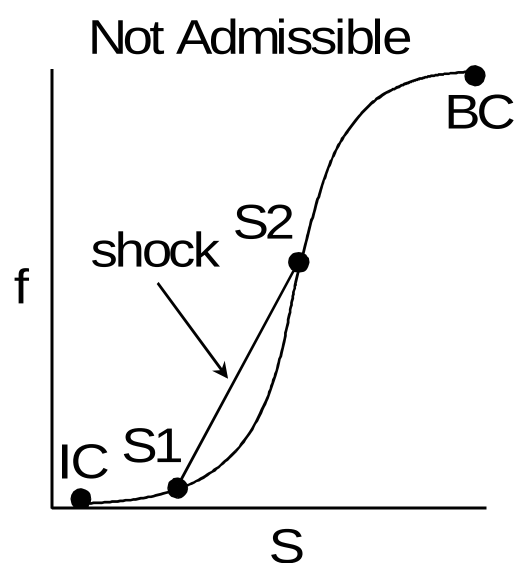
$$\phi A \Delta x \, \Delta S = u A \Delta t \, \Delta f$$

$$\left(\frac{dx_D}{dt_D} \right)_{\Delta S} = \frac{\Delta f}{\Delta S}$$

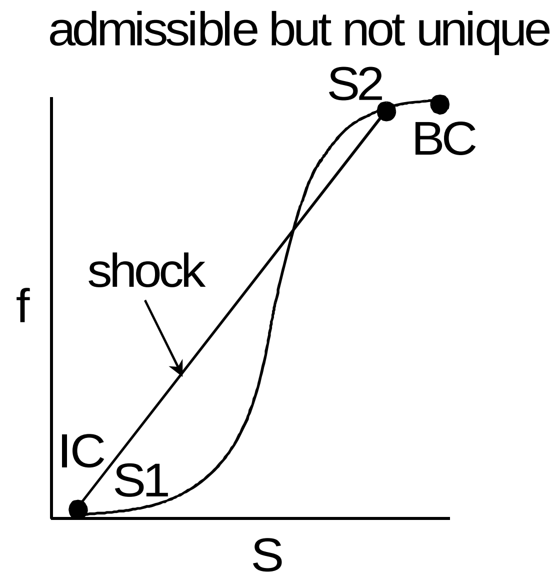


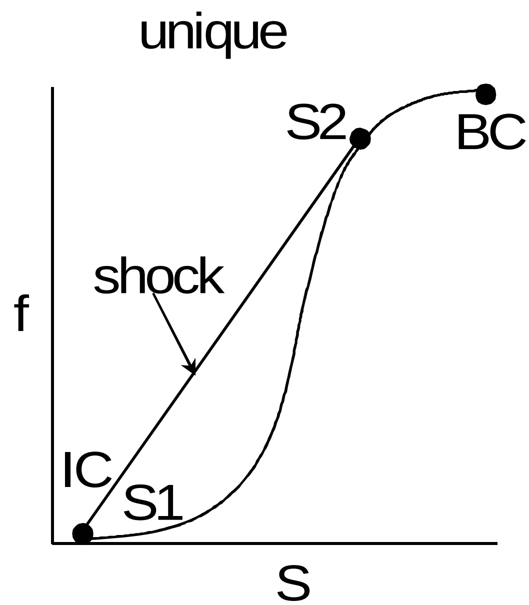
Δf/ΔS is the cord slope of the *f* versus *S* curve between S_1 and S_2.

The conservation equation for the shock shows the velocity to be equal to the cord slope between S_1 and S_2 but does not in itself determine S_1 and S_2. To determine S_1 and S_2, we must apply the rule that the waves must have non-decreasing velocity in the direction of flow. The following figure is a solution that is <u>not</u> admissible. This solution is **not admissible** because the velocity of the saturation values (slope) between the IC and S1 are less than that of the shock and the velocity of the shock (cord slope) is less than that of the saturation values immediately behind the shock.

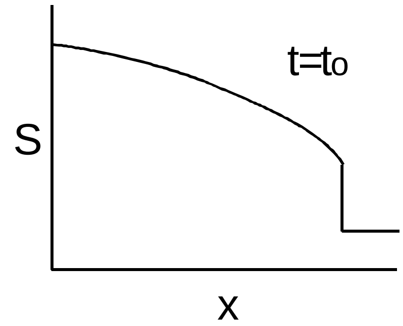


This solution is admissible in that the velocity in nondecreasing in going from the BC to the IC. However, it in **not unique**. Several values of S2 will give admissible solutions. Suppose that the value shown here is **a** solution. Also suppose that dispersion across the shock causes the presence of other values of S between S1 and S2. There are some values of S that will have a velocity (slope) greater than that of the shock shown here. These values of S will overtake S2 and the shock will go the these values of S. This will continue until there is no value of S that has a velocity greater than that of the shock to that point. At this point the velocity of the saturation value and that of the shock are equal. On the graphic construction , the cord will be tangent to the curve at this point. This is the **unique** solution in the presence of a small amount of dispersion..

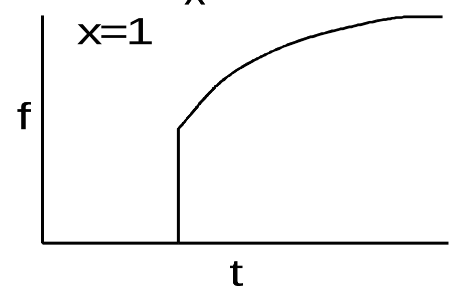




Composition (Saturation) Profile The composition (saturation) profile is the composition distribution existing in the system at a given time.



Composition or Flux History: The composition or flux appearing at a given point in the system, e.g., x=1.



Summary of Equations

The dimensionless velocity that a saturation value propagates is given by the following equation.

$$\left(\frac{dx}{dt}\right)_{dS=0} = \frac{df}{dS}(S)$$

With uniform initial and boundary conditions, the origin of all changes in saturation is at *x=0* and *t=0*. If *f(S)* depends only on *S* and not on *x* or *t*, then the trajectories of constant saturation are straight line determined by integration of the above equation from the origin.

$$x(S) = \left(\frac{dx}{dt}\right)_{dS=0} t = \frac{df}{dS}(S)\, t$$

$$x(\Delta S) = \left(\frac{dx}{dt}\right)_{\Delta S} t \quad = \frac{\Delta f}{\Delta S}\, t$$

These equations give the trajectory for a given value of *S* or for the shock. By evaluating these equations for a given value of time these equations give the saturation profile.

The saturation history can be determined by solving the equations for t with a specified value of x, e.g. x=1.

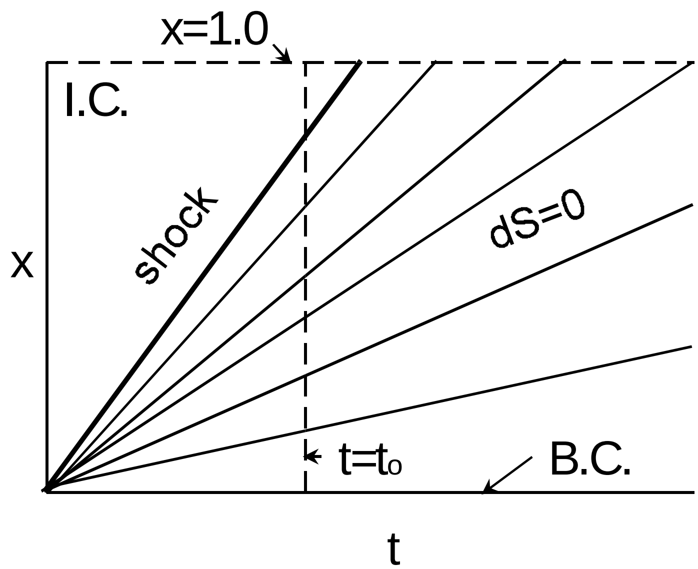
$$t(\Delta S) = \frac{x}{\frac{\Delta f}{\Delta S}}, \qquad x = 1$$

$$t(S) = \frac{x}{\frac{df}{dS}(S)}, \qquad x = 1$$

The **breakthrough time**, t_{BT}, is the time at which the fastest wave reaches *x=1.0*. The flux history (fractional flow history) can be determined by calculating the fractional flow that corresponds to the saturation history.

Summary of Diagrams

The relationship between the diagrams can be illustrated in a diagram for the trajectories. The profile is a plot of the saturation at $t=t_o$. The history at x=1.0 is the saturation or fractional flow at x=1. In this illustration, the shock wave is the fastest wave. Ahead of the shock is a region of constant state that is the same as the initial conditions.



Assignment 7.2 Calculation of Trajectories, Profile, and History

Use the fractional flow curves of the last assignment, the initial condition, *S=0*, and the boundary condition, *f=1.0*, to calculate (1) the trajectories of the waves that will exist in this system, (2) the saturation profiles at t=0.5, and (3) the fractional flow history at x=1.0.

Average Saturation and Recovery Efficiency

If the saturation profile is monotonic, then the average saturation in the system (i.e., 0<x<1) can be determined by integration by parts. This contribution to the Buckley-Leverett theory is credited the Welge (1952).

$$\begin{aligned}\overline{S} &= \int_0^1 S\, dx \\ &= S\, x\Big|_0^1 - \int_{S(x=0)}^{S(x=1)} x\, dS\end{aligned}$$

Recall that x was expressed as a function of S in the expression for the saturation profile.

$$\begin{aligned}\overline{S} &= S\left(x=1\right) - \int_{S(x=0)}^{S(x=1)} t\, \frac{df}{dS}\, dS \\ &= S\left(x=1\right) - \int_{f(x=0)}^{f(x=1)} t\ \ df \\ &= S\left(x=1\right) - t\left[f\left(x=1\right) - f\left(x=0\right)\right]\end{aligned}$$

Recall that the history at x=1 is given by t[S(x=1)].

$$\overline{S} = S\left(x=1\right) - \frac{\left[f\left(x=1\right) - f\left(x=0\right)\right]}{\frac{df}{dS}\left[S\left(x=1\right)\right]}, \qquad t > t_{BT}$$

Example: Let I.C.: S=0; B.C.: f=1. Then

$$\begin{aligned}&f\left(x=0\right) = 1.0 \\ &S(x=1) = 0, \qquad t < t_{BT} \\ &f(x=1) = 0, \qquad t < t_{BT}\end{aligned}$$

The average saturation is then

$$\begin{aligned}\overline{S} &= t, && t < t_{BT} \\ &= S - \frac{\left[f\left(S\right) - 1\right]}{\frac{df}{dS}\left(S\right)}, && t > t_{BT}\end{aligned}$$

where *S* above is *S* evaluated at *x=1*.

The **recovery efficiency** is usually defined as the fraction of the original oil in place that has been recovered. Here we will define a normalized recovery efficiency , E_R, that is normalized with respect to the original water flood movable oil in place. Since we have assumed incompressible fluids, the normalized recovery efficiency is equal to the normalized average water saturation.

$$
\begin{aligned}
E_R &= \frac{S_{oi} - \overline{S}_o}{S_{oi} - S_{or}} \\
S_{oi} &= 1 - S_{wr} \\
\overline{S}_o &= 1 - \overline{S}_w \\
E_R &= \frac{\overline{S}_w - S_{wr}}{1 - S_{or} - S_{wr}} \\
&\equiv \overline{S}
\end{aligned}
$$

With this definition, the normalized recovery efficiency has the following limiting values.

$$
\begin{aligned}
E_R\left(S_{wr}\right) &= 0 \\
E_R\left(1 - S_{or}\right) &= 1
\end{aligned}
$$

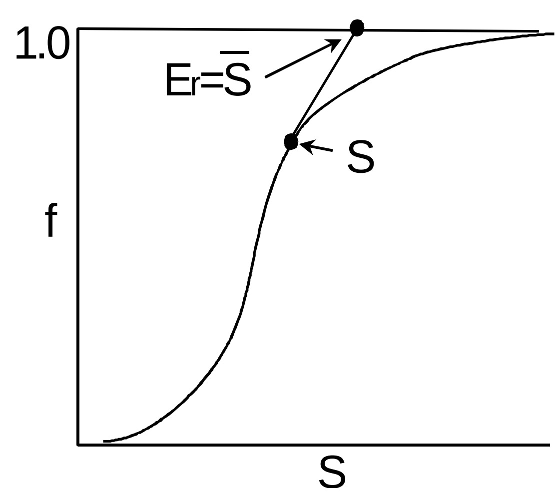
Thus

$$
\begin{aligned}
E_R &= t, & t < t_{BT} \\
&= S - \frac{\left[f(S) - 1\right]}{\frac{df}{dS}(S)}, & t > t_{BT}
\end{aligned}
$$

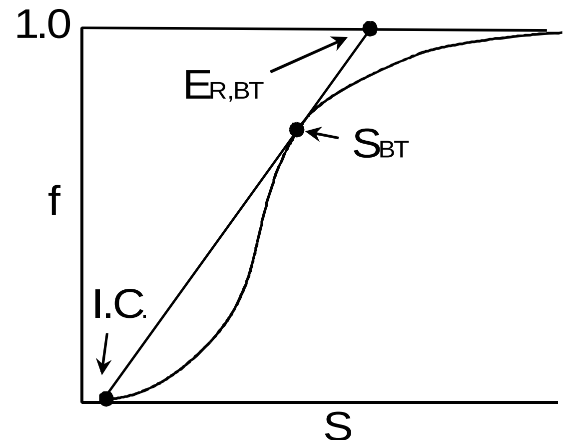
Graphical Determination of E_R

The ER can be graphically determined from the fractional flow plot. The preceding equation can be rearranged as follows.

$$
\frac{df}{dS}(S) = \frac{1 - f(S)}{E_R - S}
$$



The left side of the equation is the slope of the fractional flow curve at S. Recall that $E_R \equiv \overline{S}$. The right side of the equation is the expression for the cord slope from *f(S)* on the curve to *f=1.0* and $S = \overline{S} = E_R$. For a given value of *S*, E_R can be determined by making the slope and cord slope equal, i.e., tangent to the curve. The time is equal to the reciprocal of the slope. The recovery efficiency at breakthrough can be constructed as the intercept at f=1 of the line that intercepts the initial condition and is tangent to the fractional flow curve.



Assignment 7.3 Recovery Efficiency

Calculate the recovery efficiency as a function of time for the problem given in the last two assignments.

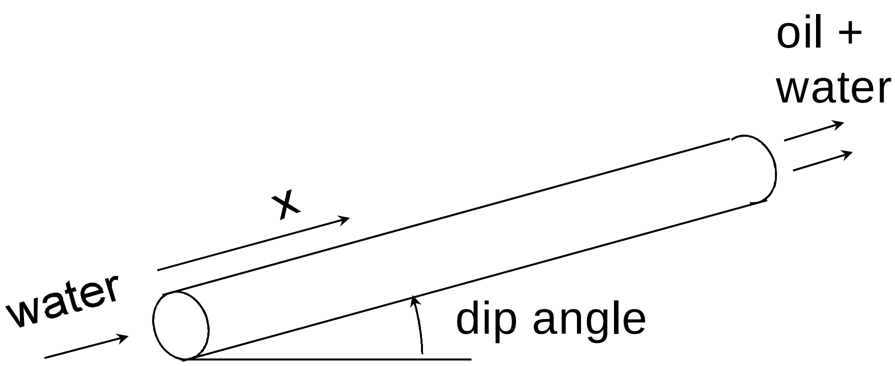
Assignment 7.4 Displacement with shock and spreading waves

Do the problem of assignments 7.1 - 7.3 but with *M*=2.0 and $n_1 = n_2 = 1.5$. Plot the tangent line from the initial condition to the fractional flow curve when plotting the fractional flow curve. When plotting *df/dS* plot also plot $\Delta f / \Delta S$ from *S*=0 to *S* of the shock. Hint: To find the tangent point use the function *fzero* to find the zero point of *f/S - df/dS*. If you are using a spreadsheet, graphical determination of the shock is OK.

Effect of Gravity on Displacement

Gravity or buoyancy always has a effect on multiphase flow in porous media if the fluids have different densities. Gravity is omnipresent and will have a greater effect if the displacement is viewed in two or three dimensions. However, we will limit ourselves to one dimensional displacement because of the ease that the effect of gravity can be quantitatively evaluated in one dimension. In one dimension we will not see the "gravity tongue" that results from gravity under or over - ride. The fractional flow formulation assumes that the saturation is uniform perpendicular (transverse) to the direction of flow. Even with uniform transverse saturation distribution, the fractional flow analysis clearly illustrates the effect of gravity assisting or retarding the displacement. Simple gravity drainage is a case where the only driving force for displacement is the buoyancy.

Displacement of oil by water in a dipping formation



The expression for the fractional flow of water (phase 1); using the notation that α is measured counter-clockwise from the horizontal is as follows.

$$f_1 = \frac{1}{\left[1 + \frac{1}{M}\frac{\left(k_{r2}/k_{r2}^o\right)}{\left(k_{r1}/k_{r1}^o\right)}\right]}\left[1 - \left(k_{r2}/k_{r2}^o\right) N_g \sin\alpha\right]$$

$$= f_1\left(S, N_g \sin\alpha\right)$$

If we use the power-law or Corey model of the relative permeability, the expression for the fractional flow is as follows.

$$f_1 = \frac{1}{\left[1 + \frac{1}{M}\frac{\left(1-S\right)^{n_2}}{S^{n_1}}\right]}\left[1 - \left(1-S\right)^{n_2} N_g \sin\alpha\right]$$

The gravity number is defined as

$$N_g = \frac{k\,k_{r2}^{o}\,g\,\left(\rho_1 - \rho_2\right)}{\mu_2\,u}$$

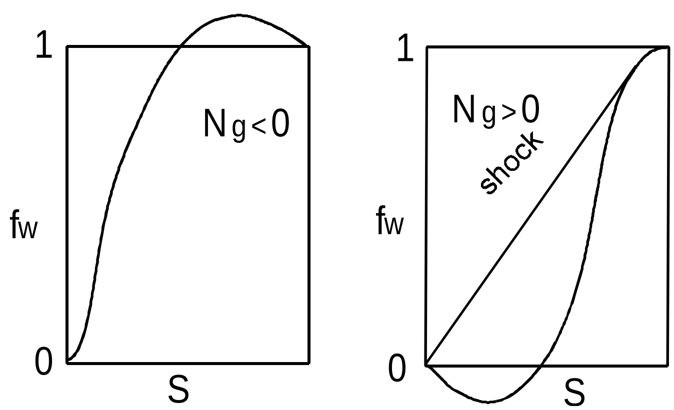
However, we will abbreviate our notation to include sin α .

$$N_g^{\alpha} = N_g \sin\alpha$$

Positive values of N_g^{α} will decrease the value of f_W, i.e., upward displacement of oil by water will decrease the fractional flow of water for a given saturation, i.e., the displacement is stabilized by gravity.

Negative values of N_g^{α} will increase the value of f_W, i.e., downward displacement of oil by water will increase the fractional flow of water for a given saturation.

Values of f_W less than zero or greater than unity can exist for some values of S and N_g^{α}. The flow of water and oil are counter-current under these conditions.



Assignment 7.5 Effect of gravity on displacement.

Do the same as assignments 7.1-7.3 except that M=2.0 and N_g^{α} = -1.0, 0, +1.0.

Inter-relation between mobility ratio and gravity number on displacement

We have examined individually the effect of mobility ratio and gravity on displacement of oil by water. We can derive an analytical expression to show the combined effect of mobility ratio and gravity if we assume that $n_1 = n_2 = 1.0$ (Hirasaki 1975). The derivative of the fractional flow for this case is as follows.

$$\frac{df_1}{dS}=\frac{N_g^{\alpha}}{\left[1+\frac{1}{M}\left(\frac{1-S}{S}\right)\right]}+\frac{\left[1-\left(1-S\right)N_g^{\alpha}\right]}{\left[1+\frac{1}{M}\left(\frac{1-S}{S}\right)\right]^2}\frac{1}{M\,S^2}$$

$$\frac{df_1}{dS}\left(S=0\right)=M\left(1-N_g^{\alpha}\right)$$

If $df/dS\left(S=0\right)\geq 1$ and $d^2f/dS^2\leq 0$, then the wave is a spreading or indifferent wave and the fastest wave is that for *S=0+*ε. The breakthrough time and thus the recovery at breakthrough can be determined from the slope at *S=0*.

$$t_{BT}=\frac{1}{\frac{df_1}{dS}\left(S=0\right)}=\frac{1}{M\left(1-N_g^{\alpha}\right)},\qquad M\left(1-N_g^{\alpha}\right)\geq 1,\quad n=1$$

$$E_{R,BT}=t_{BT}=\frac{1}{M\left(1-N_g^{\alpha}\right)},\qquad M\left(1-N_g^{\alpha}\right)\geq 1,\quad n=1$$

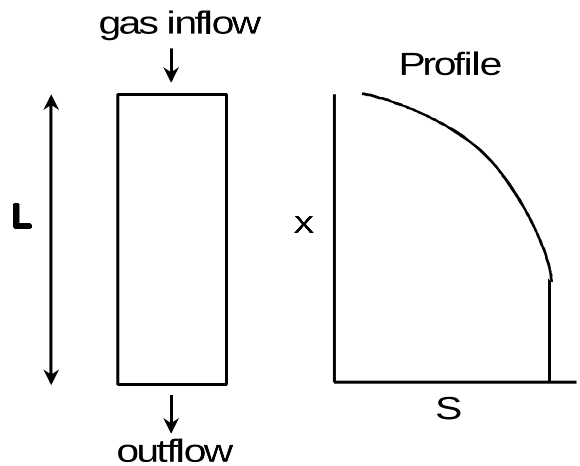
The relationship that results in piston-like displacement, i.e., $E_{R,BT} = 1.0$ is

$$N_g^{\alpha}\geq 1-\frac{1}{M},\qquad \text{or}\quad M\left(1-N_g^{\alpha}\right)\leq 1,\quad n=1$$

This relationship says that piston-like displacement will always result (with n=1 and one dimensional) if N_g^{α} >1, regardless of the mobility ratio. Also, piston-like displacement will result (with n=1 and one dimensional) with negative values of N_g^{α} if the mobility ratio is favorable enough. If you were to calculate the fractional flow curves with an equality in the above relationship, the curves will all have unit slope.

Gravity Drainage

We will now consider the case of displacement by the action of only the buoyancy, i.e., the invading phase is in hydrostatic equilibrium pressure at either end of the system. We will further simplify the problem by assuming that the invading fluid has zero viscosity. This implies that the pressure of the invading fluid is the hydrostatic pressure even inside the system. The capillary pressure will be neglected for the present analysis. We will show later that the capillary pressure often can not be neglected for short systems.



$$\nabla p_g = \rho_g g \nabla D = \rho_g g = \frac{\partial p_o}{\partial x}$$

$$\phi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k\,k_{ro}}{\mu_o}\left(\frac{\partial p_o}{\partial x} - \rho_o g \frac{\partial D}{\partial x}\right)\right]$$

$$= -\frac{k\left(\rho_o - \rho_g\right)g}{\mu_o}\frac{\partial k_{ro}}{\partial x}$$

$$k_{ro} = k_{ro}^o S^n$$

where

$$S = \frac{S_o - S_{or}}{S_{oi} - S_{or}}$$

$$dk_{ro} = \frac{k_{ro}^o n}{\left(S_{oi} - S_{or}\right)} S^{n-1} dS_o$$

$$\frac{\partial S_o}{\partial t} = -\frac{k\,k_{ro}^o\,\Delta\rho\,g}{\left(S_{oi} - S_{or}\right)\phi\,\mu_o} nS^{n-1}\frac{\partial S_o}{\partial x}$$

$$x_D = x/L$$

$$t_{Dg} = \frac{k\,k_{ro}^{o} \Delta\rho\, g\, t}{\left(S_{oi} - S_{or}\right)\phi\,\mu_o\, L}$$

$$\frac{\partial S}{\partial t_{Dg}} + nS^{n-1}\frac{\partial S}{\partial x_D} = 0$$

Drop the subscript on the dimensionless variables.

$$\left(\frac{dx}{dt}\right)_{dS=0} = -\frac{\partial S/\partial t}{\partial S/\partial x}$$
$$= nS^{n-1}$$

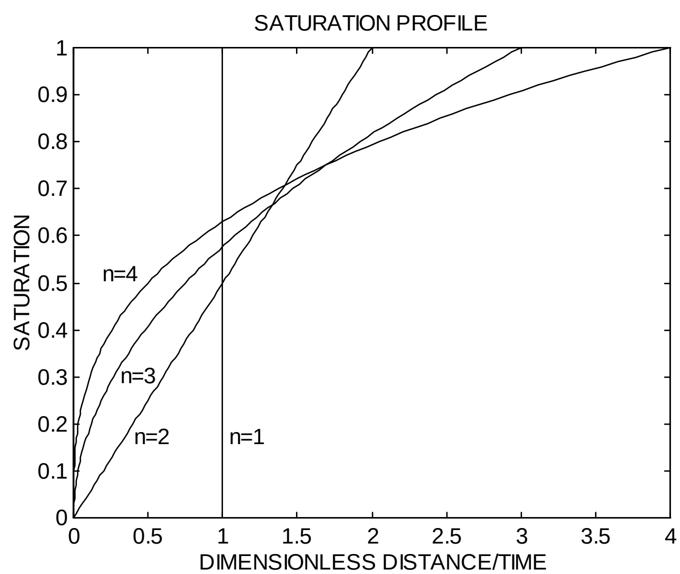
Assume uniform initial and boundary conditions. Then,

$$x(S) = nS^{n-1}\,t$$

$$S(x,t) = \left(\frac{x}{nt}\right)^{\frac{1}{n-1}}$$

or

$$\frac{x}{t} = n\,S^{n-1}$$



The profile is monotonic for *n>1*. Thus there will not be a shock. The initial condition is *S=1*. The breakthrough time is the time for the wave at *S=1-*ε to reach *x=1*.

$$t_{BT} = 1/n, \qquad n > 1$$

The recovery is found by calculating the average saturation.

$$\overline{S} = \int_0^1 S(x)\,dx$$

Prior to breakthrough the profile includes all saturations to *S=1* and a region of constant state at *S=1*. The location of the fastest wave at *S=1* is

$$x(S=1) = nt, \qquad t < t_{BT} = 1/n$$

$$\int_0^{x(S=1)} S(x)\,dx = (n-1)t, \qquad t < t_{BT}$$
$$\int_{x(S=1)}^{1} dx = 1 - nt, \qquad t < t_{BT}$$
$$\overline{S} = 1 - t, \qquad t < t_{BT}$$

After breakthrough the profile is integrated to *x=1*.

$$\overline{S} = \int_0^1 \left(\frac{x}{nt}\right)^{\frac{1}{n-1}} dx, \qquad t > t_{BT}$$
$$= \frac{(1 - 1/n)}{(nt)^{\frac{1}{n-1}}}, \qquad t > t_{BT}$$

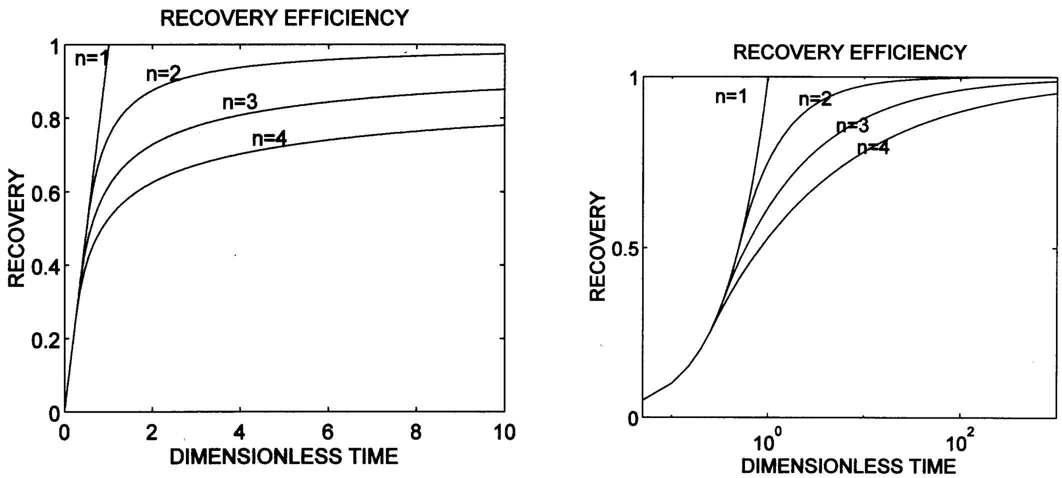
The normalized recovery efficiency is denoted by E_R.

$$E_R \equiv 1 - \overline{S}$$
$$E_R = t, \qquad t < t_{BT}$$
$$= 1 - \frac{(1 - 1/n)}{(nt)^{\frac{1}{n-1}}}, \qquad t > t_{BT}$$

The recovery efficiency at breakthrough is $E_{R,BT}$.

$$E_{R,BT} = 1/n, \qquad n \geq 1$$

This model can be used to predict the production by gravity drainage with air or gas as the invading fluid and the liquid retained by the hydrostatic saturation profile is negligible. This model has been used to estimate the relative permeability in centrifuge displacement (Hagoort 1980). To estimate the residual saturation it is necessary to normalize the recovery efficiency with respect to the pore volume rather than the movable pore volume since the latter is the quantity to be estimated.



The production as a function of dimensionless time is plotted above. In the case of n=1, the displacement is piston-like. The linear plots shows the production to become very slow after a dimensionless time of 10. However, the semilog plot shows that production continues but with a decreasing rate as the exponent becomes larger. Notice that in all cases the production is at a constant rate until breakthrough. The flux prior to breakthrough corresponds to the flux of oil at the initial relative permeability under the action of the buoyancy force.

$$u_o = -\frac{k\,k_{ro}^o}{\mu_o}\left(\frac{\partial p}{\partial x} - \rho_o g\right), \qquad t < t_{BT}$$

$$\frac{\partial p}{\partial x} = \rho_g g$$

$$u_o = \frac{k\,k_{ro}^o}{\mu_o}\left(\rho_o - \rho_g\right)g, \qquad t < t_{BT}$$

The oil flux is equal to the total flux since the gas only enters to replace the oil that is produced. Substituting the above equation into the definition of the gravity number gives the value of the gravity number prior to breakthrough.

$$N_g = +1, \qquad t < t_{BT}$$

This result is in agreement with the condition derived earlier to achieve piston-like displacement with *n=1* and infinite mobility ratio.

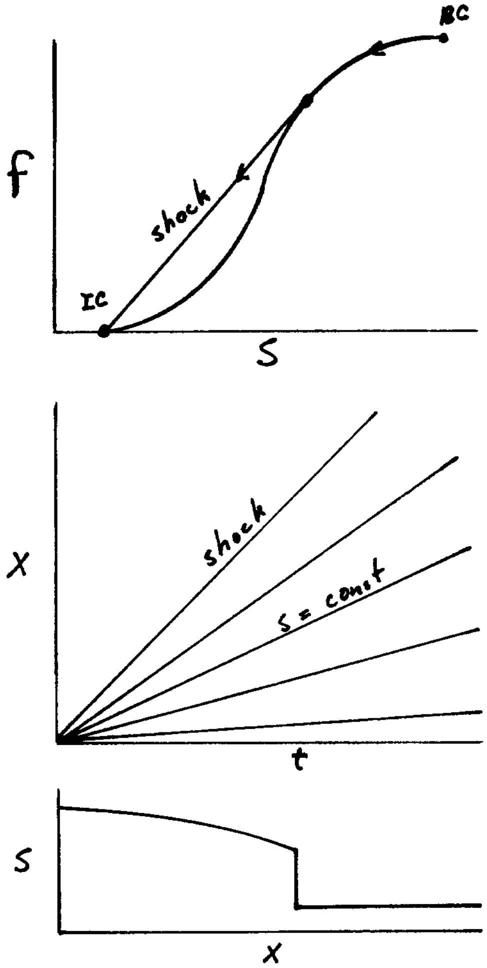
Interference of Waves

Up to now we have been solving problems with constant initial conditions and boundary conditions. This resulted in the trajectories all being straight lines originating from the origin of the (x,t) space. Now consider the case in which there is a step change in the boundary condition at time t_1.

The first figure shows the f vs S curve, distance-time diagram, and saturation profile for two phase displacement in which the initial and boundary conditions are constant. The trajectories of constant saturation are straight that originate at $(x,t)=(0,0)$. The arrows in the f versus S curve point in the direction of flow. Note that the slopes are nondecreasing in the direction of flow.

At time $t=t_1$ the boundary condition is changed to the same condition as the initial condition. The arrows show the new waves at time $t=t_1$ from the new BC to the old BC and then the same waves from the old BC to the IC. There is a spreading wave from the new BC up to a saturation $S=S_1$ and then a shock in going from S_1 to the old BC. We will call this shock the *back shock* to distinguish it from the *front shock*. The velocity of this back shock is greater that of the wave at the saturation of the old BC (which is zero velocity here). Thus the back shock will over take the slower waves and the saturation of the old BC will no longer exist. At some later time $t=t_2$, The back shock will jump to some saturation that is less than that of old BC and the saturation behind the shock will be a different value, S_2. Notice that the slope of the back shock is now steeper. The distance-time diagram shows the change in the velocity of the back shock as a trajectory with changing slope. The velocity and the saturation behind the back shock continually changes and the back shock continues to encounter different saturations ahead of it. Numerically the back shock can be calculated as a succession of short straight line segments in which the slope and saturation behind the shock is recalculated when the back shock encounters the trajectory of the next increment in saturation ahead of the back shock. The saturation profile shows the back shock "eating" into the original waves. New values of saturation behind the back shock are being created that have a velocity equal to the back shock velocity at the time the saturation value appears. Thus the trajectory of $S=S_2$ is tangent to the trajectory of the back shock at $t=t_2$. This will continue until the back shock over takes the front shock.

After the back shock over takes the front shock, there will only one shock and the saturation ahead of the shock will be the IC. The saturation behind the shock will be the saturation that existed behind the back shock. This shock will have velocities that is slower than that of the saturations behind the shock. Thus the saturation behind the shock will continue to change as well as the velocity of the shock. Again, the trajectory of the shock can be numerically calculated as a succession of straight line segments.


BC
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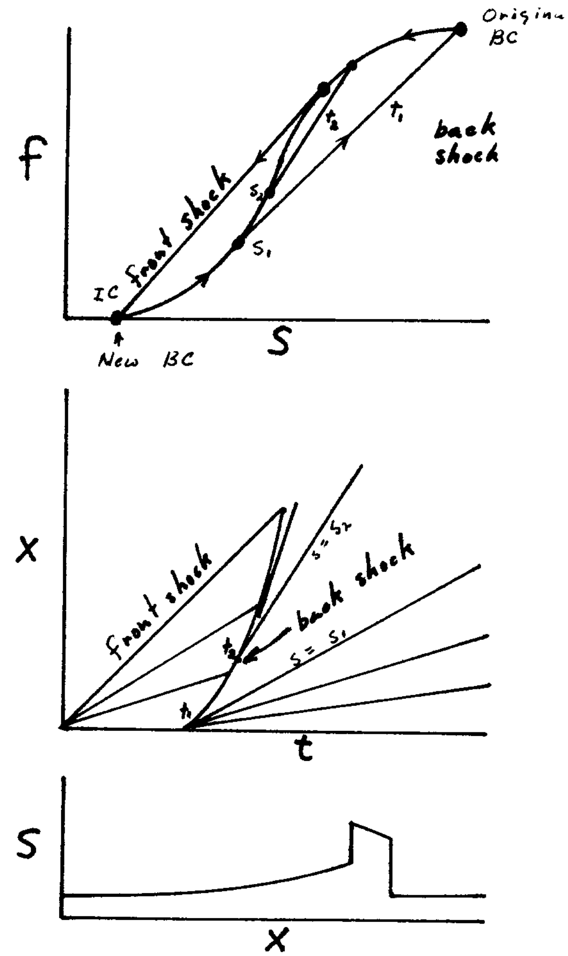
Original
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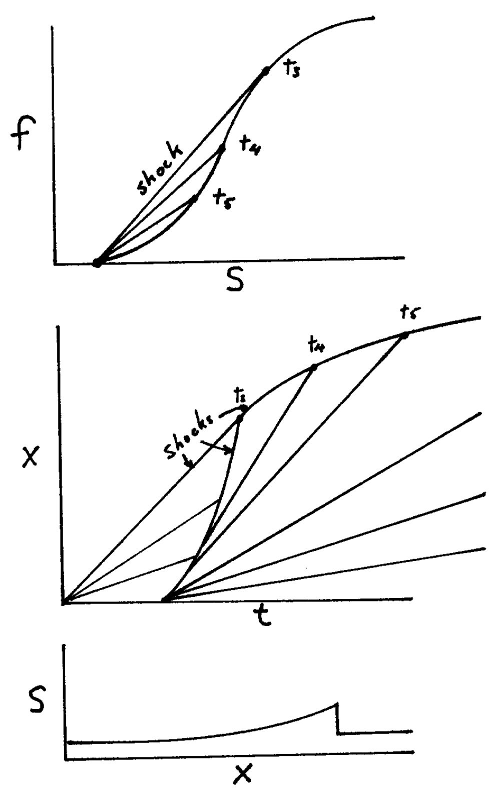
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An example of when two immiscible phases are injected as slugs is the water-alternating-gas process for mobility control for the miscible gas or CO_2 EOR process. The slugs of water reduce the gas saturation and thus the gas relative permeability. This reduces the effective mobility of the gas and improves the sweep efficiency compared to continuous injection of gas. The shape of the relative permeability (e.g. the exponent n) is an important parameter for this process. Wettability is also an important factor in this process. Water-wet conditions are not favorable for this process compared to mixed wet conditions because (1) the gas/water relative permeability ratio is larger for a given saturation compared to a mixed wet case, and (2) in water-wet conditions, the oil is trapped as isolated drops that may not be contacted by the gas while in mixed-wet cases, the oil is connected by thin films through which gas can diffuse.

Assignment 7.6: Interference of waves

For the oil-water displacement problem, let: $N_g=0$, $n=1$, and $M=0.5$, *or* 2.0. Let the first boundary condition be water injection followed by the second BC of oil injection at $t=0.1$. Plot the fractional flow with shock constructions, distance time diagram and saturation profiles for t=0.1, 0.2, 0.4, and 0.8.

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