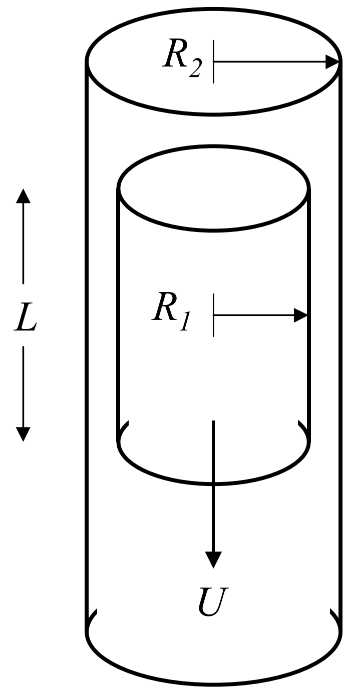
CENG 501 Examination Problem: **Estimation of Viscosity with a Falling - Cylinder Viscometer**



You are assigned to design a falling-cylinder viscometer to measure the viscosity of Newtonian liquids. A schematic diagram of the system is shown here. A solid cylinder of radius R_1 and length L falls under the influence of gravity through the test liquid in a cylindrical vessel of radius R_2 that is closed at the bottom. List the restrictions on the design of the apparatus and derive an expression for the viscosity as a function of the system parameters. Include the following steps:

1. List your assumptions. Explain consequence of assumption and how the system should be designed to satisfy the assumption.
2. Draw a schematic of the velocity profile in the annular gap between the inner and outer cylinders for the following coordinate reference frames:
   a) Coordinates fixed with respect to the outer cylinder.
   b) Coordinates moving with the inner cylinder.
3. What does the continuity equation reduce to as a result of your assumptions? What statement can be made of the velocity profile from your assumptions and the continuity equation?
4. What does the equations of motion reduce to as a result of your assumptions?
5. Express the velocity profile as a function of position in the annular gap with the potential gradient as a parameter. Let $\kappa = R_1 / R_2$.
6. Derive the relationship between the dimensionless pressure gradient and κ that results from the condition that the bottom of the vessel is closed.
7. Derive the expression for the shear stress on the inner cylinder.
8. Derive the expressions for the macroscopic forces acting on the inner cylinder.
9. Derive the expression for the dimensionless velocity of the inner cylinder as a function of κ.
10. Express the viscosity as a function of the velocity of the falling cylinder and the system parameters.

CENG 501 Final Examination, December 13-20, 2000

Instructions: The examination is closed book except for Aris, *Vectors, Tensors, and the Basic Equations* of Fluid Mechanics, Bird, Stewart and Lightfoot, *Transport Phenomena*, and your CENG 501 class notes and homework assignments. The examination problem is not to be discussed with anyone during the examination period. The time limit for the examination is three hours. It is due in Professor Hirasaki's office by the end of the examination period, December 13-20, 2000.

Time examination opened: ________________________
Time examination finished and sealed: _______________
Date: __________________

I pledge that I have followed the instructions: ____________________(signature)

CENG 501 Final Examination Problem, 2001

Flow of a fluid with a suddenly applied constant wall stress.

This problem is similar to that of flow $(-\infty < x < \infty, y > 0, t > 0)$ near a wall $(-\infty < x < \infty, y = 0)$ suddenly set into motion, except that the shear stress at the wall is constant rather than the velocity. Let the fluid be at rest before *t=0*. At time *t=0* a constant force is applied to the fluid at the wall, so that the shear stress τ_{yx} takes on a new constant value τ_o at *y=0* for *t>0*.

(a) Start with the continuity and Navier-Stokes equations and eliminate the terms that are identically zero. Differentiate the resulting equation with respect to distance from the wall and multiply by viscosity to derive an equation for the evolution of the shear stress. List all assumptions.

(b) Write the boundary and initial conditions for this equation.

(c) Solve for the time and distance dependence of the shear stress. Sketch the solution.

(d) Calculate the velocity profile from the solution of the shear stress. The following equation will be helpful.

$$\int_x^\infty (1-\operatorname{erf} u)\,du = \frac{1}{\sqrt{\pi}} e^{-x^2} - x(1-\operatorname{erf} x)$$

(e) Suppose the fluid is not infinite but rather there is another wall at a distance *2h* away from the original wall and it was also set into motion but in the opposite direction with the same wall shear stress. What are the stress and velocity profiles for the fluid between the two walls? Sketch and express as series solutions.

(f) What are the steady state stress and velocity profiles for the problem of part (e)? Sketch and express as analytical solutions.

CENG 501 Final Examination, 2001

RULES:

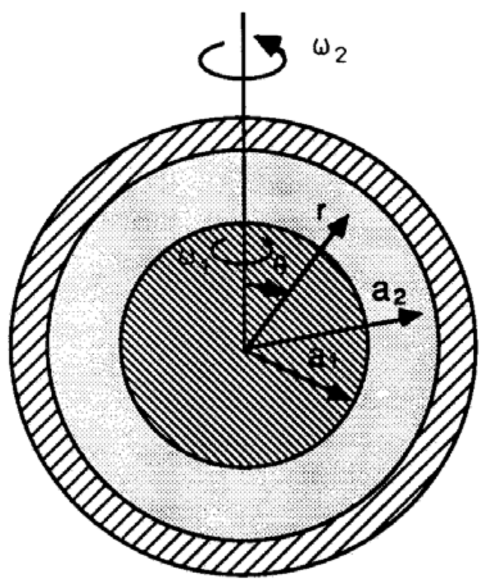
1. Open BSL, Aris, class notes and homework. Closed to all other texts or notes.
2. There is to be no discussion of the examination with anyone until after the final examination period.
3. Time period of examination: 3 hours. Write start and end time and date on examination.
4. Examination deadline: 5:00 PM, Friday, December 14.
5. Pledge that you have followed the rules.

Ph.D. Transport Qualification Examination
January 2002

Flow between Concentric Spheres Rotating about the Same Axis

A solid sphere within a fluid-filled, concentric spherical cavity is rotated about its axis at a constant angular velocity. ω_1 and ω_2 are the angular velocity of the inner and outer spheres, respectively. Derive expressions for the flow field and the torque required maintain the rotation.

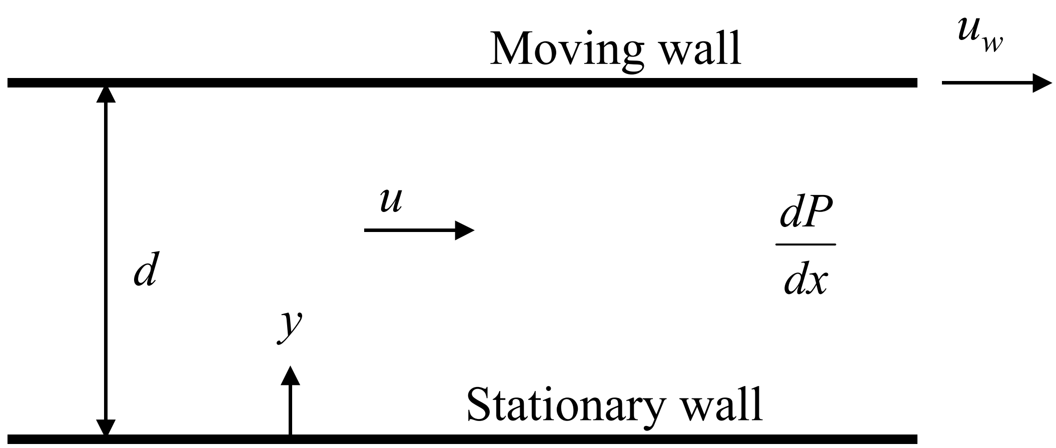
1. List the assumptions that are necessary to solve this problem during the time limit of the examination.
2. What are the differential equations governing the flow?
3. What are the boundary conditions?
4. Derive expression for the flow field.
5. Derive expression for the torque.
6. Give a qualitative explanation of the flow field that would result if the rotation rate was greater than that justified by your simplifying assumptions.



PhD Qualification Examination – Transport Phenomena

Planar Couette –Poiseuille Flow

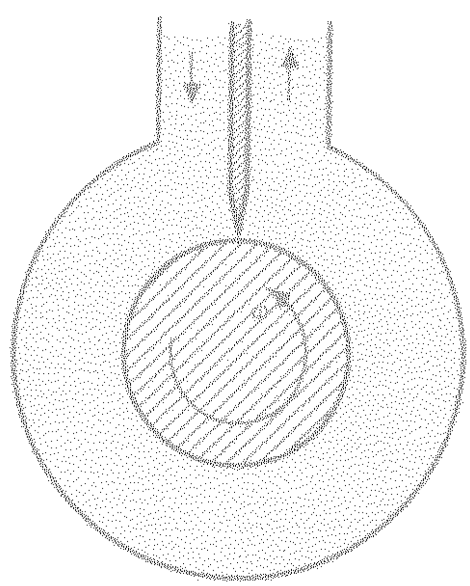
Combined forced and induced flow is called *Couette - Poiseuilli flow*. It is utilized in pumping very viscous fluids and in lubrication. Attention here is limited to the simple case of parallel plates and a Newtonian fluid. One wall is stationary and the other is moving. A pressure gradient is also imposed. The system can be treated as if infinite in extent.



a) Derive expressions for the velocity profile, average velocity, and the flux.
b) What is the flux the moving wall can produce if the pressure gradient is zero?
c) What is the pressure gradient that the moving wall can produce if the flux is zero?
d) Derive an expression for the shear stress in the fluid.
e) What is the pressure gradient that results in a zero shear stress at the stationary wall? At the moving wall?

Applications

Couette-Poiseuille pumps and extruders usually have the form of an annular channel or rotating screw as illustrated below rather than the parallel plates illustrated above.



Transport Qualifier Problem

Steady Radial Flow Between Parallel Disks

The geometry we will study is shown in Fig. 1. A viscous liquid is pumped continuously through an entrance tube to the axis of a pair of parallel disks. The flow then moves radially outward toward the circumference of the disks. The circumferential region is exposed to atmospheric pressure.

While the confined radial flow feature occurs in some injection molding processes, we will not end up with a model of injection molding here because injection molding is a *transient* process in which the liquid is pumped into a *closed* cavity against a steadily increasing pressure until the cavity is filled. Our example is considerably simplified in comparison to a commercial polymer process, but it allows us to practice working with the continuity and Navier-Stokes equations and relating physical concepts to mathematical formalism.

The goal of the analysis that follows is to find a relationship between the flow rate and the pressure forcing the flow.

Assignment:
1. List the assumptions necessary to derive a solution.
2. Derive an expression for the velocity profile.
3. Derive the relation between flow rate and pressure drop.

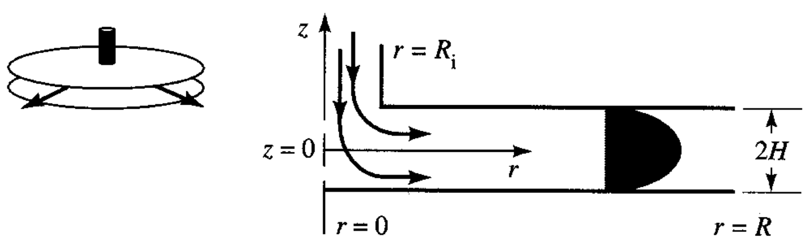


Fig. 1 Radial flow between parallel disks.

Transport Problem # 3

Floating A Disk on an Air Table

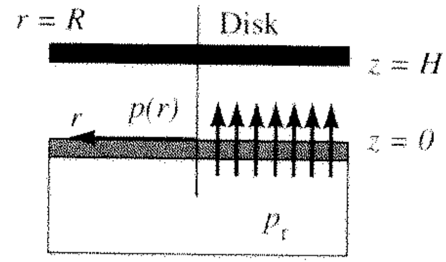


Fig. 1 Flow of air between a disk and a porous surface

A thin, solid disk sits on a porous surface, and air is blown upward through the porous surface against the face of the disk. At a high enough pressure in the reservoir, p_r, the pressure between the porous surface and the disk, $p(r)$, overcomes the weight of the disk, and the disk lifts off the surface and floats at a height *H*. **Assignment**: develop a model for the relationship of the film thickness (the floating height *H*) as a function of parameters that define this system. List all of your assumptions that are necessary to derive an analytical solution. The momentum balance equation for flow through porous media is

$$\mathbf{u} = -\frac{k}{\mu}\nabla p$$

where

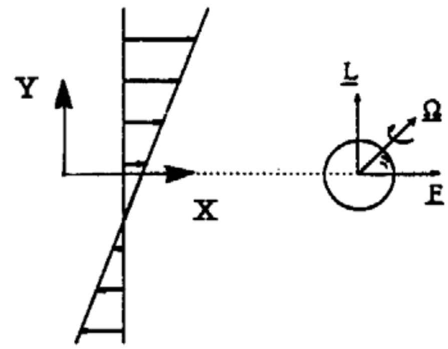
u is the superficial velocity or volumetric flux vector

k is the permeability of the porous media

μ is viscosity

Ph.D. Qualification Examination Transport Problem

Lift on a Rotating Sphere



Consider a solid sphere rotating about the *z*-axis with an angular velocity, Ω about its center, with the center fixed in space. The fluid at infinity has a translation relative to the sphere. In case (a) below, the fluid at infinity also has a simple shear flow imposed on the translation. The force on the sphere can be decomposed into a drag force parallel to the translation, *x*-direction, and a lift force in the direction perpendicular to the translation, *y*-direction. For the rotation direction and the velocity field as illustrated in the figure, determine whether the lift will be in the positive or negative y direction or zero for the following two cases:

a) The flow is creeping flow.
b) The Reynolds number is significant. The flow at infinity is only translation, i.e., no shear.

Give your reasoning. No credit will be given for guess.

Transport Phenomena Ph.D. Qualifying Examination

Gravity drainage of liquid from vertical wall

A semi-infinite smooth, vertical wall with a thin, Newtonian liquid film is surrounded by an inviscid fluid of density, ρ_g. The wall initially has a liquid film of uniform thickness, h_o. There is zero flux at the top end of the wall, i.e., $z=0$. Examine the transient drainage of this film by the action of gravity.

1) Prepare schematic diagram of system showing coordinates, initial condition, and boundary condition. (10% of total grade)
2) List the assumptions necessary to derive an analytical solution. (10%)
3) Simplify the equation of continuity and equation of motion for this system. (10%)
4) Derive the differential equation for the film thickness, $h(z,t)$. (20%)
5) Derive the analytical solution for the film thickness. (20%)
6) What is the velocity of the fastest wave (change in dependent variable)? What is the velocity of the slowest wave? (10%)
7) Find the similarity transformation that expresses the solution with a single independent variable. Express the dependent and independent variables as dimensionless variables such that solutions can be express by a single curve. (10%)
8) Prepare a sketch of the film thickness profile at different times. Does the slope of the film thickness increase or decrease with distance? (10%)

See relate problem, 2D.2 in BSL, 2^{nd} ed.

Ph.D. Qualification Examination: Transport #1
A Biomedical Flow Device

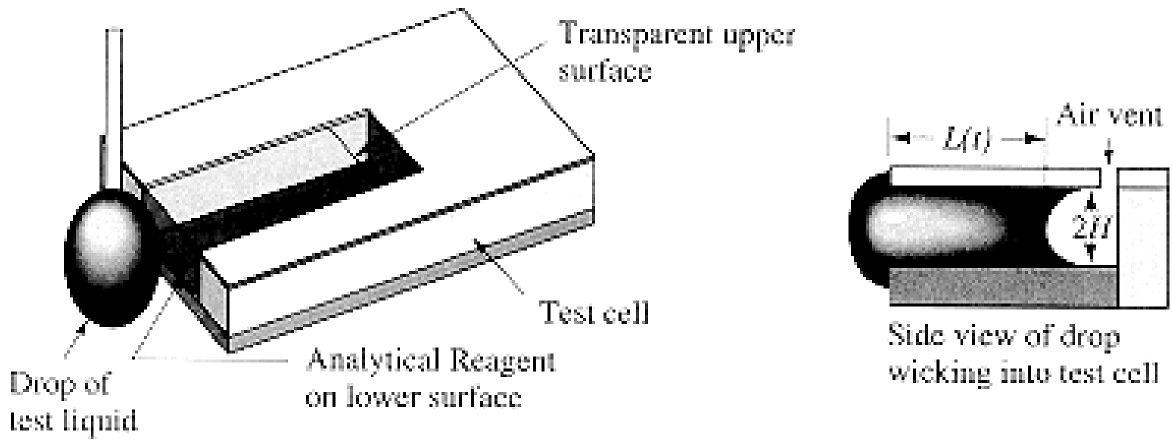


Fig. 1 Device in which a drop is drawn into a narrow planar channel by capillary action (wicking).

Many commercial systems for analyzing blood and urine samples are based on the principles of wicking and steady planar flow. Fig. 1 will help to define the fluid dynamics of wicking. In a typical flow device used in biomedical analysis, a drop of the sample to be tested is brought into contact with an opening in one end of the test cell. Capillary action is sufficient to draw the sample into the test chamber because the height of the opening is small, of the order of a millimeter. An air vent at the other end of the cell permits displaced air to escape and maintains atmospheric pressure at the advancing surface of the sample. The test chamber is essentially a region bounded by a pair of parallel planes. A pair of parallel sharp rails (Fig. 2) controls the position at which the drop stops. One of the planes is coated with an analytical reagent that dissolves into the sample and causes its color to change. An optical system "reads" the color of the sample through a transparent region of the second (upper) plane. The fluid dynamic issue in the design of this system is the rate at which the sample liquid is drawn into the test cell. We want to model this process so we can estimate that rate.

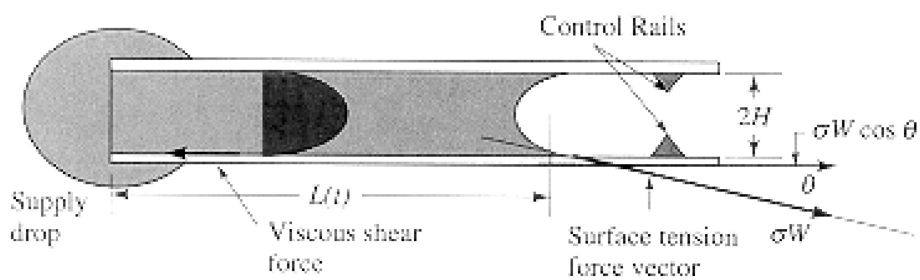


Fig. 2 Force diagram for wicking of a drop into the channel.

Our first step is to think about the physics of this system. Assuming that the plane of the device is horizontal, the only force available to "drive" the liquid into the cell arises from surface tension. The inflow is retarded by the viscous resistance of the liquid to deformation. If we sketched the flow region and labeled the various forces acting on the liquid, we would come out with a drawing like Fig. 2.

This is a very complex flow, largely because of the free surfaces at the two ends. In addition, it is a *transient* flow, because as the liquid intrudes into the channel, the area over which the viscous stresses are exerted increases. We might anticipate that the liquid will enter rapidly but slow down as viscous effects grow larger.

How does the surface tension force manifest itself, and "draw" the liquid into the channel? With reference to Fig. 2, we see that there is a force acting to the right, with a component in the plane of the channel given by

$$\frac{F_\sigma}{W} = 2\sigma\cos\theta$$

The factor of 2 accounts for the forces on the top and bottom surfaces. We write the force per unit width by using the width W of the channel in the direction normal to the page. We will assume that W >> H.

Opposing this force is the viscous resistance, but how are we to determine this term without first solving for the flow field? We will assume that we can approximate the flow field by taking the solution to a simpler problem that has some of the key features of this unsteady channel flow. The problem we select is that of *fully developed steady* flow of a viscous liquid through a parallel plate channel. The width and thickness of the channel is completely filled with the liquid. Our thinking is that when the penetration length L has become large compared to H the end effects may not be very important and the steady flow model may provide at least an approximation to the viscous resistance of the unsteady case.

Assignment:

1. Derive an expression for the distance the meniscus has traveled into the channel as a function of time after the drop was introduced to the opening of the device. Derive the expression for the time to reach the end of the channel.
2. Start with the equation of continuity and the equations of motion. State the assumptions made at each step. Draw sketch of the system at each step.
3. When may the assumptions not be strictly valid?