Problem 1

a) The transfer function of this process can be expressed as the product of three first order lag transfer functions. The AR and phase angles of a general 1st order lag are:

$$AR = \frac{K}{\sqrt{\tau^2\omega^2+1}} \text{ and } \phi = \tan^{-1}(-\tau\omega) \tag{S1.1}$$

Thus, applying the principle of superposition we get:

$$AR = \frac{3}{\sqrt{64\omega^2+1}}\frac{1}{\sqrt{4\omega^2+1}}\frac{1}{\sqrt{\omega^2+1}} \tag{S1.2}$$

$$\phi = \tan^{-1}(-8\omega)+\tan^{-1}(-2\omega)+\tan^{-1}(-\omega) \tag{S1.3}$$

b) Asymptotically as w goes to infinity, the AR is approximated by

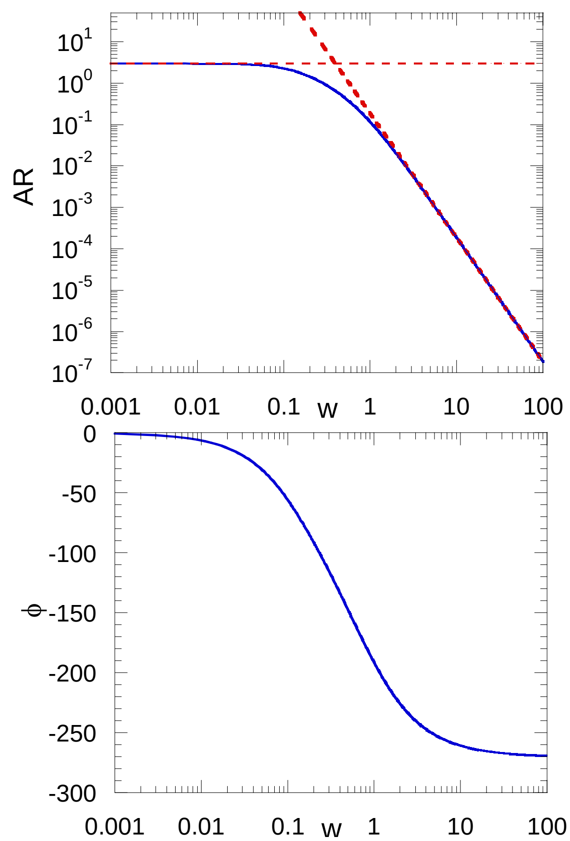
$$AR = \frac{3}{8\omega}\frac{1}{2\omega}\frac{1}{\omega} \tag{S1.4}$$

while for ω going to zero, AR goes to 3. Thus, the corner frequency will be obtained by solving the equations

$$3 = \frac{3}{8\omega}\frac{1}{2\omega}\frac{1}{\omega} \quad \rightarrow \quad \omega = 0.397 \tag{S1.5}$$

Taking logarithms in the asymptotic expression for AR, the asymptote slope is –3.

c) The Bode plots are obtained computationally (i.e. give an array of values for ω and find the corresponding phase angle and amplitude ratios from the above formulae). They are shown in figure 1.



Problem 1: Bode plots

Problem 2

a) The transfer function of the first process can be viewed as a product of 3 transfer functions: one 1st order lead and two 1st order lags. The transfer function of the second process can be viewed as a product of 3 functions: two 1st order lags and one transfer function with a positive zero. The AR and phase angles of a general 1st order lag are:

$$AR = \frac{K}{\sqrt{\tau^2\omega^2+1}} \text{ and } \phi = \tan^{-1}(-\tau\omega) \tag{S2.1}$$

The AR and phase angles of a general 1st order lead (G(s)=τs+1) are:

$$AR = \sqrt{\tau^2\omega^2+1} \text{ and } \phi = \tan^{-1}(\tau\omega) \tag{S2.2}$$

The AR and phase angles of a general process with one positive zero (G(s)=-τs+1) are:

$$AR = \sqrt{\tau^2\omega^2+1} \text{ and } \phi = \tan^{-1}(-\tau\omega) \tag{S2.3}$$

Applying the principle of superposition to the first process one obtains:

$$AR = \frac{1}{\sqrt{0.3^2\omega^2+1}}\frac{1}{\sqrt{0.3^2\omega^2+1}}\sqrt{10^2\omega^2+1} \tag{S2.4}$$

$$\phi = \tan^{-1}(-0.3\omega)+\tan^{-1}(-0.3\omega)+\tan^{-1}(10\omega) \tag{S2.5}$$

Applying the principle of superposition to the second process we get:

$$AR = \frac{1}{\sqrt{0.3^2\omega^2+1}}\frac{1}{\sqrt{0.3^2\omega^2+1}}\sqrt{10^2\omega^2+1} \tag{S2.6}$$

$$\phi = \tan^{-1}(-0.3\omega)+\tan^{-1}(-0.3\omega)+\tan^{-1}(-10\omega) \tag{S2.7}$$

b) In both cases the AR is the same. As ω goes to zero, AR goes to 1. As ω goes to infinity AR goes to $AR = \frac{1}{0.3\omega}\frac{1}{0.3\omega}10\omega = \frac{1000}{9\omega}$. From this we see that the asymptote slope is –1 in a log-log plot, while the corner frequency is obtained by solving the equation:

$$AR = 1 = \frac{1}{0.3\omega}\frac{1}{0.3\omega}10\omega = \frac{1000}{9\omega} \quad \rightarrow \quad \omega = \frac{1000}{9} \tag{S2.8}$$

c) The Bode plots are obtained computationally (i.e. give an array of values for w and find the corresponding phase angle and amplitude ratios from the above formulae). They are illustrated in figure 2.

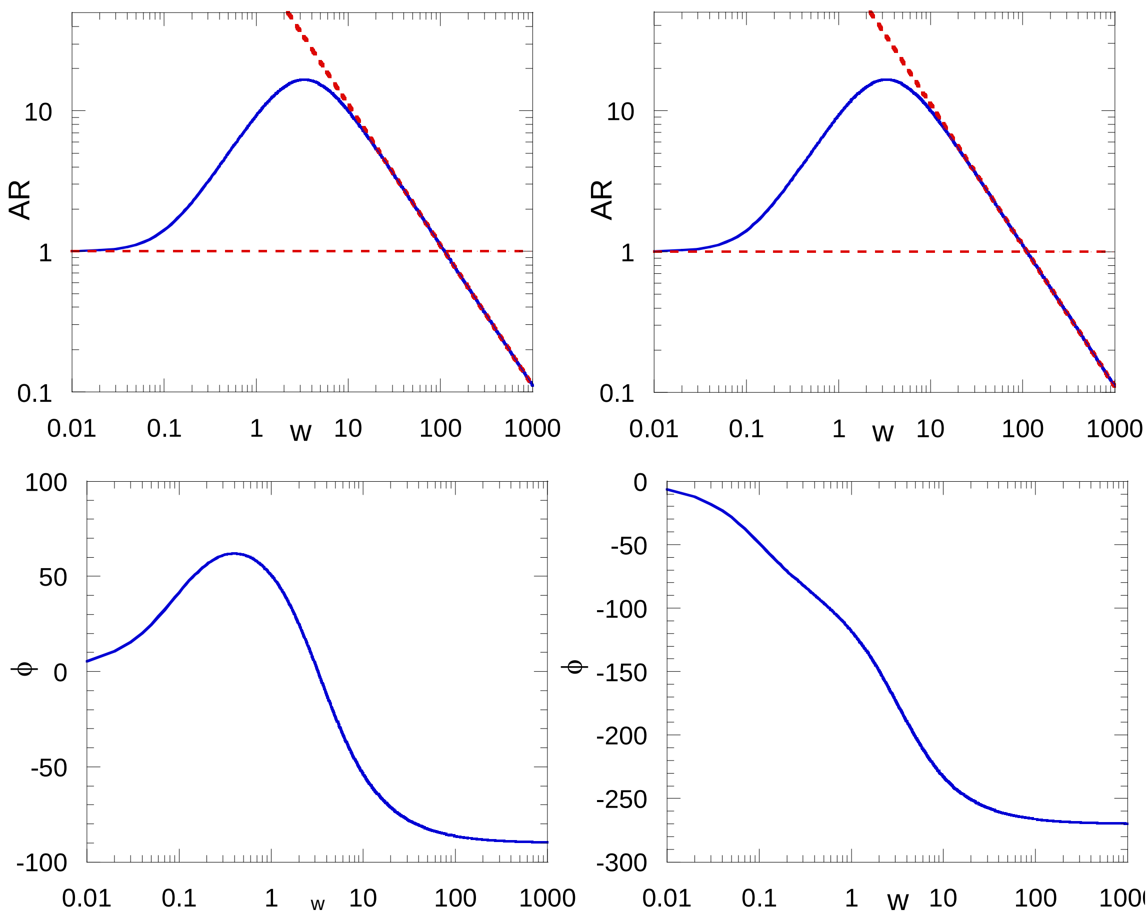


Figure 2: Bode plots

Problem 3

This process is a 1st order lag with time delay. The AR and phase angles of a general 1st order lag are:

$$AR = \frac{K}{\sqrt{\tau^2\omega^2 + 1}} \text{ and } \phi = \tan^{-1}(-\tau\omega) \tag{S3.1}$$

The AR and phase angles of a pure general dead time process are:

$$AR = 1 \text{ and } \phi = -t_d w \tag{S3.2}$$

Applying the superposition principle and substituting τ=10, t_d=5, K=1 one obtains:

$$AR = \frac{1}{\sqrt{100\omega^2 + 1}} \tag{S3.3}$$

$$\phi = \tan^{-1}(-10\omega) - 5\omega \tag{S3.4}$$

Solving (S3.3) for ω provides the crossover frequency ω_{CO}:

$$\omega_{CO} = 0.36733 \text{ rad/min} \tag{S3.5}$$

Substituting (S3.5) into (S3.3) provides the AR which at the crossover frequency:

$$AR(\omega_{CO}) = 0.2629 \tag{S3.6}$$

Thus, the ultimate period and ultimate gain are respectively

$$P_u = \frac{2\pi}{\omega_{CO}} = 17.105 \text{ min/cycle} \tag{S3.7}$$

and:

$$K_u = \frac{1}{AR(\omega_{CO})} = 3.80 \tag{S3.8}$$

Therefore, according to the Ziegler-Nichols controller tuning technique, the settings for the PI controller are:

$$K_c = K_u/2.2 = 1.729 \text{ and } \tau_I = P_u/1.2 = 14.254 \tag{S3.9}$$

while for the PID controller they are:

$$K_c = K_u/1.7 = 2.2375 \text{ and } \tau_I = P_u/2 = 8.5525 \text{ and } \tau_D = P_u/8 = 2.138 \tag{S3.10}$$

Implementing these two controllers using Simulink (see the block diagram in figure 3) for a unit step change in the set-point yields the following ISE, IAE and ITAE indices:

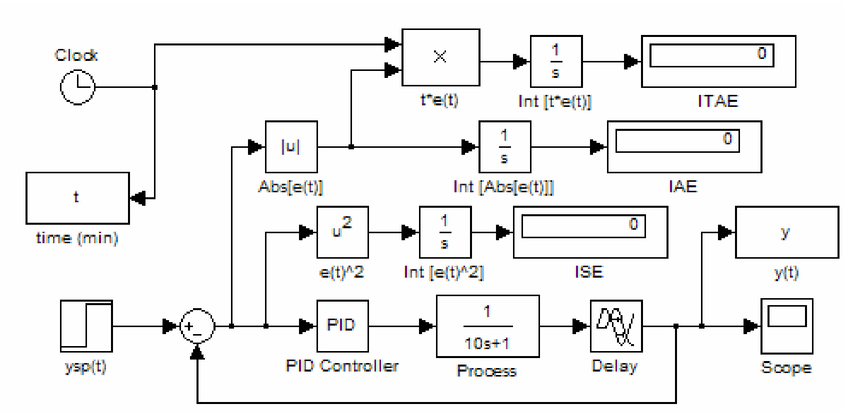


Figure 3: Simulink block diagram

	ISE	*IAE*	*ITAE*
PI	7.315	10.74	113.7
PID	6.373	9.542	84.99