Surface Area

The specific surface area is a dominant parameter in models for permeability and in the transport of a species that can adsorb on the mineral surfaces. The specific surface area is usually expressed as square meters of surface per gram of solid. Here we will factor out the grain density and express the specific surface area as square meters per cubic centimeter of solid. (Later we will express the specific surface as a ratio of pore surface/pore volume.) The solid will be modeled as an oblate spheroid. This is a solid of revolution of an ellipse about its minor axis. The minor radius is *b* and the major radius is *a*.

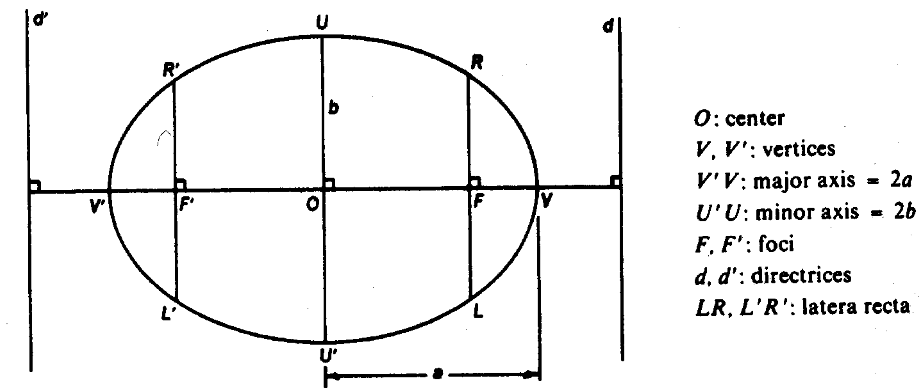


Fig. 3.31 Parameters of an ellipse (CRC Standard Mathematical Tables, 1987)

The ratio, *Sb/V*, is given by the following formula. (Mensuration formulas)

$$\left(\frac{S\,b}{V}\right)=\frac{3}{2}+\frac{3}{4}\left(\frac{1-\varepsilon^2}{\varepsilon}\right)\log_e\left(\frac{1+\varepsilon}{1-\varepsilon}\right)$$

where the eccentricity is

$$\varepsilon=\frac{\sqrt{a^2-b^2}}{a}$$

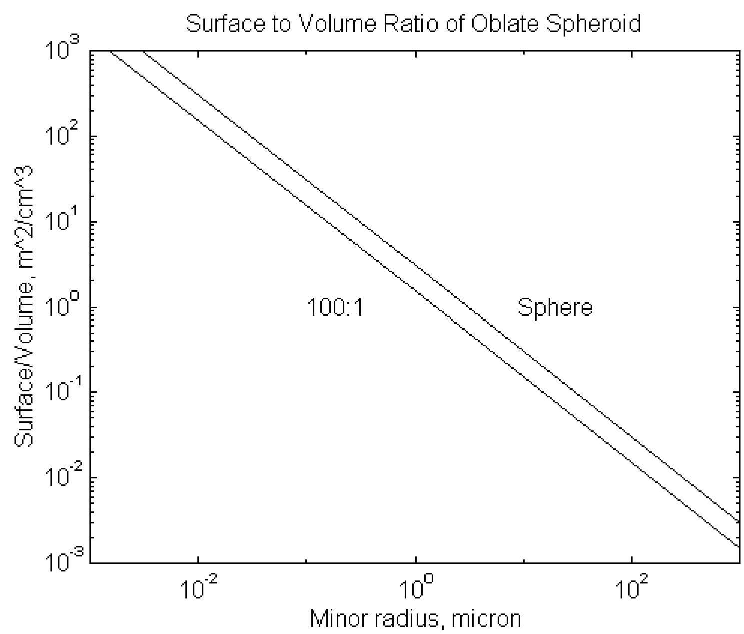


Fig 3.32 Surface to volume ratio of oblate spheroid

The group, *(Sb/V)*, will have consistent units if *S* is in square meters, *V* is in cubic centimeters, and b is in microns. Figure 3.32 plots the specific surface as a function of the minor radius, *b*. The grain density (2.65 gm/cm^3 for quartz) has been factored out and the specific surface area is expressed as per unit cm^3 rather than gram. A upper coarse sand grain has a radius of about 10^3 microns (one millimeter) and it has a surface area of about 10^{-3} m^2 /cm^3. A silt or clay particle with a minor radius of about 1.0 micron has a surface area of about 1.0 m^2/cm^3. A smectite sheet with a thickness of about 10^{-3} micron (1.0 nm) will have a surface area of about 10^3 m^2/cm^3. (Note: Is something is wrong here? The sphere appears to have a greater specific area than an oblate spheroid. A sphere should be a body of minimum area for a given volume. Answer: For the same volume, an oblate spheroid will have its surface to volume ratio increasing in proportion to the 2/3 power of the aspect ratio. The specific surface is plotted as a function of the radius of the minor axis. The major axis is greater than the minor axis ratio by the aspect ratio.)

When evaluating adsorption, the specific surface area of sand grains usually is not of much interest compared to the clays contained in the rock. For example the following table illustrates the range of specific areas that can be expected from clays (Corey 1990)

Clay type	Area, m^2/gram
kaolinite	45
illite	175
montmorillonite	800

In addition to the importance of the surface/volume ratio to adsorption on porous media, the ratio of surface area to pore volume will be shown later to be an important parameter in models of permeability and NMR relaxation of fluids in the pore space. The expression for the specific surface shows the surface to pore volume ratio to be inversely proportional to the length of the minor axis, *b*, for a given eccentricity. The constant of proportionality is 3 for a sphere and is equal to 3/2 for a thin disk. (Note: I think it should be 2 for a thin disk.)

Porosity

Porosity is the fraction (or percent) of the rock bulk volume occupied by pore space. The porosity may be divided into ***macro porosity*** and ***micro porosity*** in rocks that have a bimodal pore size distribution. Some examples include: (1) sandstones with a significant amount of clays, (2) sandstones with microporous chert grains, i.e., ***interparticle*** and ***intraparticle*** porosity, (3) carbonate rocks with vuggy porosity (caverns are an extreme case) and matrix porosity, (4) carbonate rocks with moldic porosity and matrix porosity, (5) carbonate rocks with interparticle porosity and intercrystalline porosity, (6) fracture porosity and matrix porosity. The total porosity can also be divided into ***effective*** porosity and ***ineffective*** porosity. Ineffective pores are pores with no openings or zero coordination number. Effective porosity can be divided into Cul-de-sac or dead-end pores with a coordination number of one and catenary pores with coordination number of two or more. These types of porosity are illustrated in Fig. 3.33.

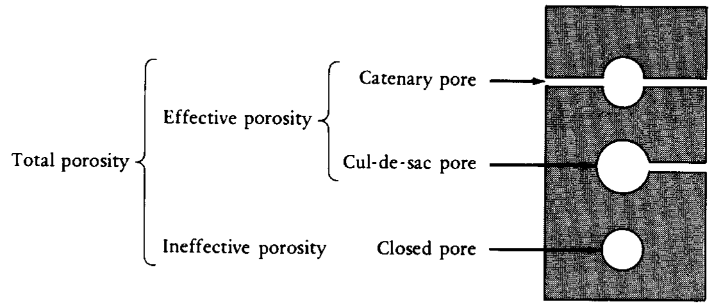


Fig. 3.33 The three basic types of porosity. (Selley 1985)

Sandstones

The rules for the factors governing the magnitude of the porosity is different for clastic (sandstone) and carbonate rocks. The following relationships between porosity and textural properties apply for sandstones (Jorden and Campbell 1984). Also see Fig. 3.7.

1. Porosity is independent of grain size for the same sorting.
2. Porosity decreases as sorting becomes poorer. See Fig. 3.7 and 3.34.
3. Porosity increases as grain sphericity (shape) decreases and as grain angularity (roundness) decreases. The general, though not universal, tendency is for diagenesis to reduce original porosity of clastic rocks.

The alteration of porosity through diagenesis is illustrated in Fig. 3.35. The porosity of the original sediment may originally be 40%-50%. In regions of rapid sedimentation such as in a river delta, ***compaction*** is the primary diagenetic alteration mechanism. Subsidence may accompany the compaction. Dissolution of some minerals and precipitation can result in consolidation of the rock and reduction of porosity by the process of ***cementation***.

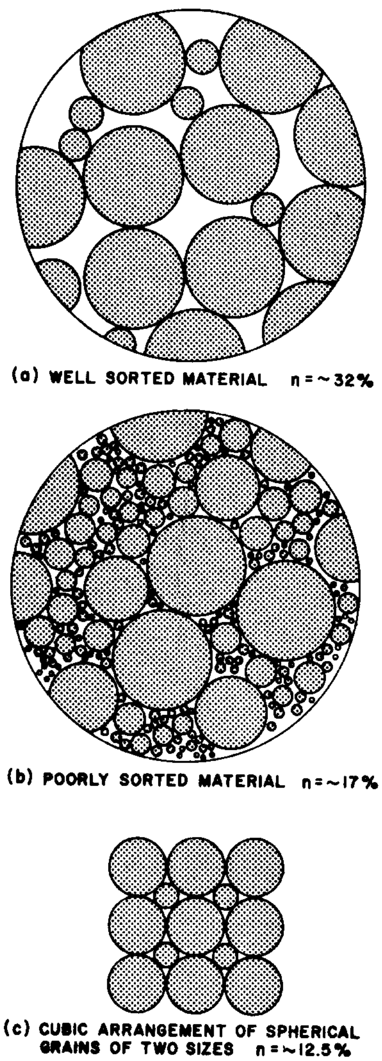


Fig. 3.34 Effect of sorting on porosity (Bear 1972)

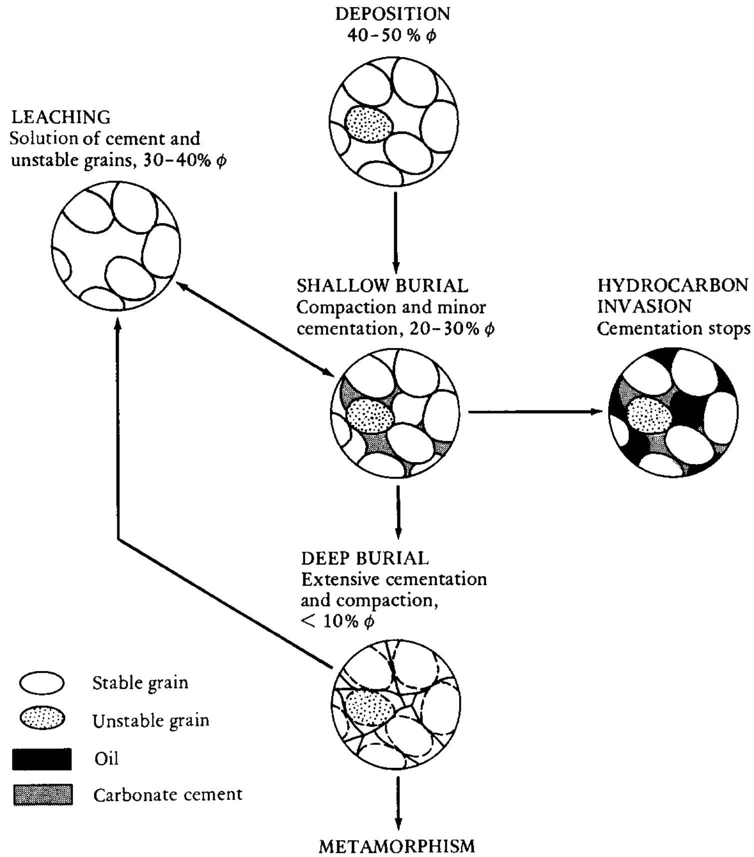


Fig. 3.35 Diagenetic pathways of sandstones (Selley, 1985)

Carbonate rocks

At deposition, carbonate sediments are highly to very highly porous. Some sediments have porosity ranging from 0.40 to 0.78 at deposition. The following relationships between porosity and textural properties apply to carbonates (Jorden and Campbell 1984).

1. Porosity is not correlated strongly with either median grain size or sorting.
2. Porosity is controlled largely by the amount of fines present -i.e., the larger the precent fines, the larger the porosity.
3. Diagenesis of carbonate rocks can result in porosity that is either significantly less or greater than original porosity.

Permeability

The mobility, denoted by λ, is a transport coefficient of the porous medium for the volumetric flux of a fluid just as electrical conductivity and thermal conductivity are transport coefficients for the flow of electrical current and heat, respectively. This transport coefficient was divided by Darcy into two factors ($\lambda = k/\mu$), the permeability, k, which is a property of the porous medium and the viscosity, μ, which is a property of the fluid. The permeability was originally conceived as a constant of a particular medium. However, in reality the permeability is generally not spatially uniform, i.e., porous media are usually heterogeneous, depends on direction, i.e., is not isotropic, depends on the current stress conditions and past stress history, is a function of the electrolyte composition of the fluids, and depends on the amount and distribution of the fluid phases, i.e., depends on relativity permeability. It is because of this highly variable nature of permeability that we need to know the factors that govern the value of permeability. We will describe two models of the permeability. They are both based on a bundle of capillary tubes model. However, one is based on a packed bed of spherical particles and the other is based on a pore size distribution model.

Packed Bed of Spherical Particles

One model for relating the flow resistance of porous media to the dimensions of the pores or particles is the Blake-Kozeny model (Bird, Stewart, and Lightfoot 1960). This model represents the pore network of the porous medium as a bundle of capillary tubes with an average or equivalent radius, *R*, and an average length, *L'*, that is somewhat longer than the system length. The effective radius is related to a particle diameter, D_p, by applying the hydraulic radius concept and assuming that the porous medium is a bed of uniform particles. The resulting expression is then compared with Darcy's law to determine an expression for the permeability of the medium in terms of the particle diameter and porosity.

Darcy's law is an empirical relationship between the flux and the driving force for laminar, single phase flow through porous media.

$$
\begin{aligned}
u &= q / A \\
&= -\lambda \frac{\Delta P}{L} \\
&= -\frac{k}{\mu}\frac{\Delta P}{L}
\end{aligned}
$$

where

$$P = p - \rho\, g\, z$$

The constant of proportionality between the flux and the driving force, commonly known as the *mobility*, λ, is directly proportional to the ***permeability*, *k*,** which is a property of the porous medium, and inversely proportional to the *viscosity,* μ, which is a property of the fluid.

The porous medium is modeled as a bundle of capillary tubes with a length *L'*, that is greater than the system length, *L*, due to the ***tortuosity*** of the pore network. $\tau = \left(L'/L \right)^2$ It has been empirically determined that this tortuosity factor can be approximated by the factor 25/12.

$$\tau = \left(L'/L \right)^2 = 25/12$$

The average velocity in a capillary tube is given by the Hagen-Poiseuille law.

$$< v >= \frac{R^2 \left(P_o - P_L \right)}{8\mu L'}$$

The average velocity in the bundle of tubes is greater than the average velocity in the pore space of the medium because of the greater length traversed in the tortuous capillary. Alternatively, it can be argued that the fluid it the porous medium must also traverse a greater length but the transverse components of velocity cancel in averaging over the porous medium and thus the average velocity in the pores of the medium is less than the average velocity in a tortuous capillary.

$$v_{pore} = \langle v \rangle_{capillary} \left(L / L' \right)$$

The average velocity of the fluid in the pores (***v****, the* ***interstitial velocity***) is related to the flux ***(u*, *superficial velocity***, *filtration velocity, or* ***Darcy velocity***) by the porosity of the porous medium (ϕ, pore volume/bulk volume). If the porous medium is random, then the fraction of the cross-sectional area open to pores is equal to the porosity. Thus the flux is

$$
\begin{aligned}
u &= \frac{q}{A} \\
&= \phi\, v \\
&= \phi < v > \left(L / L' \right) \\
&= \frac{\phi\, R^2 \left(P_o - P_L \right)}{8\mu\, L' \left(L'/ L \right)} \\
& \frac{\phi\, R^2 \left(P_o - P_L \right)}{8\mu\, L\, \tau} \\
&= \frac{3\phi\, R^2 \left(P_o - P_L \right)}{50\mu\, L}, \qquad \tau = \frac{25}{12}
\end{aligned}
$$

By comparing the above equation with Darcy's law we have,

$$
\begin{aligned}
k &= \frac{3\phi\, R^2}{50} \\
R &= \sqrt{\frac{50k}{3\phi}}
\end{aligned}
$$

The above equation is an expression for the equivalent pore radius of the porous medium assuming a bundle of capillary tubes model with a tortuosity of 25/12.

The wetted surface of a porous medium can be related to the permeability and porosity by introducing the concept of the *hydraulic radius*. For flow in a capillary, the hydraulic radius is related to the radius as follows.

$$
\begin{aligned}
R_h &= \frac{\pi\, R^2}{2\pi\, R} \\
&= \frac{R}{2}
\end{aligned}
$$

In porous media, the hydraulic radius can be determined as follows:

$$
\begin{aligned}
R_h &= \frac{\text{cross section available for flow}}{\text{wetted primeter}} \\
&= \frac{\text{volume available for flow}}{\text{total wetted surface}} \\
&= \frac{\left(\text{volume of voids}\right) / \left(\text{volume of bed}\right)}{\left(\text{wetted surface}\right) / \left(\text{volume of bed}\right)} \\
&= \frac{\phi}{a}
\end{aligned}
$$

Note: The specific surface area in this equation is per bulk volume rather than grain volume as discussed earlier. We can eliminate the hydraulic radius between the last two equations to express the equivalent pore radius in terms of porosity and specific area.

$$R = \frac{2\phi}{a}$$

Substituting into the equation we derived earlier for the flux through a bundle of capillary tubes, we have

$$u = \frac{6\phi^3 \left(P_o - P_L\right)}{25\, a^2 \mu\, L}$$

Comparing this equation with Darcy's law, we have

$$k = \frac{6\phi^3}{25a^2}$$

$$a = \sqrt{\frac{6\phi^3}{25k}}$$

This equation relates the wetted area of the porous medium to the permeability and porosity.

If we assume the porous medium to be a packed bed of uniform spheres, the particle diameter, D_p, can be related to the permeability and porosity. The specific area (per unit bulk volume) for a spherical bead pack with a porosity ϕ is

$$a = \frac{\pi\, D_p^2}{1/6\,\pi\, D_p^3}(1-\phi)$$

$$= \frac{6}{D_p}\left(1-\phi\right)$$

This specific area is the surface area per unit volume of bed. The surface area per unit volume of solid can be determined by dividing by the matrix volume/bed volume. This is the same as the ratio of the area and volume of a sphere.

$$\frac{a}{1-\phi} = \frac{6}{D_p}$$

By eliminating the specific area between the last two equations, we have an equation for the permeability as a function of the particle diameter.

$$k = \frac{\phi^3 D_p^2}{150(1-\phi)^2}$$

$$D_p = \sqrt{\frac{150(1-\phi)^2 k}{\phi^3}}$$

Symbols and conversion to consistent (SI) units (The SI Metric System of Units and SPE Metric Standard, SPE, 1984)

Quantity	Symbol	SI units	Customary units	multiply customary units by
specific area (/ bulk vol)	a	m^2/m^3	m^2/cm^3	1.0 E+06
area	A	m^2	ft^2	9.2903 E-02
particle diameter	D_p	m	mm	1.0 E-03
			μm	1.0 E-06
permeability	k	m^2	μm^2	1.0 E-12
			darcy	9.8692 E-13
			md	9.8692 E-16
length	L	m	ft	3.048 E-01
pressure	p	Pa	kPa	1.0 E+03
			psi	6.8947 E+03
flow rate	q	m^3/s	cm^3/s	1.0 E-06
radius	R	m		
superficial velocity	u	m/s	ft/D	3.5278 E-06
interstitial velocity	v	m/s		
volume	V	m^3	ft^3	2.8317 E-02
			bbl	1.5899 E-01
viscosity	μ	Pa·s	cp	1.0 E-03
porosity	ϕ			
surface or interfacial tension	σ	N/m	mN/m	1.0 E-03
	σ	N/m	dyne/cm	1.0 E-03

The following table lists the permeability and porosity of some sand packs as a function of grain size and sorting.

Permeability (darcies) of artificially mixed and wet-packed sand [Jorden and Campbell 1984 (Beard and Weyl 1973)]

	Size							
	Coarse		Medium		Fine		Very Fine	
Sorting	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
Extremely well sorted	475.	238.	119.	59.	30.	15.	7.4	3.7
Very well sorted	458.	239.	115.	57.	29.	14.	7.2	3.6
Well sorted	302.	151.	76.	38.	19.	9.4	4.7	2.4
Moderately sorted	110.	55.	28.	14.	7.	3.5		
Poorly sorted	45.	23.	12.	6.				
Very poorly sorted	14.	7.	3.5					

Porosity of artificially mixed and wet-packed sand [Jorden and Campbell 1984 (Beard and Weyl 1973)]

	Size							
	Coarse		Medium		Fine		Very Fine	
Sorting	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
Extremely well sorted	0.431	0.428	0.417	0.413	0.413	0.435	0.423	0.430
Very well sorted	0.408	0.415	0.402	0.402	0.398	0.408	0.412	0.418
Well sorted	0.380	0.384	0.381	0.388	0.391	0.397	0.402	0.398
Moderately sorted	0.324	0.333	0.342	0.349	0.339	0.343	0.356	0.331
Poorly sorted	0.271	0.298	0.315	0.313	0.304	0.310	0.305	0.342
Very poorly sorted	0.286	0.252	0.258	0.234	0.285	0.290	0.301	0.326

Assignment 3.4 Calculation of Permeability as a Function of Grain Size

Calculate and plot the permeability (darcy) as a function grain size (mm) and porosity for grain size in the range (10^{-4} mm to 10 mm) and porosity of (0.2, 0.3. 0.4, 0.5). Also plot the measured values for the extremely well sorted sand packs listed above. Post the average value of the sand pack porosity. Use Fig. 2.8 to determine the grain size. For a porosity of 0.4 tabulate the approximate grain size (in descriptive scale, mm, and μm) that will result in a permeability of 100 darcy, 1 darcy, and 1 md.

Estimation of Permeability from Pore Size Distribution

Rapid methods to estimate rock permeability has always been a high priority in the petroleum industry. Mercury porosimetry for measuring capillary pressure and calculation of permeability therefrom was introduced by Bob Purcell of Shell Oil Co. in 1949. The method treats the porous medium as a bundle of capillary tubes with the pore size distribution quantified by the mercury-air capillary pressure curve. The tortuosity is an empirical factor that brings the calculation into correspondence with measured permeability.

The average velocity in a capillary tube of radius R_i is described by the Hagen-Poiseuille law.

$$\langle v \rangle_i = \frac{R_i^2 \, \Delta P}{8\mu \, L}$$

The capillary radius can be determined for the relation of the capillary pressure to an equivalent pore radius.

$$\left(P_c\right)_i = \frac{2\sigma \cos\theta}{R_i}, \qquad R_i = \frac{2\sigma \cos\theta}{\left(P_c\right)_i}$$

Thus the average velocity in a capillary tube can be expressed in terms of the capillary pressure at which that capillary is being entered be a nonwetting fluid.

$$\langle v \rangle_i = \frac{\left(\sigma \cos\theta\right)^2 \, \Delta P}{2\mu \, L} \frac{1}{\left(P_c\right)_i^2}$$

Let $S(P_c)$ denote the fraction of the pore space that is occupied by the wetting phase when the capillary pressure is equal to P_c. Then dS is the incremental fraction of the pore space corresponding to P_c and P_c-dP_c. The interstitial velocity is the integral over all pores.

$$v = \int_0^1 \langle v \rangle \, dS$$

The superficial velocity (q/A) is then as follows.

$$u = \phi \, v = \frac{\left(\sigma \cos\theta\right)^2 \, \Delta P \phi}{2\mu \, L} \int_0^1 \frac{dS}{P_c^2\left(S\right)}$$

This equation can be compared with Darcy's law.

$$u = \frac{k}{\mu} \frac{\Delta P}{L}$$

By comparing the last two equation, an expression can be derived for the permeability.

$$k = \frac{(\sigma \cos\theta)^2 \phi}{2} \int_0^1 \frac{dS}{P_c^2(S)}$$

Tortuosity has not yet been considered to this point. Purcell introduced a factor, called the "lithology factor" to bring the calculated permeability into correspondence with the measured air permeability. We will use the tortuosity factor here to parallel the nomenclature for the packed bed.

$$k = \frac{(\sigma \cos\theta)^2 \phi}{2\tau} \int_0^1 \frac{dS}{P_c^2(S)}$$

Purcell observed that τ ranged from 2.8 for 1500 md sandstone to 12 for 1 md sandstone. This may be compared with the value of 25/12 ≈ 2 for a packed bed of spheres.

Thomeer (1960) refined the method by introducing a model for fitting the measured capillary pressure data.

Mercury capillary pressure curves can be measured from drill cuttings when cored samples are not available. Swanson (1981) observed that the low pressure portion of the capillary pressure curve was often different between measurements with small samples (e.g. drill cuttings) and larger core samples. This difference is thought to be due to the sample surface roughness and/or the accessibility of pores to the external surfaces. The low pressure portion corresponds the larger pores which contribute the most to permeability. Thus he suggested using a point on the capillary pressure curve that is independent of sample size. This point is the point of tangency of the curve of P_c versus mercury volume as a percent of bulk

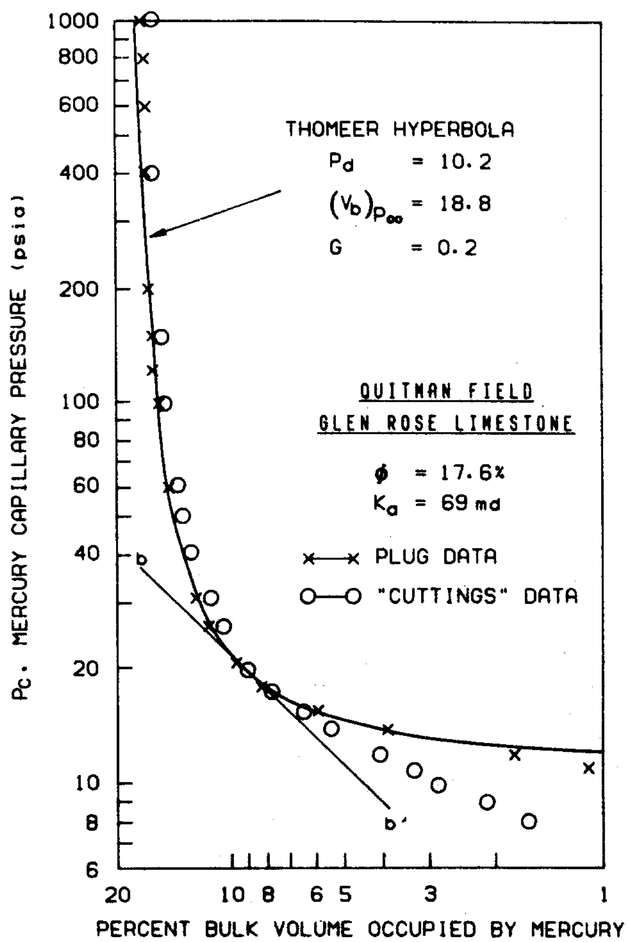


Fig. 3.36 Comparison of capillary pressure measured on plugs and cuttings (Swanson, 1981)

sample volume with the 45° line on a log-log scale. This method as well as the departure of the cuttings data is shown in Fig. 3.36. Using this method, the correlation for both clean sands and carbonates is as follows.

$$k_w = 355\left(\frac{S_b}{P_c}\right)_A^{2.005}$$

This correlation is compared with measurements in Fig. 3.37. In the event Thomeer parameters are known this equation can be expressed as follows.

$$k_w = 355\left[10^{-2\sqrt{G/2.303}}\left(\frac{BV_{P\infty}}{P_d}\right)\right]^{2.005}$$

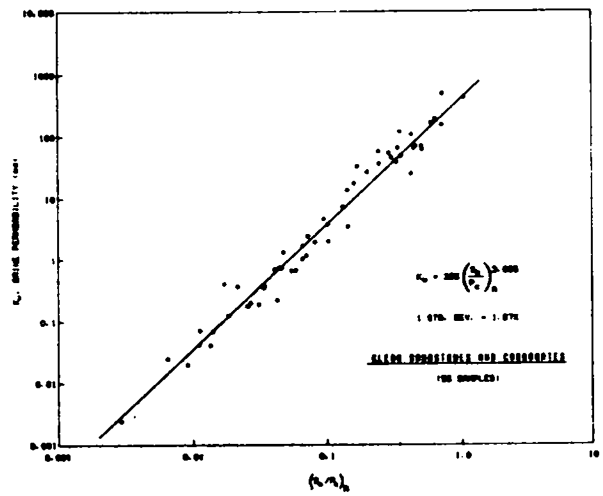


Fig. 3.37 Correlation of brine permeability with capillary pressure data (Swanson, 1981)

where

$BV_{P\infty}$ Thomeer percent bulk volume occupied by mercury at infinite mercury pressure (approximated by porosity)

G Thomeer pore geometrical factor

k_w brine permeability

P_c mercury capillary pressure, psi

P_d Thomeer mercury/air extrapolated displacement pressure, psi

S_b mercury saturation in percent of bulk volume (approximated by the product of porosity and mercury saturation)

$(S_b/P_c)_A$ correlating parameter taken at the point A (tangent to 45° line) of capillary pressure curve

Note that the exponent for this correlation agrees with the value predicted from Purcell's theoretical model using Poiseuille's equation. Ma, Jiang, and Morrow (1991) give a recent review of estimating the permeability from capillary pressure data.

Estimation of Permeability from Grain Size Distribution

We saw earlier (Beard and Weyl, 1973) that the parameters from the grain size distribution (grain size and sorting) has been used to correlate the porosity and permeability of clay-free unconsolidated sands. Now that we have a model (Kozeny) for the permeability, we are in a position do develop a correlation for predicting permeability from the grain size distribution.

Sorting. Sorting is usually expressed as qualitatively ranging from extremely well sorted to very poorly sorted. The earlier section on *Grain Size Distribution* listed the range of the sorting coefficient, S_o, that corresponds to the qualitative measure of sorting. Here we will use the arithmetic mean of the range to correspond to the qualitative measure of sorting. Now we will express the sorting in terms of the standard deviation, σ, of the distribution of the logarithm of the grain size. The sorting coefficient is defined as follows.

$$S_o = \left(d_{25} / d_{75}\right)^{1/2}$$

Assume that the grain size can be described by a log normal distribution. The grain size is then expressed as follows.

$$y = \log d$$
$$y = \mu + \sqrt{2}\,\sigma\,\mathrm{erfinv}\left[2P\{y\} - 1\right]$$
$$\text{or}$$
$$y = \mu + \sqrt{2}\,\sigma\,\mathrm{erfinv}\left[1 - 2P\{y\}\right]$$

where

μ is the median of the distribution (log mean or geometric mean grain diameter)

σ is the standard deviation of the log normal distribution

The choice of the two expressions depends on whether the cumulative probability corresponds to less than grain size *d* or greater than grain size *d*. The sorting coefficient is now expressed in terms of the logarithm of the grain diameter at the 25 and 75 percentile..

$$S_o = \exp\left(\frac{y_{25} - y_{75}}{2}\right)$$
$$y_{25} - y_{75} = \sqrt{2}\,\sigma\left[\mathrm{erfinv}\left(0.5\right) - \mathrm{erfinv}\left(-0.5\right)\right]$$
$$S_o = \exp\left(0.6744\,\sigma\right)$$
$$\sigma = \frac{\log S_o}{0.6744}$$

The transformation from the qualitative sorting, to the sorting coefficient (Beard and Weyl, 1973), and to the standard deviation of the log normal distribution is summarized in following table.

Sorting	S_o	σ
Extremely well sorted	1.05	0.072
Very well sorted	1.15	0.207
Well sorted	1.3	0.389
Moderately sorted	1.7	0.787
Poorly sorted	2.35	1.267
Very poorly sorted	4.2	2.128

Correlation of porosity with sorting. The porosity data of Beard and Weyl was correlated with the standard deviation (of the logarithm grain size distribution), Fig. 3.38.

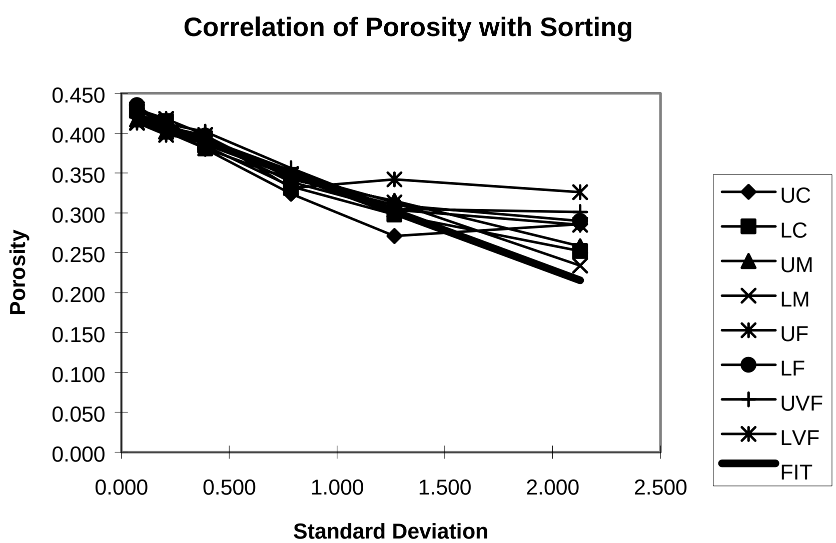


Fig. 3.38 Correlation of porosity with sorting (R^2 =0.93, excluding very poorly sorted data)

The regression of porosity with standard deviation excluding the *Very poorly sorted* data gives the following linear relationship.

$$\phi = 0.428 - 0.0998\sigma$$

Tortuosity. The Blake-Kozeny model determined a value of 25/12 for the tortuosity of a bed of uniform spherical particles. We will let the tortuosity, τ, be a function of the sorting. The Carman-Kozeny model is as follows.

$$k = \frac{\phi^3 \; D_p^2}{72\,\tau\,(1-\phi)^2}$$

or

$$\tau = \frac{\phi^3 \; D_p^2}{72\,k\,(1-\phi)^2}$$

Grain size	D_p
Upper coarse	1.30
Lower coarse	0.70
Upper medium	0.40
Lower medium	0.30
Upper fine	0.20
Lower fine	0.13
Upper very fine	0.10
Lower very fine	0.07

The tortuosity required to fit the Carman-Kozeny equation to the measured permeability of Beard and Weyl was calculated from the above equation. The grain size was estimated by transforming from the qualitative grain size to diameter in mm. The calculated tortuosity and the regression excluding the coarse sand data are illustrated in Fig. 3.39.

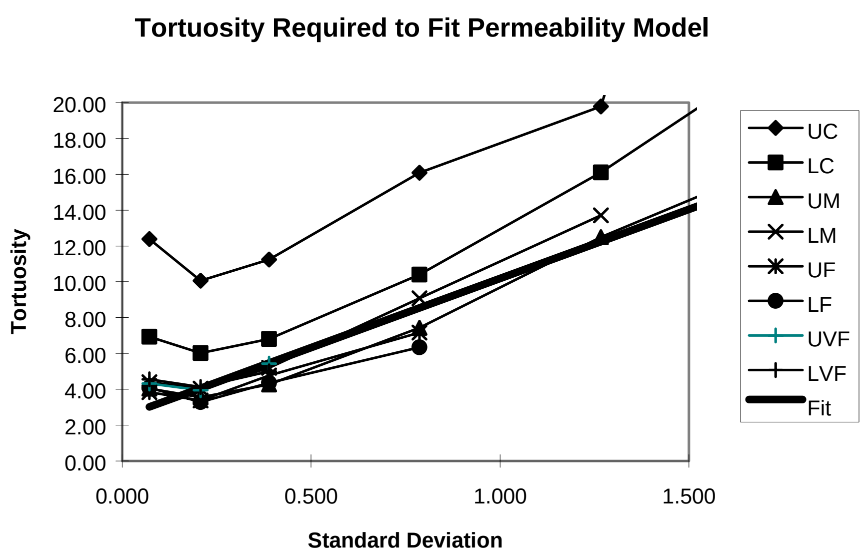


Fig. 3.39 Tortuosity required to fit Kozeny model for permeability (R^2=0.93, excluding coarse sand data)

Note that the tortuosity extrapolates to 2.5 for zero standard deviation, a value very close to the 25/12 determined by Blake for a spherical bead back. The linear regression, excluding the coarse sand data, give the following result.

$$\tau = 2.46 + 7.72\,\sigma$$

Since both the porosity and tortuosity are a function of the sorting, one would expect a cross-correlation between tortuosity and porosity. The cross-correlation of the linear correlations for porosity and tortuosity and of the porosity and tortuosity of the individual sands are shown in Fig. 3.40. The equation for the cross-correlation of the porosity and tortuosity correlations with sorting is as follows.

$$\tau = 35.6 - 77.3\phi$$

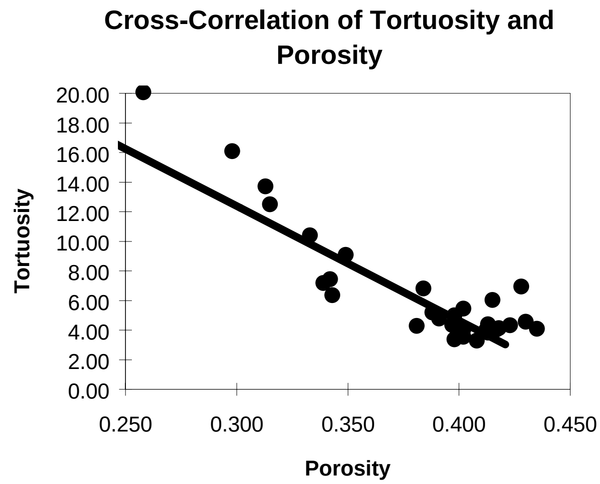


Fig. 3.40 Cross-correlation of porosity and tortuosity (upper coarse sand data omitted)

Permeability predicted from porosity, grain diameter, and sorting. The permeability predicted from the Carman-Kozeny model using the correlation for tortuosity given above is compared with the measured permeability of Beard and Weyl in Fig. 3.41. The predicted values for the upper coarse sand were much larger than the measured values and some are off the figure (i.e., >1000 darcy).

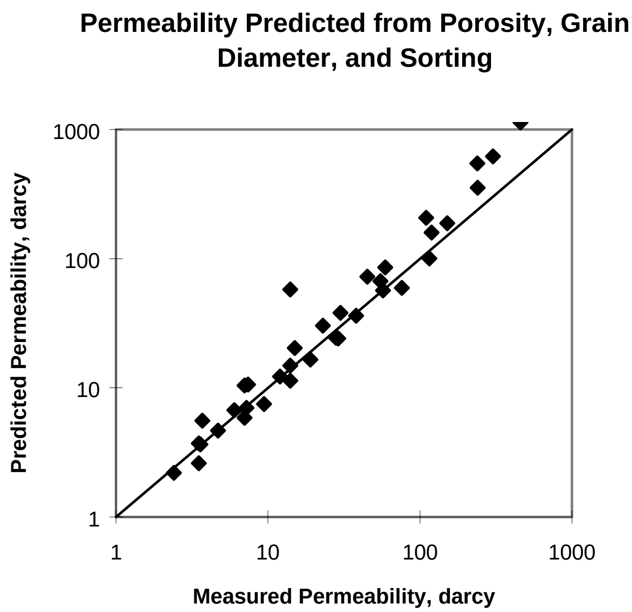


Fig. 3.41 Permeability predicted from porosity, grain size, and sorting (R^2=0.87)

If one has no other measurement other than the grain size distribution, then a measured porosity will not be available to use in the Kozeny model. In this case we can use the correlation of porosity with sorting derived above. The permeability correlated from the grain size and sorting is shown in Fig. 3.42.

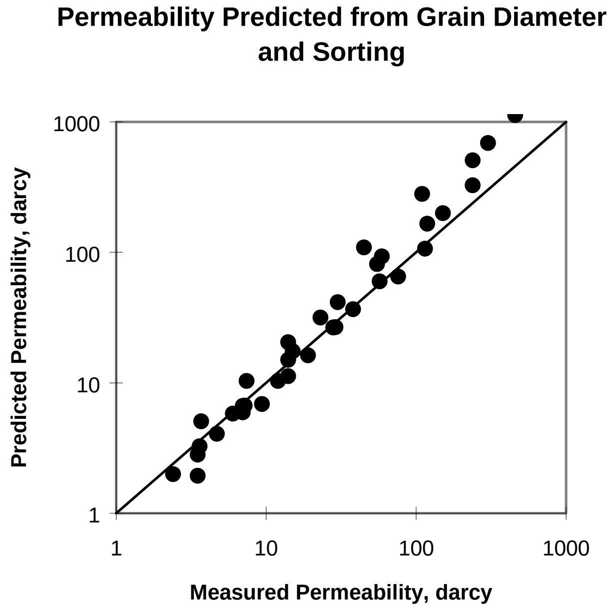
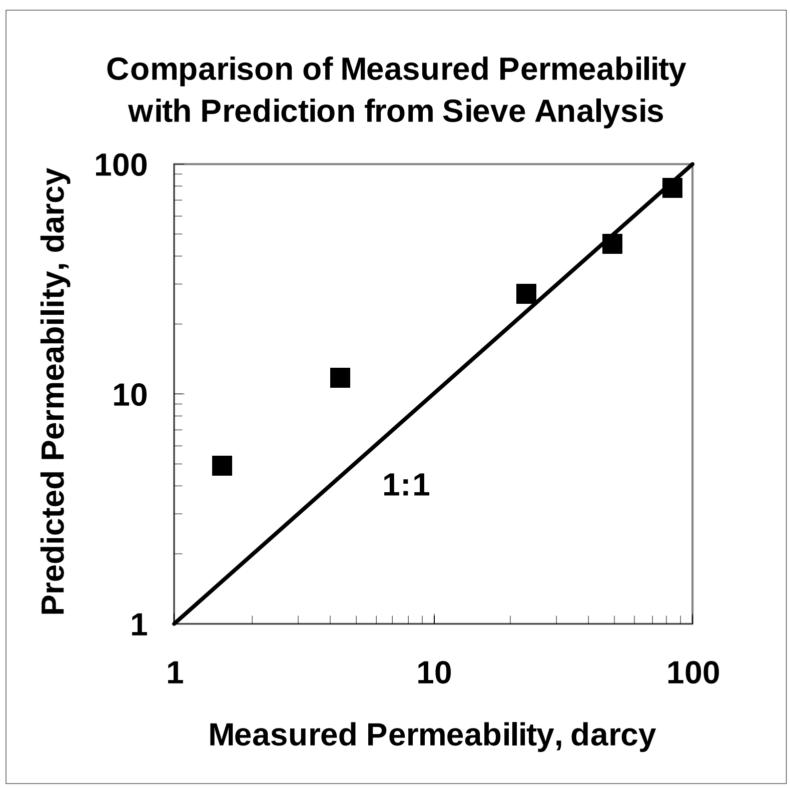


Fig. 3.42 Permeability predicted from the Kozeny model using only grain diameter and sorting (R^2=0.90)



Independent test of correlation. The comparisons between measurement and prediction above are biased because the correlations were derived from the measurements. The correlations were used to predict the permeability distribution of an aquifer at Hill Air Force Base in Utah. Permeabilities were measured on selected samples to test the correlations. The comparison is shown in Fig. 3.43. The comparison is good for the high permeability samples but the deviation increases for the lower permeability samples. It was discovered that some of the samples containing clays had clumps of sand grains that were interpreted as a large sand grain. Thus the predicted permeability was too high for these samples. A clay sample was analyzed to have a median grain diameter of 1.11 mm. When the error was pointed out to the service company, they further pulverized the sample and reported a median grain diameter of 0.18 mm, apparently the result of only smaller clay aggregates. Thus sieve analysis of clay containing sediments will not be accurate without adequate pulverization of the aggregates.

Assignment 3.5 Estimation of permeability and porosity from sieve analysis.
Estimate the permeability and porosity of sample SB9-71 from the sieve analysis data in the file, *sb9_71.txt*. Cut and paste the rows with the grain size distribution into a file with the sample name and an extension of *dat*. Process the data to estimate the median grain diameter and standard deviation with the MATLAB file, *sieve2.m*. Show a plot of the fit of the lognormal distribution to the data. The measured values of permeability and porosity are 48.3 darcy and 0.395, respectively.

Other correlations. There is a frequent need to estimate permeability from grain size distribution. A survey of the literature has not been made here. Recently Panda and Lake (1994) estimated permeability from the grain size distribution. Unfortunately, they used the parameters of the distribution of grain diameter rather than the logarithm of the grain diameter. Their analysis is much more complex and they have to use a parameter for the skewness. A distribution that is a normal distribution (not skewed) in the logarithm of the grain size has skewed distribution of the grain size. Ostermeier (1995) recently developed a correlation using parameters of the grain size distribution (not the logarithm of grain size).