EXERCISE 2

1. Determine the order of magnitude of each of the following sequences of real numbers, as $t \to \infty$:
   
   (a) $y_t = t$
   (b) $y_t = 3 + t^2$
   (c) $y_t = 3 + 1/t^2$
   (d) $y_t = t + t^{1/2}$
   (e) $y_t = (1 + 2t + t^2)/(1 + 2t^2)$

2. Suppose $x_t \sim \text{i.i.d.}$, $E[x_t] = \mu$, and $V[x_t] = \sigma^2$ then determine the order in probability of the following sequences of random variables:
   
   (a) $x_1 - \mu$
   (b) $\bar{X}_n = \sum_{i=1}^{n} x_t/n$
   (c) $\bar{X}_n - \mu$
   (d) $(\bar{X}_n)^2 - \mu^2$
   (e) $\sum_{t=1}^{n} (x_t - \mu)^2/n - \sigma^2$

3. Suppose $x_t \sim N(\mu, \sigma^2) \sim \text{i.i.d.}$
   
   (a) Show that $\bar{X} = x_1 + 1/n$ is an asymptotically unbiased estimator of $\mu$ but is not consistent.
   (b) Determine the limiting distribution of $[n^{1/2}(\bar{X}_n - \mu)]$.
   (c) Show that $\bar{X}_n$ is asymptotically efficient relative to $\bar{X}$.
   (d) Determine the limiting distribution of $[n^{1/2}(\bar{X}_n + \bar{X}/n - \mu)]$.
   (e) Determine the limiting distribution of $[(\bar{X}_n - \mu)/(s^2/n)^{1/2}]$ if $\text{plims}_{n \to \infty} s^2 = \sigma^2$.

4. Suppose $x_t \sim \text{i.i.d.}$ with $E[x_t] = \mu$ and $V[x_t] = \sigma^2$ but not necessarily normal
   
   (a) Show that $\text{plim}_{n \to \infty} \bar{X}_n = \mu$.
   (b) Prove that $s^2 = \sum_{t=1}^{n} (x_t - \bar{X}_n)^2/(n - 1)$ is consistent for $\sigma^2$.
   (c) Determine the limiting distribution of $[n^{1/2}(\bar{X}_n - \mu_0)]$ under both $H_0 : \mu = \mu_0$ and $H_1 : \mu = \mu_1 = \mu_0 + \gamma/\sqrt{n}$.
   (d) Determine the limiting distribution of $[(\bar{X}_n - \mu)/(s^2/n)^{1/2}]$. 