EXERCISE 5: Answers

1. Duesenberry considered the following model for aggregate consumption:
   \[ C_t = \alpha + \beta D_t + u_t \]
   where \( u_t \) is an unknown disturbance term.
   a. Run an OLS regression based on this specification and report \( \hat{\alpha} \) and \( \hat{\beta} \). \( OLS \) regression yields: \( \hat{\alpha} = -2257.28 \) (3823.05) and \( \hat{\beta} = 0.892956 \) (0.01815) with \( SSE = 8.306 \times 10^7 \).
   b. Making explicit the assumptions you are making, what are the properties of your estimates \( \hat{\alpha} \) and \( \hat{\beta} \). The assumptions are (i) \( E[u_t] = 0 \), (ii) \( E[u_t^2] = \sigma^2 \), (iii) \( E[u_t u_j] = 0 \) for \( j \neq i \), (iv) \( D_t \) nonstochastic, (v) \( D_t \) nonconstant, [(vi) \( u_t \sim i.i.d. N(0,\sigma^2) \). Under these assumptions \( (\hat{\alpha},\hat{\beta}) \) are unbiased, consistent, BLUE, [MLE, and BUE].
   c. Test the null hypothesis that the marginal propensity to consume is one. Under \( H_0 : \beta = 1 \) and the assumptions made, \( (\hat{\beta} - 1)/s.e.(\hat{\beta}) = (0.8930 - 1)/0.01815) = -5.895 \) is a realization from \( t_{11} \). Consulting the t-tables we find this to be to the left of the .005 LHS tail value of -3.106. So we reject at the 1% level.
   d. Discuss the properties of your test under the alternative that \( \beta \) is less than one. Under \( H_1 : \beta = \beta_1 < 1 \) and the assumptions made, \( (\hat{\beta} - 1)/s.e.(\hat{\beta}) = (\beta - \beta_1)/s.e.(\hat{\beta}) + (\beta_1 - 1)/s.e.(\hat{\beta}) \) with the second term being a negative shift factor. The larger the shift, the more likely an extreme negative value (less than the critical value and thereby leading to rejection) is to occur. The shift term and hence the probability of rejection will increase with: sample size, the difference between the alternatives, and the signal-noise ratio.

2. An alternative model is based upon saving behavior and can be written:
   \[ S_t = \alpha + \beta D_t + u \]
   where \( S_t = D_t - C_t \).
   a. Obtain OLS estimates of \( \alpha \) and \( \beta \). \( OLS \) regression yields: \( \hat{\alpha} = 2257.28 \) (3823.05) and \( \hat{\beta} = 0.107044 \) (0.01815) with \( SSE = 8.306 \times 10^7 \).
   b. What is the relationship between the slope coefficient here and in the preceding problem? The slope coefficient is \( 1 - \hat{\beta} \) from the previous regression. Note that intercept is the negative of the value from above, the standard errors are the same, as are the \( SSE \) values.
   c. Rank (if possible) this regression and the previous regression. It is tempting to rank the first regression above the second, since its \( R^2 \) is higher. However, the regressions are essentially identical - if you know one you know the other with some very simple math. The \( R^2 \) statistic is appropriate for ranking regressions only if both regression have the same dependent variable. Otherwise we are comparing apples and oranges in a very real sense.
   d. Test the hypothesis that \( \beta \) in this model is one. Under \( H_0 : \beta = 1 \) and the assumptions made above, \( (\hat{\beta} - 1)/s.e.(\hat{\beta}) = (0.107044 - 1)/0.01815) = -49.207 \) is a realization from \( t_{11} \). Consulting the t-tables we find this to be to the left of the .005 LHS tail value of -3.106. So we reject at the 1% level. Note that this is the same as testing for \( \beta = 0 \) for the first regression.
   e. Calculate a confidence interval for \( \hat{\beta} \) under the null hypothesis of unity. What is the rela-
tionship of this interval to the hypothesis test. For a 95% interval we have \( \{\widehat{\beta} \pm t_{11.025} \times s.e.(\widehat{\beta})\} = \{.107044 \pm 2.201 \times .02443\} = \{.0533, .1608\} \). This interval represents the set of values that are acceptable as a null hypothesis. Since neither zero nor one are included we can reject the null that MPS is zero or one.

3. Ando and Modigliani considered the model:

\[ C_t = \alpha + \beta D_t + \gamma A_t + u \]

Run this regression and report \( \alpha \), \( \beta \), and \( \gamma \). OLS regression yields: \( \hat{\alpha} = 6076.89 \) (4315.62), \( \hat{\beta} = 0.486864 \) (0.149970), and \( \hat{\gamma} = 0.0664655 \) (0.0244318) with \( SSE = 4.77329 \times 10^7 \).

a. Test whether \( \gamma \), the coefficient for the “wealth” effect, is different from zero. Under \( H_0 : \gamma = 0 \) and the assumptions made above, \( (\hat{\gamma} - 1)/s.e.(\hat{\gamma}) = (0.0665 - 0)/0.0244) = 2.720 \) is a realization from \( t_{10} \). Consulting the t-tables we find this to be to the left of the .025 RHS tail value of 2.228 but to the right of the .005 RHS tail value of 3.169. So we reject at the 5% level but not at the 1% level.

b. Calculate the prob-value for \( \hat{\gamma} \) under the null of \( \gamma = 0 \). What is the relationship of this value to the hypothesis test. The prob-value, which is the probability of obtaining a value as extreme or more extreme than the realization is clearly less than .01. Using value provided by GAUSS we have .0215 as the prob-value. If we choose an \( \alpha \)-value or significance value larger than the prob-value we will reject the null hypothesis.

c. Test the joint hypothesis that \( \beta = 1 \) and \( \gamma = 0 \). The restricted regression is \( C_t - D_t = \alpha + u \) which yields \( SSE_r = 3.45792 \times 10^8 \). Thus \( [(SSE_r - SSE_u)/2]/(SSE_u/10) = [(345792000 - 47732900)/2]/4773290 = 31.2216 \). Alternatively, after subtracting \( D_t \) from both sides, we have \( S_t = \alpha + \beta D_t + \gamma A_t + u \) and the null hypothesis becomes \( \beta = 0 \) and \( \gamma = 0 \) and the \( F \)-value output by GAUSS, which is the same, may be used.

d. Compare and rank (if possible) this regression with the regression from (1). Both regressions have the same dependent variable, so we may compare on the basis of \( R^2 \), which is lower for the last regression. In addition the variable which was added \( A_t \) is significant which suggests it improves the regression. We should choose the last regression as best.