Oscillations

"Physics is experience, arranged in economical order."

E. Mach

OBJECTIVES

To study some simple oscillatory systems.

THEORY

Typical dictionary definitions of the verb "oscillate" are "to move or travel back and forth between two points" and "to vary above and below a mean value". Physics uses the word in essentially the same way, and recognizes a wide range of mechanical and electrical systems that oscillate. One of the simplest possibilities is a mass connected to a spring in such a way that the mass can move back and forth if pushed away from an equilibrium position. This system contains the two essential elements of an oscillator: Something to move back and forth, and some mechanism to provide a restoring force. Another mechanical example would be a solid bar, clamped at one end. It will vibrate if struck, but here the moving mass and the spring are the same material. A capacitor and an inductor provide an electrical realization of exactly the same equation as the mass and spring.

In this exercise we will study the motion of a mass suspended from a springy material, as shown schematically in Fig. 1. We assume that the mass can only move vertically, and take the origin at the point where the spring is not stretched at all. The equation of motion, including both the gravitational and spring forces, is

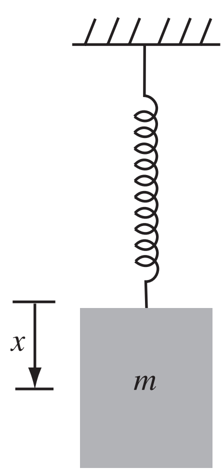

Fig. 1 Idealized mass and spring oscillator.

$$\frac{d^2x}{dt^2} = g - \frac{F(x)}{m} \tag{1}$$

When the mass is not moving it will be at an equilibrium position x_e given by the solution of

$$F(x_e) = mg \tag{2}$$

If the mass moving we expect it to oscillate around the equilibrium position, and if we knew the form of *F(x)* we might be able to solve Eq. 1 to find a description of the motion.

Physics textbooks usually consider the case of a "simple harmonic oscillator". The simple harmonic oscillator is constructed with an ideal massless spring, for which the force is a linear function of the stretch

$$F(x) = kx \tag{3}$$

The constant of proportionality k is called the spring constant. This assumption is made mostly because it is fairly easy to solve Eq. 1 for this force, although it is also possible to manufacture springs which are approximately linear. The equation of motion is then

$$\frac{d^2x}{dt^2} = g - \frac{k}{m}x \tag{4}$$

The solution for the stretch as a function of time is

$$x = A\sin(\omega t + \phi) + x_e \tag{5}$$

where A and ϕ are constants determined by the initial conditions, x_e is the equilibrium position, and ω is the angular frequency of oscillation, given by

$$\omega = \sqrt{k/m} \tag{6}$$

A derivation of the result is in your text, or you can show that it is correct by substituting x from Eq. 5 into Eq. 4.

When *F(x)* is not so simple it may or may not be possible to write down a general analytic solution to Eq. 1. We can still make progress by considering small-amplitude oscillations, for which x remains close to x_e. Expanding $F(x)$ in a Taylor series around x_e gives the expression

$$F(x) = F(x_e) + k_e(x - x_e) \tag{7}$$

where we have neglected all terms of the infinite series beyond the first two. The constant k_e is given by

$$k_e = \left.\frac{dF(x)}{dx}\right|_{x_e} \tag{8}$$

Substituting this approximation for F(x) into Eq. 1, and simplifying with relation 2, we get the approximate equation of motion

$$\frac{d^2x}{dt^2} = \frac{k_e x_e}{m} - \frac{k_e}{m}x \tag{9}$$

Since we kept only the linear variation of the force, the equation of motion has exactly the same form as for simple harmonic motion, and will have the same solution. We therefore expect that small oscillations about the equilibrium position will occur with frequency

$$\omega = \sqrt{k_e / m} \tag{10}$$

when the effective spring constant k_e is found from the slope of the $F(x)$ curve at x_e according to Eq. 8.

One final complication is that the system may have friction of some sort. This could be due to air resistance or to some internal process in the spring material. Whatever the cause, the conversion of mechanical energy to some other form will cause the oscillations to gradually decrease in amplitude. For many circumstances, the decrease is exponential, leading to a solution of the form

$$x = Ae^{-\gamma t}\sin(\omega t + \phi) + x_e \tag{11}$$

where γ is a constant determined by the nature of the energy loss. There will also be a decrease in the oscillation frequency, but it is usually small if the damping is weak enough that oscillation persists for at least several cycles.

EXPERIMENTAL PROCEDURE

1. Data acquisition and analysis

The overall experimental setup is shown in Fig. 2. A small bucket to hold various weights hangs from a spring. The spring, in turn, is held by a force probe on a support arm which is clamped to a lab table. Stretch is measured by a sonic ranger which sits on the floor directly under the weight bucket, pointing up. A box with a wire mesh window protects the ranger from falling weights.

Data acquisition is accomplished with LoggerPro software. Start the program by double-clicking an icon labeled Oscillators.cmbl, or load the file after the program is running. This will give you a good starting point for this exercise.

To test the process, you can suspend the bucket, loaded with a few hundred grams, from the helical metal spring. If you displace the bucket vertically and then start data acquisition you should get a plot of stretch vs time showing decreasing oscillations, and a plot of force vs position. The plots may become irregular if the sonic ranger is confused by other obstacles, if it is not under the bucket, or if the bucket swings sideways too much.

Before taking data, calibrate the force probe on the 10N range. It is also convenient to set the zero of the position plot at the equilibrium position for the mass and spring. Go to Experiment > Zero... or click on the zero icon on the tool bar, unselect the force probe, and click OK. The sonic ranger will run very briefly, and the menu will close. The distance plotted is now relative to the equilibrium position of the bucket.

Once you have a clear and properly-zeroed display of the oscillation, you can do a quantitative comparison with Eq. 11. Using the Damped sine function

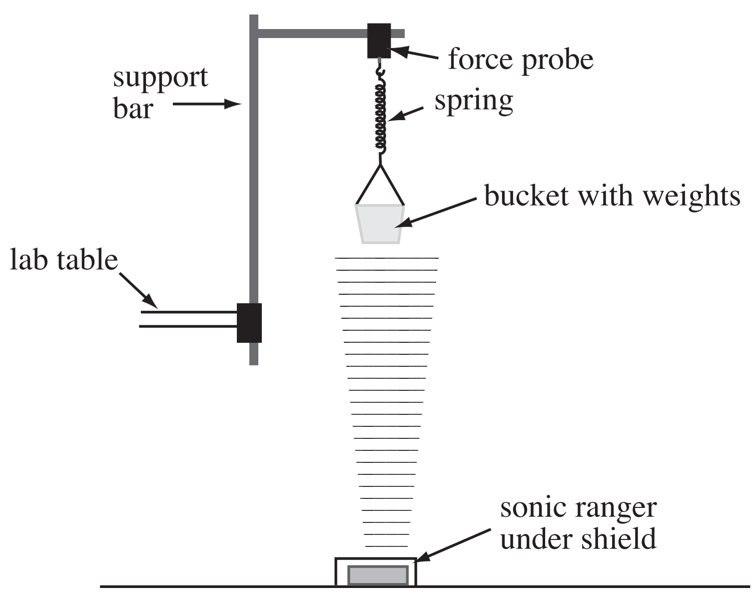

Fig. 2 Physical arrangement for oscillation measurements.

$$A*\exp(-B*t)*\sin(C*t+D)+E \tag{12}$$

in the automatic curve fit, click Try Fit as usual to let the program optimize the parameters shown. If the results are unreasonable, edit the parameters to give a more reasonable starting point, click Automatic, and Try Fit again. When the fit is satisfactory, record the oscillation frequency for later analysis.

You will also want to measure the slope of the force vs position curve to get an effective spring constant to compare with your oscillation frequencies. To do this, simply determine the slope of the force-position graph for each equilibrium position you use.

2. Measurement program

Two springs, with different characteristics, are available. One is a wire helix, for which the force is supposed to be approximately linear. The other is a large rubber band. You should determine the oscillation frequencies and effective spring constants of each spring for several weights, corresponding to a range of x_e. You can then compare the prediction of Eq. 10 that

$$k_e = m\omega^2 \tag{13}$$

with the values of k_e that you measure directly from the force vs position curve. If you choose to use rather small masses the agreement might be improved by accounting for the mass of the spring, a portion of which is in motion. The usual claim is that you add 1/3 of the mass of the spring to the overall mass in Eq. 13.

For the metal spring the maximum stretched length to avoid damage is about 80 cm, while the rubber band will only stretch about 75 cm before breaking. Without exceeding those limits, determine the oscillation frequency and effective spring constant for several different masses for both types of spring.

Your data will probably show that k_e is nearly constant for the metal helix, but varies significantly with x_e for the rubber band. Obtain a force vs position curve for the full range of positions from unstretched to maximum stretch for each spring, and use the curves to explain the different behaviors of k_e.

For the rubber band you will probably also note that the force vs position curve is different for extension and contraction, a phenomenon called hysteresis. Since the mechanical work done is different for the two directions of motion, mechanical energy will be converted to heat over an oscillation cycle, accounting for the strong damping you observed. The metal helix has little or no hysteresis and correspondingly small damping.