Physics 102 Spring 2006: Test 2—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and 15 multiple-choice questions.
- The test is graded on a scale of 100 points; the free-response questions are worth a total of 70 points, and the multiple-choice questions account for 30 points (two points each).
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

***.

Show your work for the free-response problems, including neat and clearly labeled figures, in your blue book.

Answers without explanation (even correct answers) will not be given credit.

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I. (35 pts) The sketch below shows a simple RC circuit. The capacitor is initially uncharged, and at $t = 0$ the switch S is closed. Express your answers in terms of V_0, R_1, R_2, and C.

(a) Determine the current through each resistor immediately after the switch is closed.

(b) Determine the charge on the capacitor $Q(t)$ as a function of time after the switch is closed. What are the initial charge $Q(t = 0)$, the final charge $Q(t \to \infty)$, and the time constant? Sketch $Q(t)$ vs. t.

(c) Determine the current through the capacitor $I(t)$ as a function of time after the switch is closed. What are the initial current $I(t = 0)$, the final current $I(t \to \infty)$, and the time constant? Sketch $I(t)$ vs. t.

(d) Determine the current through each resistor a long time after the switch is closed.

(e) After the switch has been closed for a long time, it is opened again. Determine the current through the capacitor $I(t)$ as a function of time after the switch is opened. What are the initial current $I(t = 0)$, the final current $I(t \to \infty)$, and the time constant in this case? Sketch $I(t)$ vs. t.
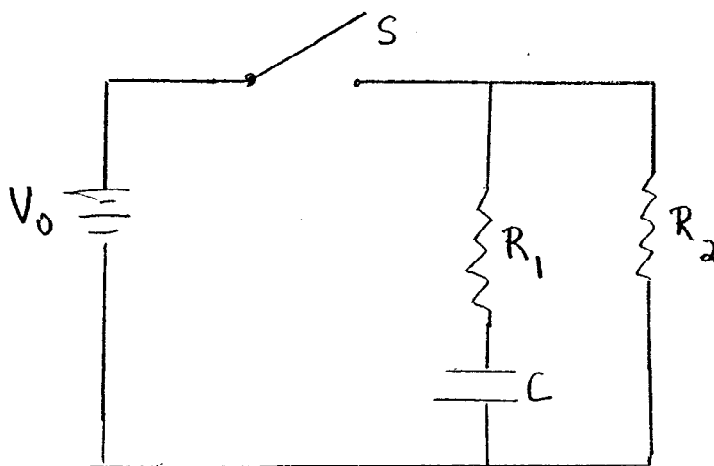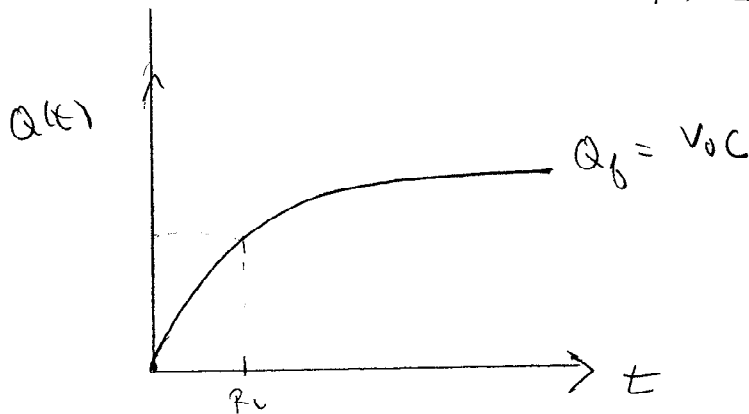
$$\ln(Q - V_0 C) = -t/R_1 C + A$$

$$Q(t) = V_0 C + B e^{-t/R_1 C}$$

$$Q(t=0) = 0 \implies B = -1$$
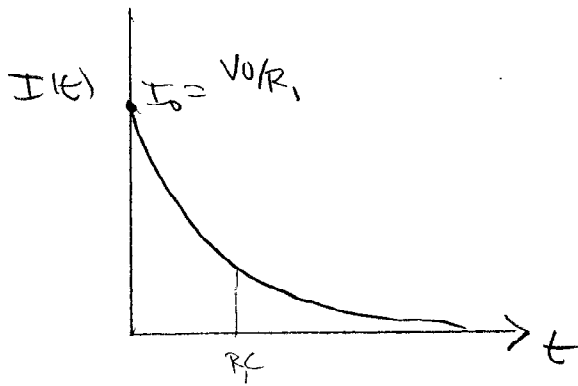
$$\boxed{Q(t) = V_0 C (1 - e^{-t/R_1 C})}$$

$$\boxed{\begin{array}{l} Q(t=0) = 0 \\ Q(t \to \infty) = V_0 C \end{array}}$$

$Q(t)$ $Q_0 = V_0 C$ $R_1 C$ t

(C) $\quad I = \dfrac{dQ}{dt} = -V_0 C \left(\dfrac{-1}{R_1 C}\right) e^{-t/R_1 C}$

$$\boxed{I(t) = \dfrac{V_0}{R_1} e^{-t/R_1 C}}$$

$$\boxed{\begin{array}{l} I(t=0) = \dfrac{V_0}{R_1} \\ I(t \to \infty) = 0 \end{array}}$$

$I(t)$ $I_0 = V_0/R_1$ $R C$ t

(d) A long time after the switch is closed, C acts like an open circuit:

$$\boxed{I_1(t \to \infty) = 0} \quad \boxed{I_2(t \to \infty) = \frac{V_0}{R_2}}$$

The current in R_2 is unchanged.

(e) When S is opened again, the capacitor discharges through both R_1 & R_2 in series
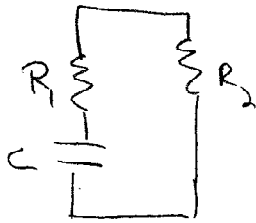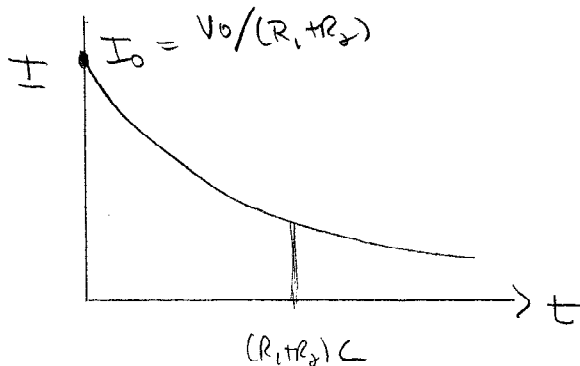

$$R_{eff} = R_1 + R_2 \qquad \tau = (R_1 + R_2)C$$

$$I(t) = I_0 e^{-t/(R_1+R_2)C}$$

Before S is opened, C is charged to V_0.

$$\boxed{I(0) = \frac{V_0}{R_1 + R_2}} \quad \text{is the initial current}$$

$$\boxed{I(t) = \frac{V_0}{R_1+R_2} e^{-t/(R_1+R_2)C}}$$

$$\boxed{I(t \to \infty) = 0}$$


The time constant is longer for discharging.

35 II. (35 pts) Three very long parallel are arranged as shown below. Two wires, which carry current I out of the page, pass through the points $y = 0$, $x = \pm a$. The third wires carries a current I into the page and is located at the point $x = 2a$, $y = -a$.

8 (a) Determine the magnetic field at the location of the top left wire ($x = 0, y = a$) due to the other two wires.

5 (b) Determine the force per unit length on the top left wire.

9 (c) Determine the magnetic field $\vec{B}_{12}(x)$ due to the two wires located at $x = 0$, $y = \pm a$ at an arbitrary point P on the x-axis, a distance x from the origin.

10 (d) Determine the magnetic field $\vec{B}_3(x)$ at the point P due to the single wire at $x = 2a, y = -a$.

3 (e) From (c) and (d), determine the total magnetic field at the point P.
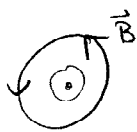
II.
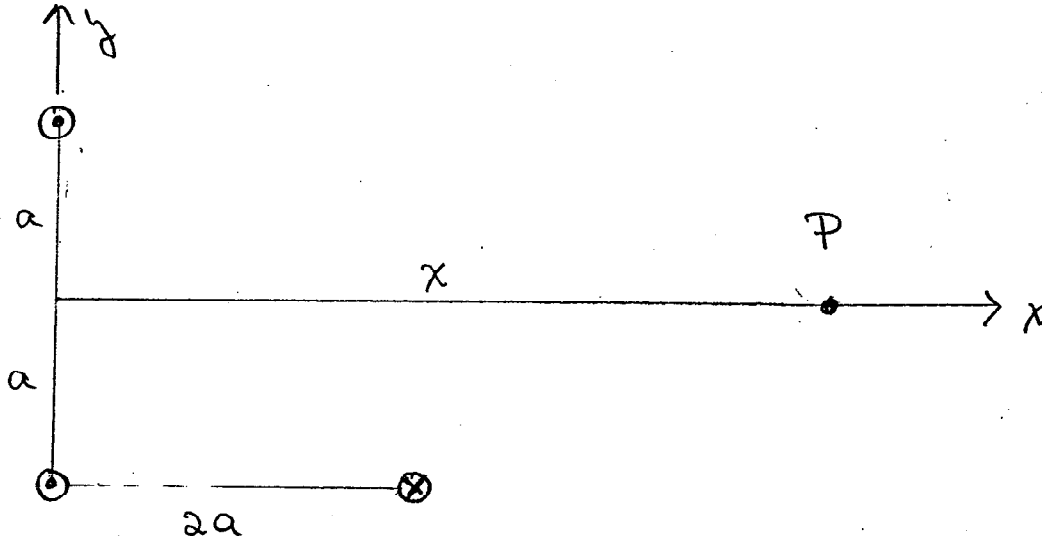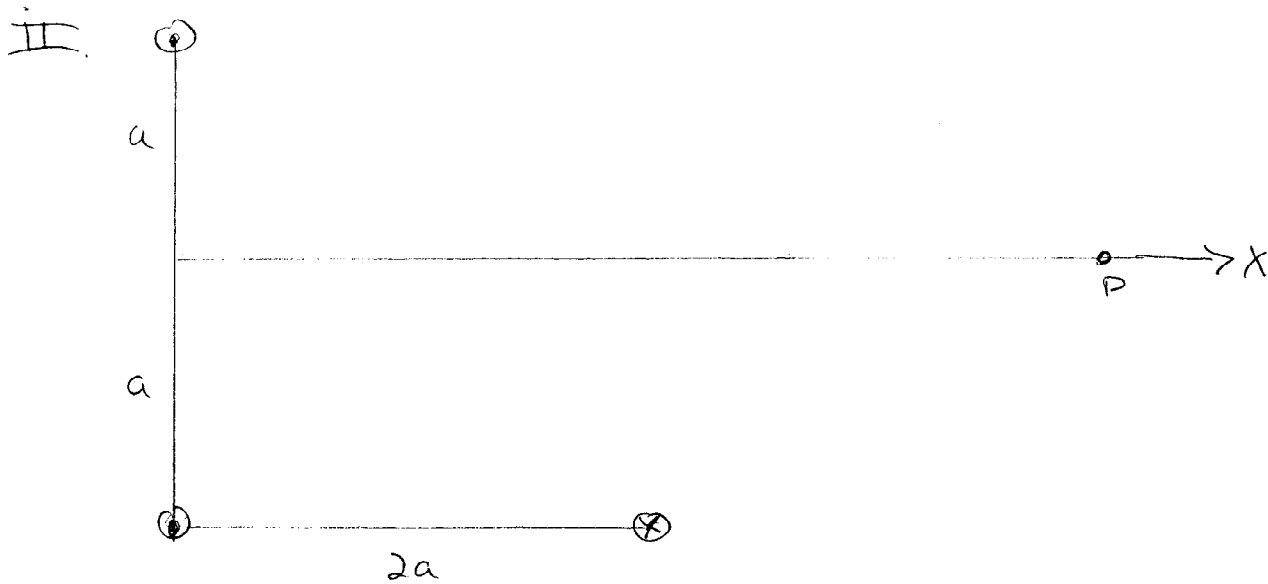
(a) $\vec{B}$ at top left due to the other two wires
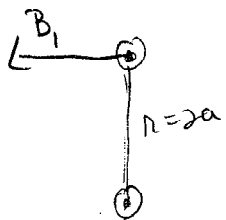
from Ampere's law we know that the field due to a long wire forms concentric loops
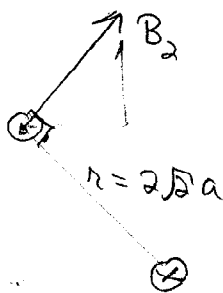
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

This gives $\vec{B}_1$ (due to lower left wire) $= \frac{\mu_0 I}{4\pi a} (-\hat{\imath})$

$\vec{B}_2$ due to the lower right wire will be at $45°$:

$$\vec{B}_2 = \frac{\mu_0 I}{4\pi (\sqrt{2} a)} \left(\frac{1}{\sqrt{2}} \hat{\imath} + \frac{1}{\sqrt{2}} \hat{\jmath} \right)$$

$$\boxed{\vec{B}_{TOT} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi a} \left(-\frac{1}{2} \hat{\imath} + \frac{1}{2} \hat{\jmath} \right)}$$
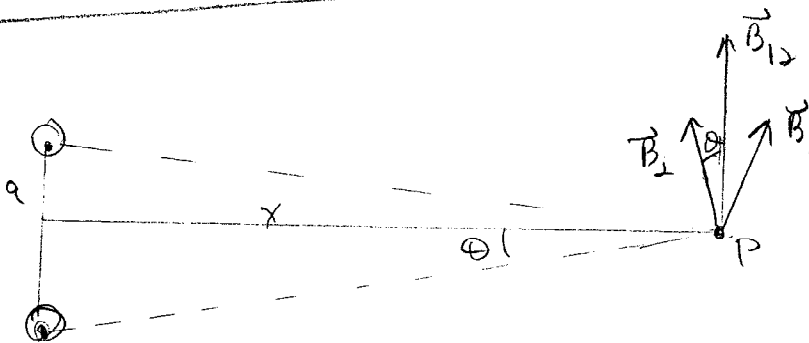
(b) F/L on top left wire

$$\vec{F} = I\vec{L} \times \vec{B} \qquad \vec{L} = L\hat{z} \quad \text{current in } +z \text{ direction}$$

$$\vec{F} = IL\left(\frac{\mu_0 I}{4\pi a}\right)\left(\underbrace{-\tfrac{1}{2}\hat{z} \times \hat{x}}_{-\tfrac{1}{2}\hat{y}} + \underbrace{\tfrac{1}{2}\hat{z} \times \hat{y}}_{-\tfrac{1}{2}\hat{x}}\right)$$

$$\boxed{\frac{\vec{F}}{L} = \frac{\mu_0 I^2}{4\pi a}\left(-\tfrac{1}{2}\hat{x} - \tfrac{1}{2}\hat{y}\right)}$$

(c)

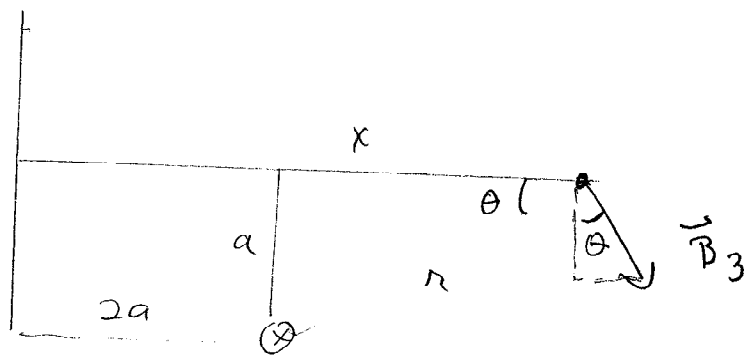The horizontal components cancel — vertical components add

$$B_{2y} = B_{1y} = B_1 \cos\theta \qquad\qquad \cos\theta = \frac{x}{\sqrt{x^2 + a^2}}$$

$$|B_1| = \frac{\mu_0 I}{2\pi (x^2 + a^2)^{1/2}}$$

$$\boxed{\vec{B}_{12}(x) = \frac{2\mu_0 I\, x}{2\pi (x^2 + a^2)}\hat{y} = \frac{\mu_0 I\, x}{\pi (x^2 + a^2)}\hat{y}}$$

factor of 2 comes from adding the two contributions

(d)

$$|B_3| = \frac{\mu_0 I}{2\pi \Lambda} \qquad \text{where } \Lambda = \left(a^2 + (x-2a)^2\right)^{\frac{1}{2}}$$

$\vec{B_3}$ has both x & y components

$$B_{3x} = B_3 \sin\theta \qquad\qquad B_{3y} = -B_3 \cos\theta$$

$$\sin\theta = \frac{a}{\left[(x-2a)^2 + a^2\right]^{\frac{1}{2}}}$$

$$\cos\theta = \frac{x-2a}{\left[(x-2a)^2 + a^2\right]^{\frac{1}{2}}}$$

$$\vec{B_3} = \frac{\mu_0 I}{2\pi \left[(x-2a)^2 + a^2\right]^{\frac{1}{2}}} \left[\frac{a}{\left[(x-2a)^2 + a^2\right]^{\frac{1}{2}}}\hat{i} + \frac{x-2a}{\left[(x-2a)^2 + a^2\right]^{\frac{1}{2}}}(-\hat{j})\right]$$

$$\boxed{\vec{B_3} = \frac{\mu_0 I}{2\pi \left[(x-2a)^2 + a^2\right]} \left[a\hat{i} - (x-2a)\hat{j}\right]}$$

Note that the y-component changes sign for $x < 2a$, our expression works for both cases.

(e) Superposition tells us we just add the contributions

$$|\vec{B_{TOT}}| = \frac{\mu_0 I}{2\pi} \left[\frac{a}{(x-2a)^2+a^2}\hat{i} + \left(\frac{x}{x^2+a^2} - \frac{x-2a}{(x-2a)^2+a^2}\right)\hat{j}\right]$$

Physics 102 Spring 2006: Exam #2 —Multiple-Choice Questions

1. A charged particle, q, is moving with speed v perpendicular to a uniform magnetic field. A second identical charged particle is moving with speed $2\,v$ perpendicular to the same magnetic field. The time to complete one full circular revolution for the first particle is T_1. The time to complete one full circular revolution for the particle moving with speed $2\,v$ is

   (a) $\frac{T_1}{4}$.

   (b) $\frac{T_1}{2}$.

   (c) T_1.

   (d) $2\,T_1$.

   (e) $4\,T_1$.

2. Four free-particles with the same **_negative_** charge and the same initial speed are incident on the same region of constant magnetic field pointed in the $+y$-direction (as shown below). Rank the z-component of the force due to the magnetic field on the particles from greatest to least. (The $+z$-direction points out of the page.)

   (a) $F_{z_{v1}} = F_{z_{v2}} = F_{z_{v3}} = F_{z_{v4}}$.

   (b) $F_{z_{v1}} = F_{z_{v4}} > F_{z_{v3}} > F_{z_{v2}}$.

   (c) $F_{z_{v2}} > F_{z_{v3}} > F_{z_{v1}} = F_{z_{v4}}$.

   (d) $F_{z_{v1}} > F_{z_{v2}} > F_{z_{v3}} > F_{z_{v4}}$.

   (e) $F_{z_{v4}} > F_{z_{v3}} > F_{z_{v2}} > F_{z_{v1}}$.
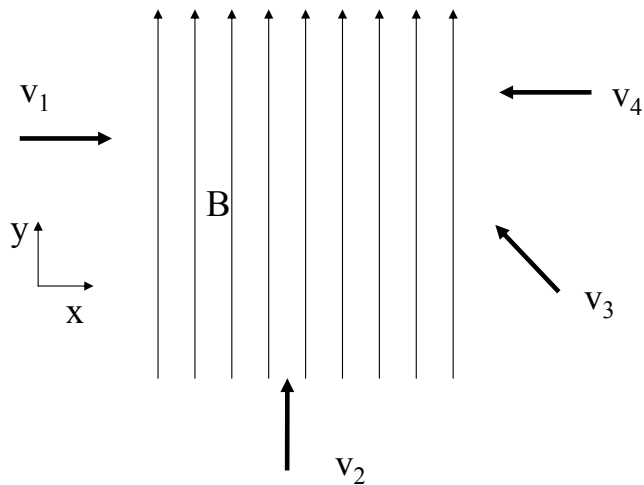
For questions **3**, refer to the figure below, depicting a positively charged particle deflected upward a distance d in a region of length L. Uniform magnetic and electric fields are contained in the region specified by the box. The magnetic field is directed perpendicular to the plane of the page and is directed into the page. The electric field is directed down the page.


$\vec{B}$ (into the page)
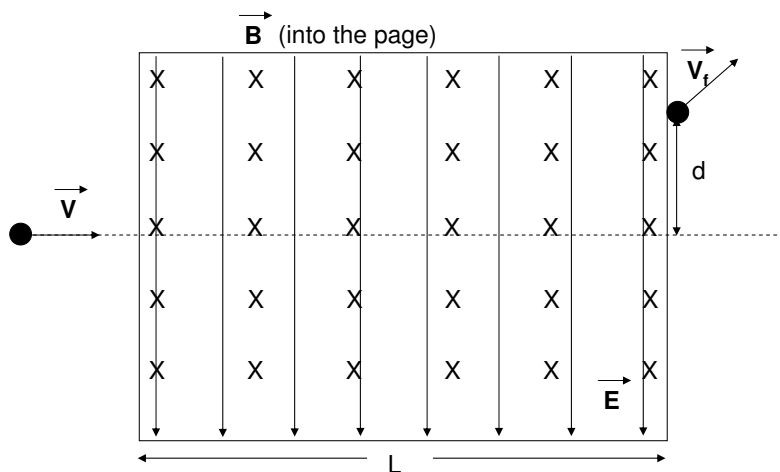
3. Which of the following statement(s) is (are) true?

    I. The electric field does positive work.

   II. The electric field does no work.

  III. The electric field does negative work.

  IV. The speed of the particle is such that $v > \frac{E}{B}$.

   V. The speed of the particle is such that $v = \frac{E}{B}$.

 VI. The speed of the particle is such that $v < \frac{E}{B}$.

  (a) only I and IV are true

  (b) only III and IV are true

  (c) only III and VI are true

  (d) only II, III and IV are true

  (e) only I, III, and IV are true

4. In the circuit depicted below, R_V is a variable resistor. If the resistance of R_V is increased, what happens to V_1 (the potential difference across R_1) and V_2 (the potential difference across R_2).
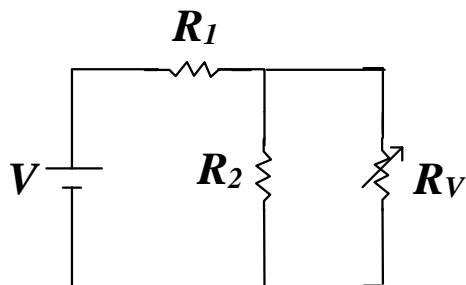

(a) V_1 decreases and V_2 increases.

(b) V_1 decreases and V_2 decreases.

(c) V_1 increases and V_2 increases.

(d) V_1 increases and V_2 decreases.

5. If the potential difference across a capacitor of capacitance C doubles, the capacitance of the capacitor is

(a) $4\ C$.

(b) $2\ C$.

(c) C.

(d) $1/2\ C$.

(e) $1/4\ C$.

6. An ideal ammeter A is connected between points a and b in the circuit below, in which the four resistors are identical. The current through the ammeter is
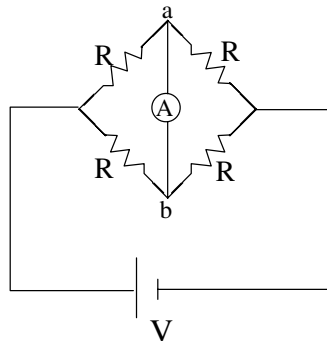

   (a) I.

   (b) $I/2$.

   (c) $I/4$.

   (d) 0.

7. Consider four equal currents going into or out of the page as indicated in the figure below. Rank the line integral of the magnetic field $\oint \vec{B} \cdot \vec{dl}$ (from greatest to least) taken in the clockwise direction.

   (a) $A = B = C = D$

   (b) $A > C > B > D$

   (c) $D > B > C > A$

   (d) $B = C > D > A$

   (e) $A > D > C = B$
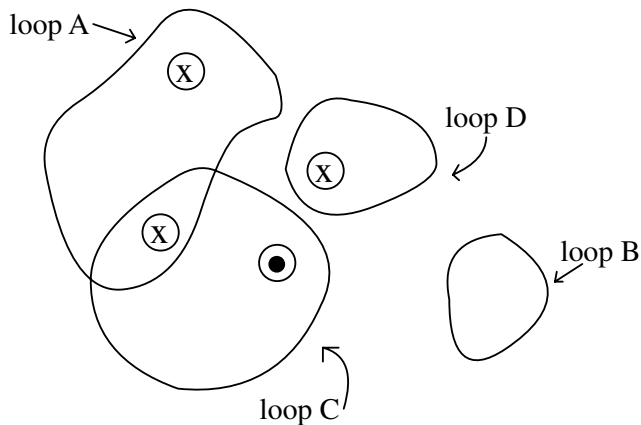
8. Two identical capacitors are connected in series as shown in the figure below. A dielectric slab ($\kappa > 1$) is placed between the plates of one capacitor, and the battery remains connected. Which of the following statement(s) is (are) correct following the insertion of the dielectric?
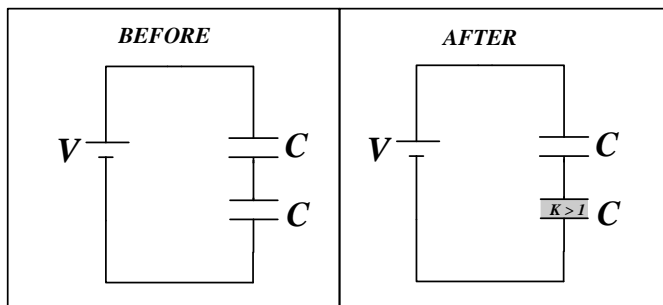

   I. The charge supplied by the battery decreases.

  II. The charge supplied by the battery does not change.

 III. The capacitance of the system increases.

 IV. The capacitance of the system decreases.

  V. The electrostatic potential energy decreases.

(a) Only I is correct.

(b) Only II is correct.

(c) Only III is correct.

(d) Only IV is correct.

(e) III, and V are correct.

9. A negative charge $-q$ is moving parallel to a long straight wire carrying a constant current as illustrated in the figure below. Which arrow correctly describes the direction of the magnetic force experienced by the charge?

(a) ⊗(into the page)

(b) ⟵

(c) ⟶
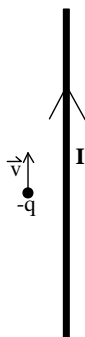
(d) ⊙(out of the page)

(e) 0

10. The figure below shows four *different* sets of wires that cross each other *without actually touching*. The magnitude of the current is the same in all four cases, and the directions of current flow are as indicated. For which configuration will the magnetic field at the center of the square formed by the wires be equal to zero?
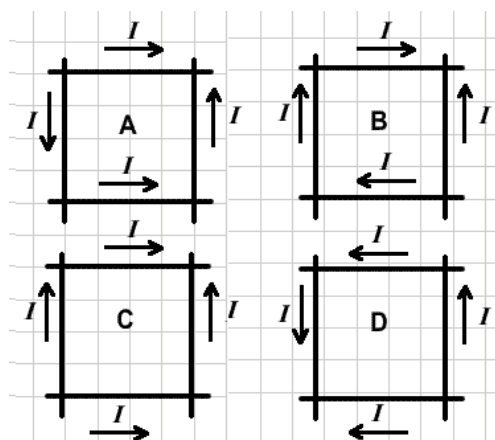
(a) A.
(b) B.
(c) C.
(d) D.

11. The figure below shows two long wires carrying equal currents I_1 and I_2 flowing in opposite directions. Which arrow correctly represents the direction of the magnetic field at the origin of the coordinate system defined in the figure? Point **P** is located at an equal distance d from each wire.
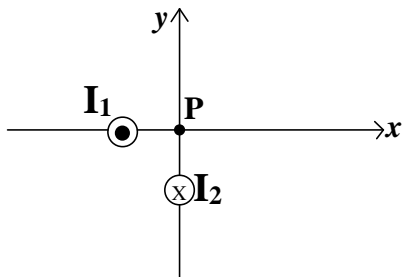

(a) $\uparrow$

(b) $\longrightarrow$

(c) ↙

(d) ↗

(e) 0

12. A rectangular loop of wire of length l and width w carrying current I is placed in a uniform magnetic field $\vec{B}$ as shown in the figure below. The magnetic field is parallel to the plane containing the loop. Which of the following statement(s) is (are) correct?
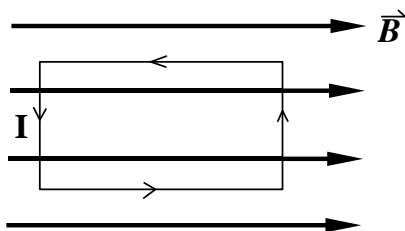

 I. The current loop experiences a net force.

 II. The current loop experiences no net force.

 III. The magnetic dipole moment of the current loop, $\vec{\mu}$, points into the page.

 IV. The magnetic dipole moment of the current loop, $\vec{\mu}$, points out of the page.

 V. The current loop experiences no net torque.

 VI. The current loop experiences a net torque.

 (a) Only II and V are correct.

 (b) Only I and VI are correct.

 (c) Only I, III and IV are correct.

 (d) Only II, IV and V are correct.

 (e) Only II IV and VI are correct.

13. A metal spring can be used as a solenoid. The spring is stretched slightly, and a current is passed through it. The resulting magnetic field will cause the spring to

 (a) collapse.

 (b) stretch out further.

 (c) not change because nothing happens.

14. A positively charged disk is rotated clockwise as shown in the figure below. What it the direction of the magnetic field at point **A** in the plane of the disk?

(a) ⊗(into the page)

(b) ⟶

(c) ⟵
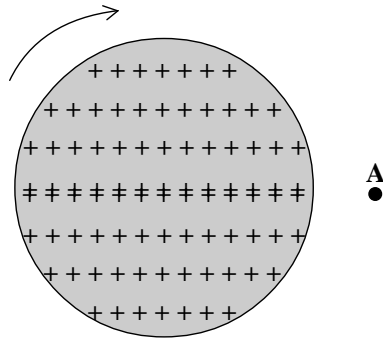
(d) ⊙(out of the page)

(e) 0

15. A wire of radius a carries a current I uniformly distributed throughout its interior. Which graph below best represents the magnitude of the magnetic field as a function of r, the distance from the center of the wire?
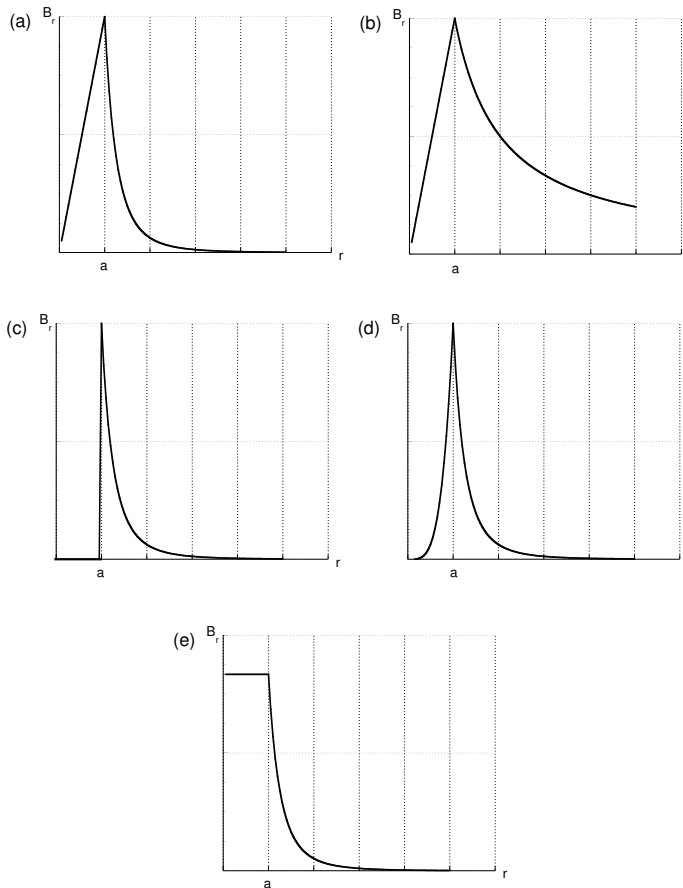
Physics 102 Spring 2006: Exam #2 —Multiple-Choice Answers

	A	B	C	D	E
1			X		
2					X
3		X			
4	X				
5			X		
6				X	
7					X
8			X		
9		X			
10			X		
11				X	
12					X
13	X				
14				X	
15		X			