Last Name: First Name:

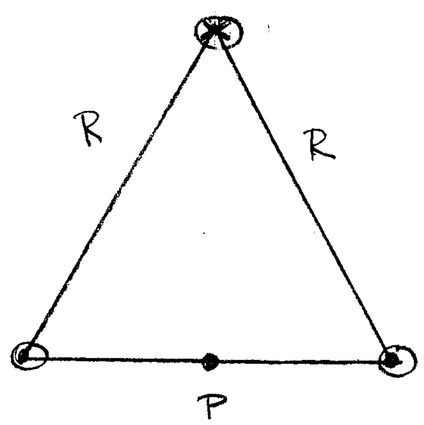
Physics 102 Spring 2007: Exam 2—Free Response and Instructions

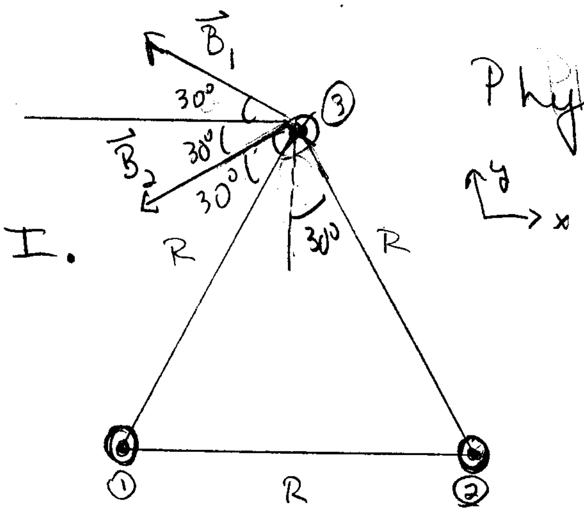
- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and 15 multiple-choice questions.
- The test is graded on a scale of 100 points; free-response question I counts for 30 points, free-response II counts for 25 points, and the multiple-choice questions account for 45 points (three points each).
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

I. (30 pts) Three very long current carrying wires are arranged to form an equilateral triangle with side R, as shown in the figure below. The bottom two wires carry current I out of the page, and the top wire carries current I into the page.

8 (a) Determine the magnetic field at the location of the top wire due to the bottom two wires.
4 (b) Determine the force per unit length on the top wire due to the bottom two.
8 (c) Determine the magnetic field at the location of the lower left wire due to the other two.
5 (d) Determine the magnetic field at the point P midway between the lower two wires.
2 (e) A particle with charge $+q$ is released from rest at point P. Determine the force on this particle due to the magnetic field immediately after it is released.
3 (f) A particle with charge $+q$ is released with initial velocity in the x-direction, $\vec{v} = v_o\hat{j}$. Determine the force on this particle due to the magnetic field immediatley after it is released.





Phys 102 - Exam 2

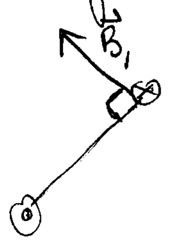
I. Three wires form an equilateral triangle - bottom two carry current out of page; top wire carries current I into page.

(a) $\vec{B}$ at the location of the top wire has two contributions, $\vec{B}_1$ and $\vec{B}_2$. From Ampere's law we know that the magnetic field around a current-carrying very long wire is concentric loops.

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

The direction of $\vec{B}_1$ at the top wire will be $\perp$ to the line from wire ① to the top wire ③.



Likewise, $\vec{B}_2$ is $\perp$ to the line from wire ② to wire ③.

from the figure, it is clear that the y-components cancel and the x-components add.

$B_x = B\cos\theta$ where $\theta = 30^\circ$ $\quad |B| = \dfrac{\mu_0 I}{2\pi R}$

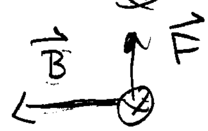
$$\vec{B}_{at\,3} = \frac{\mu_0 I}{2\pi R}\,(2\cos 30^\circ)\,(-\hat{\imath}) \qquad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\vec{B}_{at\,3} = \frac{\mu_0 I}{2\pi R}\,\frac{2\sqrt{3}}{2}\,-\hat{\imath}$$ direction is $-x$

$$\boxed{\vec{B}_{at\,3} = -\frac{\sqrt{3}\,\mu_0 I}{2\pi R}\,\hat{\imath}}$$

(b) Force per unit length on (3):

$$\vec{F} = I\vec{L} \times \vec{B}$$

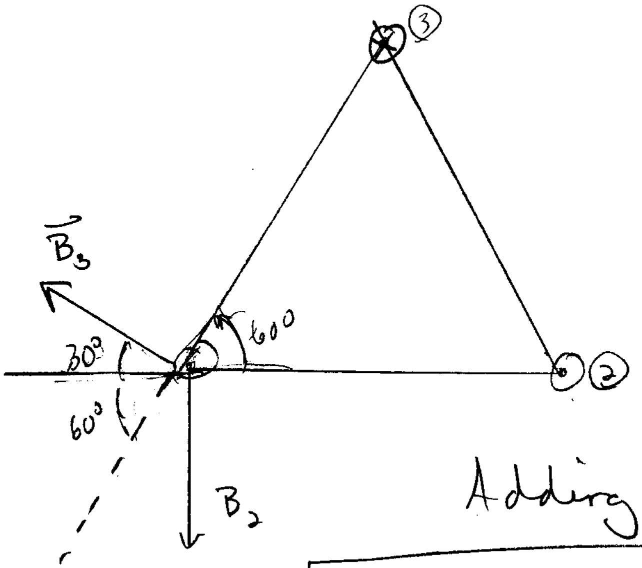


direction of $\vec{F}$ is up

$$\vec{F} = IL\left(\frac{\sqrt{3}\,\mu_0 I}{2\pi R}\right)\hat{\jmath}$$

$$\boxed{\frac{\vec{F}}{L} = \frac{\sqrt{3}\,\mu_0 I^2}{2\pi R}\,\hat{\jmath}}$$ - Force is upward

(C) $\vec{B}$ at lower left wire



$$\vec{B}_2 = \frac{\mu_0 I}{2\pi R}(-\hat{j})$$

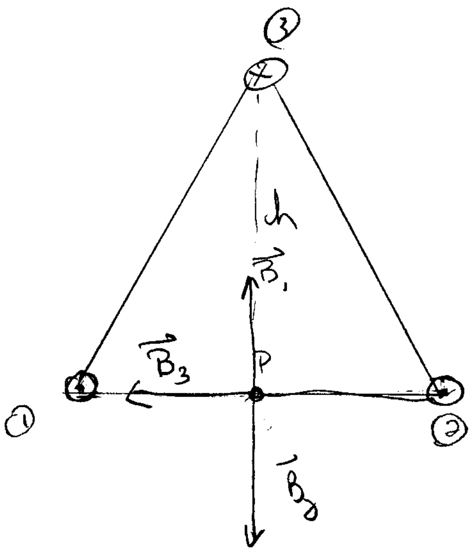
$$\vec{B}_3 = \frac{\mu_0 I}{2\pi R}\left[\cos 30^\circ - \hat{i} + \sin 30^\circ \hat{j}\right]$$

$$\vec{B}_3 = \frac{\mu_0 I}{2\pi R}\left[-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

Adding the two contributions we get

$$\boxed{\vec{B}_{at\,1} = \frac{\mu I}{2\pi R}\left[-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right]}$$

(d) $\vec{B}$ at point P, midway between ① and ②



The contributions from ① & ② cancel!

$$\vec{B}_3 = \frac{\mu_0 I}{2\pi h}(-\hat{i})$$

where $h^2 + \left(\frac{R^2}{4}\right) = R^2$

$$h^2 = \frac{3R^2}{4}$$

$$h = \frac{\sqrt{3}R}{2}$$

$$\vec{B}_{at\,P} = \frac{\mu_0 I\, 2}{2\pi\sqrt{3}R}(-\hat{i})$$

$$\boxed{\vec{B}_{at\,P} = -\frac{\mu_0 I}{\sqrt{3}\pi R}\hat{i}}$$

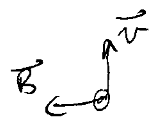
(e) $\vec{F}_q = q\vec{v} \times \vec{B}$

For a particle released from rest, $\boxed{\vec{F} = 0}$

(f) $\vec{v} = v_0 \hat{j}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q v_0 |B| (\hat{j} \times -\hat{i})$$



F in +z direction

$$\vec{F} = q v_0 \left(\frac{\mu_0 I}{\sqrt{3}\pi R} \right) \hat{k}$$

$$\boxed{\vec{F}_{\text{on } q} = \frac{\mu_0 q v_0 I}{\sqrt{3}\pi R} \hat{k}}$$

Phys. 102 Exam 2 - Grading Criteria

I. 30 pts

(a) 8 pts

Amperes Law 2

B for single wire 2

Take components correctly 2

Correct final result 2

(b) 4 pts

$F = I\vec{L} \times \vec{B}$ 2

Correct F/L 2

(c) 8 pts

B due to bottom wire 2

B due to top wire 2

Components 2

Correct sum 2

(d) 5 pts

Fields from bottom wires cancel 2

Correct field from top wire 3

(e) 2 pts

$\vec{F} = q\vec{v} \times \vec{B}$ 1

$F = 0$ 1

(f) 3 pts

$\vec{F} = q\vec{v} \times \vec{B}$ 1

Correct F 2

II.(25 pts) In the mass spectrometer shown below, singly charged Cl^+ ions are passed through a velocity selector and then enter a region of uniform magnetic field, with the magnetic field pointing out of the page. The goal is to separate the isotopes Cl^{35} and Cl^{37} by at least 2cm in the mass spectrometer after they have traveled a semicircular path as shown. The magnetic field in the mass spectrometer is 1T. Take the mass of the Cl ions to be 35amu and 37amu where 1 amu (atomic mass unit)=1.66×10^{-27}kg. The charge of the electron is 1.6×10^{-19}C.

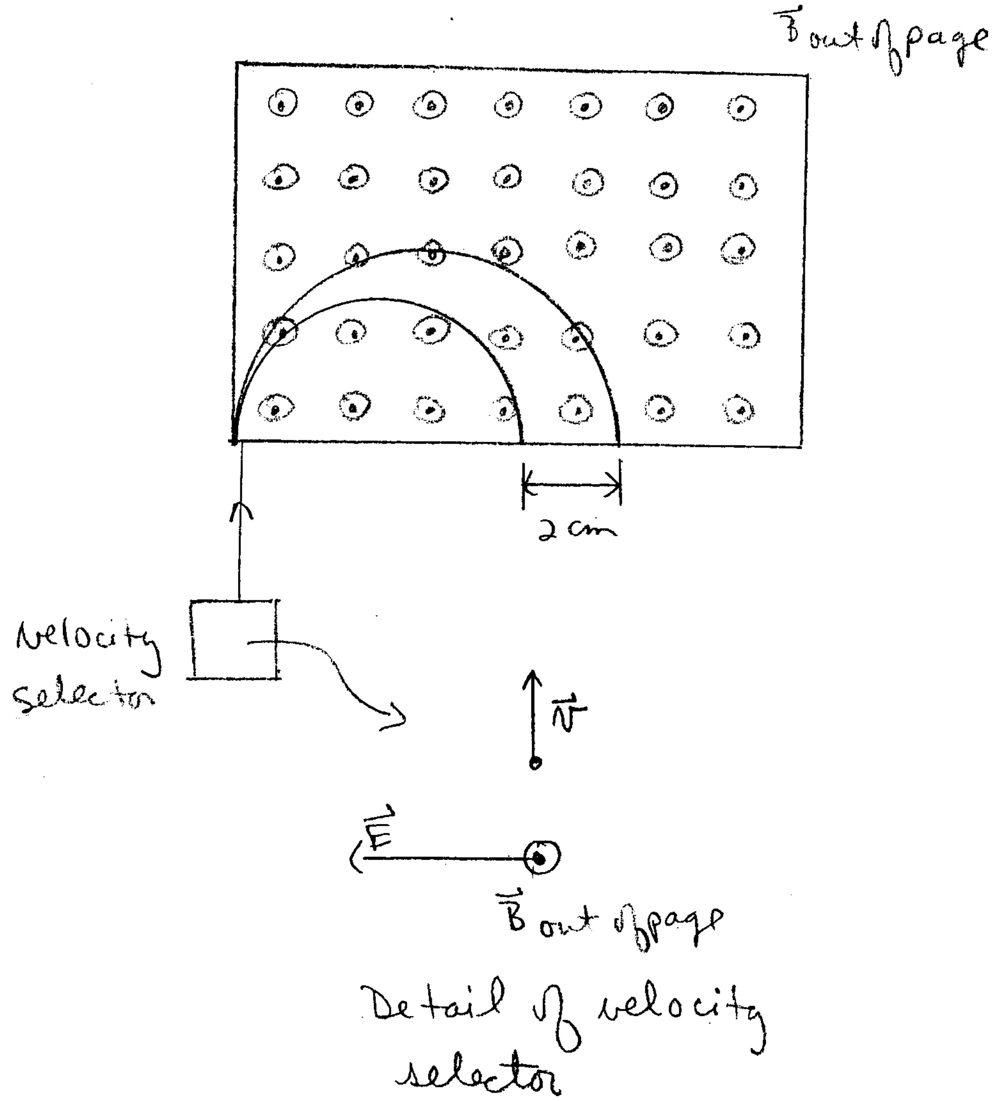
5 (a) Show that the radius of curvature of a particle of charge q, velocity v, and mass m in a uniform magnetic field B is $R = \frac{mv}{qB}$.

7 (b) Determine the velocity that the ions must have as the enter the mass spectrometer so that they land 2cm apart after they have traveled a complete semicircle.

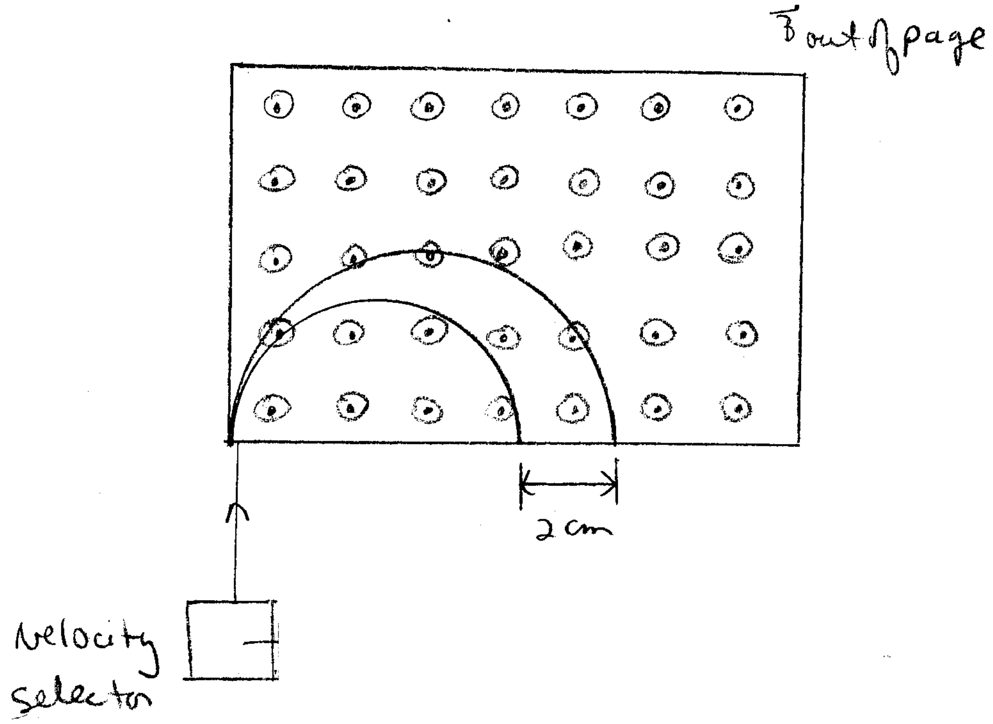
4 (c) At that velocity, what is the radius of curvature of the Cl^{35} ions?

5 (d) The velocity selector consists of perpendicular $\vec{E}$ and $\vec{B}$ fields which are both perpendicular to the direction of the incoming ions. Show that when a charged particle enters such a region of crossed $\vec{E}$ and $\vec{B}$ fields, there is one value of the velocity v_o for which the force on the charged particle is zero.

4 (e) If the magnitude of $\vec{B}$ in the velocity selector is 1T, what magnitude of the electric field $\vec{E}$ will select the velocity you found in (a)?



II.



(a) To determine the radius of curvature, set the magnetic force equal to the centripetal force needed for uniform circular motion.

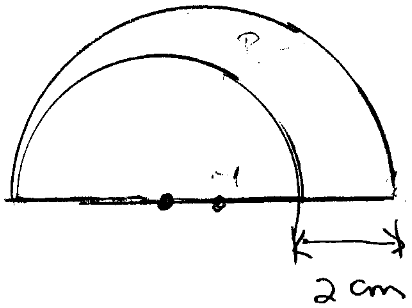
$$\frac{mv^2}{R} = |q\vec{v} \times \vec{B}|$$

In this case $\vec{v} \perp \vec{B}$ so $\vec{v} \times \vec{B} = vB$ and we get

$$\frac{mv^{2}}{R} = qvB$$

$$\boxed{R = \frac{mv}{qB}}$$

(b)



The difference in diameters is 2cm,

$2R_1 - 2R_2 = .02\,m$

$$2R_1 - 2R_2 = \frac{2(m_1 - m_2)v}{qB} = .02\,m$$

Now we can solve for v

$1\ \text{Tesla} = \frac{N\text{-}s}{C\text{-}m}$

$$v = \frac{(.02\,m)(qB)}{2(m_1 - m_2)}$$

$$v = \frac{(2 \times 10^{-2}\,m)(1.6 \times 10^{-19}\,C)\left(1\ \frac{N\text{-}s}{C\,m}\right)}{2(1.66 \times 10^{-27}\,kg)}$$

$$v = 0.482 \times 10^{6}\ \frac{kg\text{-}m}{s^2} \cdot \frac{s}{kg}$$

$$\boxed{v = 4.82 \times 10^{5}\ m/s}$$

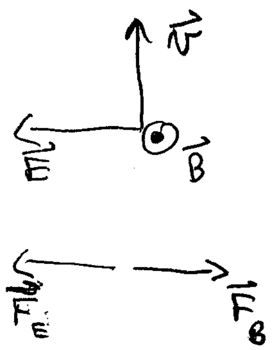
(c)

$$R = \frac{mv}{qB} = \frac{35\,(1.66\times10^{-27}\,kg)(4.82\times10^{5}\,m/s)}{(1.6\times10^{-19}\,C)(1\,\frac{N s}{C m})}$$

$$R = 175\times10^{-3}\ \frac{kg\ m\ s^2\ m}{(kg\cdot m)\,s^2}$$

$$\boxed{R = .175\,m}$$

(d)



$$\vec{F}_E = q\vec{E} = qE(-\hat{i})$$

$$\vec{F}_B = q\vec{v}\times\vec{B} = qvB(+\hat{i})$$

The directions of the electric & magnetic forces is opposite, for the net force to be zero the magnitudes must be equal

$$|\vec{F}_B| = |\vec{F}_E|$$

$$qv_0B = qE$$

$$\boxed{v_0 = E/B}$$

(d)

$$E = v_0 B = (4.82 \times 10^5 \; \mathrm{m/s})\left(1 \; \frac{\mathrm{N\,s}}{\mathrm{C\,m}}\right)$$

$$\boxed{E = 4.82 \times 10^5 \; \mathrm{N/C}}$$

Grading Criteria -

II. 25pts

(a) 5pts

magnetic force = centripetal force. 4

correct result for R 1

(b) 7 pts

(c) 4 pts

correct R 4

(d) 5pts

F_E - 2pts

F_B - 2pts

$v = E/B$ - 1 pt

(e) 4pts

Correct E