Last Name: First Name:

Physics 102 Spring 2005: Exam 1—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; each free-response question accounts for 35 points, and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

35

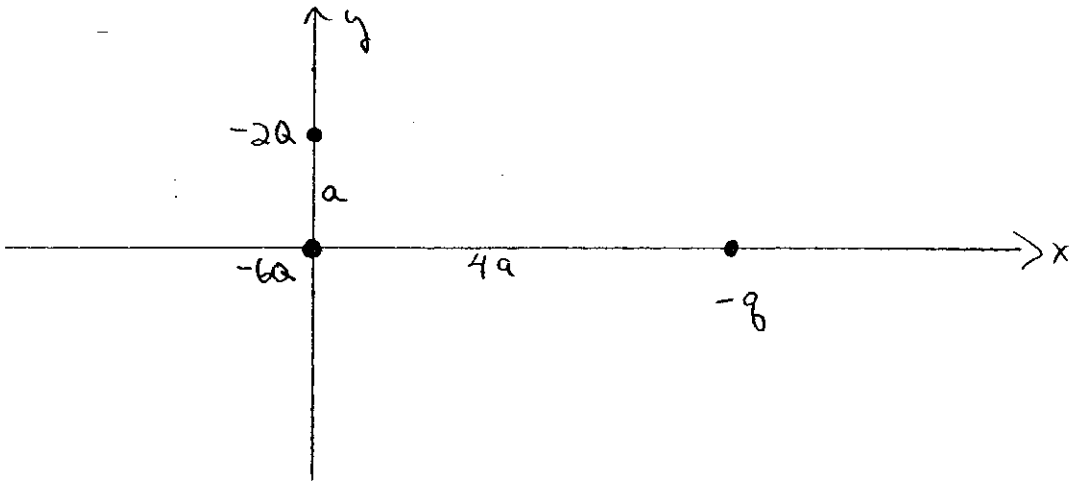
I. As shown in the figure below, three point charges are arranged as follows: a negative charge $-6Q$ is located at the origin; a second negative charge $-2Q$ is located on the y-axis, a distance a from the origin; a third negative charge $-q$ is located on the x-axis, a distance $4a$ from the origin. The electrostatic potential energy is zero when the charges are infinitely far from each other. The electrostatic potential is zero at infinity.

5 (a) Determine the total electrostatic potential energy of this charge system.

10 (b) The charge $-q$ is released and moves off to infinity. What is its kinetic energy when it is infinitely far from the origin? The other two charges remain fixed.

(c) Determine an expression for the electrostatic potential $V(x, y)$ at an arbitrary point $P(x, y)$ in the x-y plane due to the two remaining charges.

(d) Determine an expression for the the x-component of the electric field E_x due to the two remaining charges, as a function of x for an arbitrary point on the x-axis.



35 II. An insulating sphere of radius R is centered at the origin. It carries a positive uniform volume charge density ρ. In addition, a very long, thin insulating rod runs parallel to the y-axis at $x = 4R$. The rod carries a negative uniform linear charge density $-\lambda$. Express your answers in terms of ρ, R, λ, and possibly other constants.

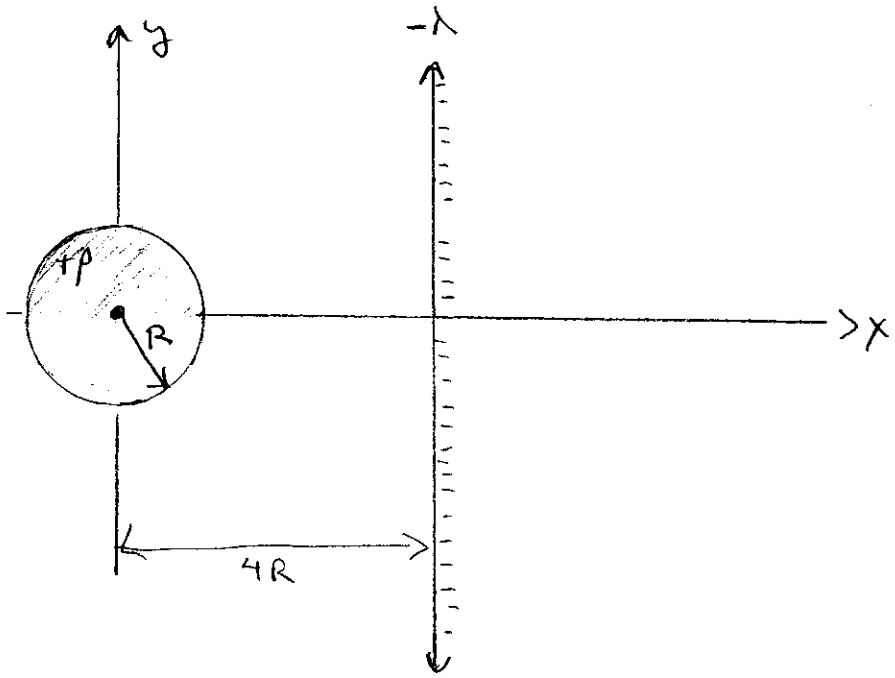
7 (a) Determine the electric field $\vec{E}$ at the point $x = 2R, y = 0$.

7 (b) Determine the electric field $\vec{E}$ at the point $x = 0, y = 3R$.

8 (c) Determine the contribution to the x-component of the electric field, E_x, due to the rod only, as a function of position x on the x-axis. Sketch this contribution to E_x.

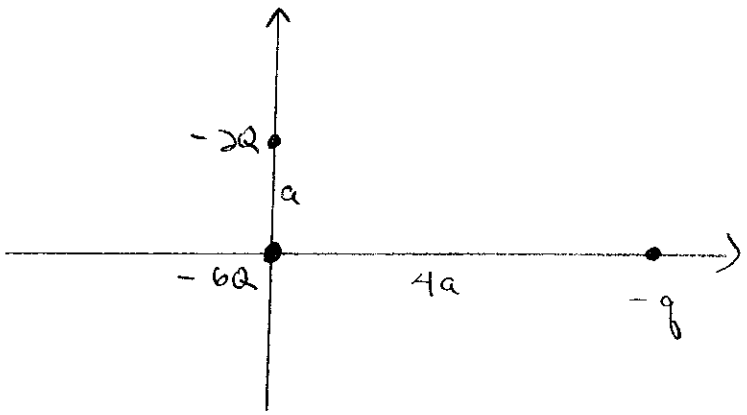
8 (d) Determine the contribution to the x-component of the electric field, E_x, due to the sphere only, as a function of position x on the x-axis. Sketch this contribution to E_x.

5 (e) Determine the electric flux Φ_E through a cube of side $\frac{1}{3}R$ centered at $x = 0, y = 2R$.



Phys 102
Exam 1

I.



(a) $U = \frac{k(-2Q)(-6Q)}{a} + \frac{k(-6Q)(-q)}{4a} + \frac{k(-2Q)(-q)}{\sqrt{16a^2+a^2}}$

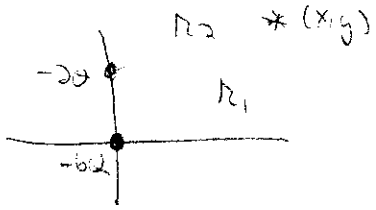
$$U = \frac{12kQ^2}{a} + \frac{3kQq}{2a} + \frac{2kQq}{\sqrt{17}\,a}$$

(b) When $-q$ moves off to infinity, its kinetic energy will equal the change in U

$$U_{final} = \frac{12kQ^2}{a}$$

$$\Delta U = \frac{3kQq}{2a} + \frac{2kQq}{\sqrt{17}\,a} = KE$$

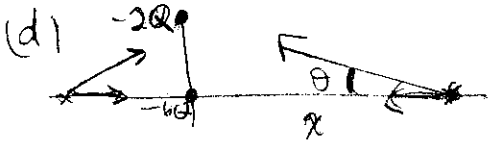
(c)



$$V(x,y) = \frac{-2kQ}{r_2} - \frac{6kQ}{r_1} \qquad r_1 = \sqrt{x^2+y^2} \qquad r_2 = \sqrt{x^2+(y-a)^2}$$

$$\boxed{V(x,y) = \frac{-2kQ}{\sqrt{x^2+(y-a)^2}} - \frac{6kQ}{\sqrt{x^2+y^2}}}$$

(d)



$$E_x = \left(-\frac{6kQ}{x^2} - \frac{2kQ}{\sqrt{x^2+a^2}}\cos\theta\right)\hat{i} \qquad \cos\theta = \frac{x}{\sqrt{x^2+a^2}}$$

$$\boxed{E_x = \left[-\frac{6kQ}{x^2} - \frac{2kQx}{(x^2+a^2)^{3/2}}\right]\hat{i}} \qquad x \geq 0$$

$$\boxed{E_x = \left[\frac{6kQ}{x^2} - \frac{2kQx}{(x^2+a^2)^{3/2}}\right]\hat{i}} \qquad x \leq 0$$

Note that the direction of E_x changes sign for $x>0$ and $x<0$.

1(d) Alternative method – $E_x = -\frac{\partial V}{\partial x}$

$$V(x,y) = \frac{-2kQ}{\left[x^2+(y-a)^2\right]^{1/2}} - \frac{6kQ}{(x^2+y^2)^{1/2}}$$ from (c)

$$\frac{\partial V(x,y)}{\partial x} = \frac{+2kQ(-\frac{1}{2})(2x)}{(x^2+(y-a)^2)^{3/2}} - \frac{6kQ(-\frac{1}{2})(2x)}{(x^2+y^2)^{3/2}}$$

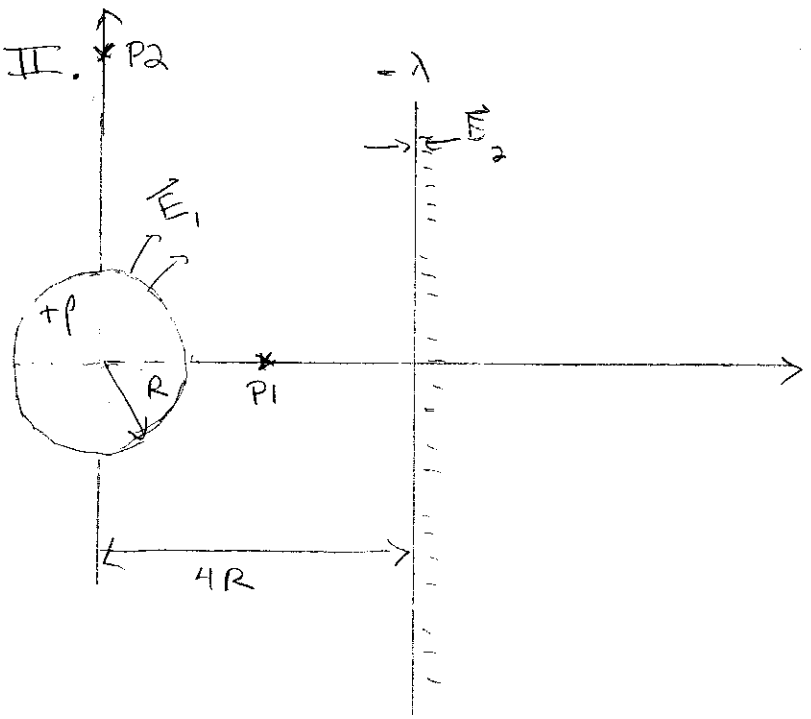
On the y-axis, $y \to 0$

$$E_x(y=0) = -\frac{\partial V}{\partial x} = \frac{-2kQx}{(x^2+a^2)^{3/2}} - \frac{6kQx}{|x|^3}$$

$$\boxed{E_x(y=0) = \left[\frac{-2kQx}{(x^2+a^2)^{3/2}} - \frac{6kQx}{|x|^3}\right]\vec{i}}$$

This expression is correct for all x, note that E_x changes sign for $x>0$ and $x<0$

II.



(a) find $\vec{E}$ at $x=2R, y=0$ (P1)

$\vec{E}_1$ due to sphere:

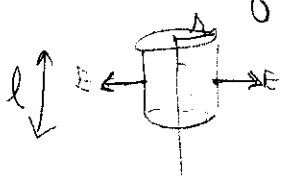
$\int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ all the charge on the sphere is enclosed.

$4\pi r^2 E_1 = \frac{Q_{TOT}}{\varepsilon_0}$ $\quad Q_{TOT} = \frac{4}{3}\pi \rho R^3$

$\vec{E}_1 = \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$ where $r = 2R$ at point P1 and $\hat{r} = \hat{\imath}$ at P1

$\vec{E}_1(P1) = \frac{\rho R}{12\varepsilon_0}\hat{\imath}$ – due to sphere. +3

$\vec{E}_2$ due to the line of charge:



$\int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ $\quad \vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$2\pi r l E = \frac{-\lambda \cancel{l}}{\varepsilon_0}$ (radially inward)

At point P1, $\hat{n} = -\hat{i}$ and $r = 2R$

Note at P1 $\vec{E}_2$ is in the $\pm x$ direction.

$$\vec{E}_2(P_1) = \frac{-\lambda}{2\pi\varepsilon_0(2R)}(-\hat{i}) = \frac{+\lambda}{4\pi\varepsilon_0 R}\hat{i} \quad +4$$

The total field is the superposition of these two contributions:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\boxed{\vec{E}(P1) = \left(\frac{\rho R}{12\varepsilon_0} + \frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i}}$$

(b) P2 $\Rightarrow$ $x = 0$; $y = 3R$

The same formula for $\vec{E}_1$ & $\vec{E}_2$

$$\vec{E}_1 = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{n} \qquad \text{at P2 } \hat{n} = \hat{j} \text{ and } r = 3R$$

Note $\vec{E}_1$ is in the $+y$ direction.

$$\vec{E}_1(P2) = \frac{\rho R^3}{3\varepsilon_0 9R^2}\hat{j} = \frac{\rho R}{27\varepsilon_0}\hat{j} \quad +3$$

$$\vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat{r} \qquad \hat{r} \text{ is still } -\hat{i},\ \vec{E}_2 \text{ is } \perp \text{ to the line of charge.}$$

$r = 4R$ - the perpendicular distance from P2 to the line $+4$

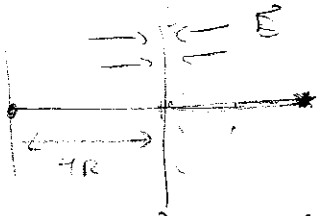
$$\vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0(4R)}(-\hat{i})$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\boxed{\vec{E}(P2) = \frac{+\lambda}{8\pi\varepsilon_0 R}\hat{i} + \frac{\rho R}{27\varepsilon_0}\hat{j}}$$

(c) $\vec{E}$ due to rod only on x-axis

$$\vec{E}_{\text{rod}} = \frac{-\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

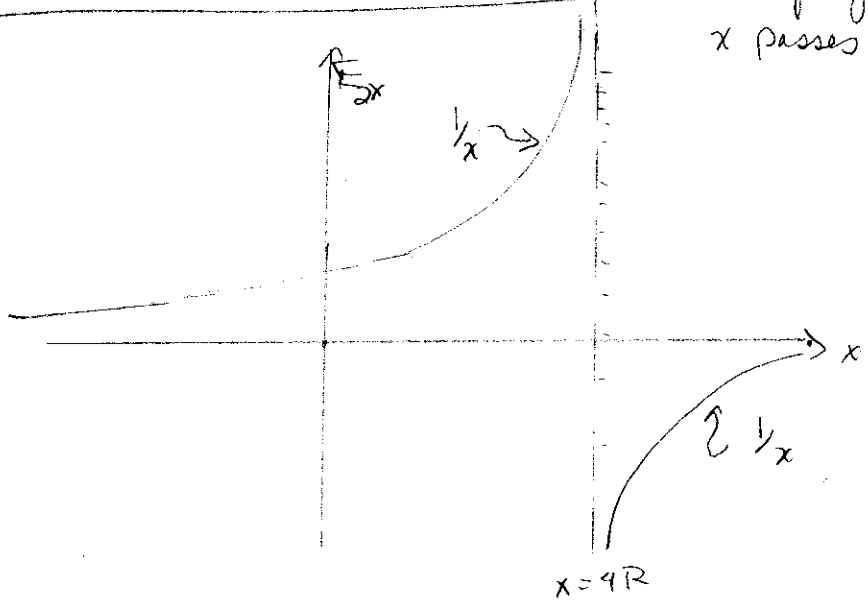


$$E_{\text{rod}}(\text{on x-axis}) = \frac{-\lambda}{2\pi\varepsilon_0 |x-4R|}\hat{\imath}$$

$\Rightarrow \vec{E} = E_x$ only, no other components

$$E_{rx}(\text{on x-axis}) = \frac{-\lambda}{2\pi\varepsilon_0 (x-4R)}\hat{\imath}$$

Correct for all values of x – note change of sign as x passes through 4R



(d) $\vec{E}$ due to sphere only, on x-axis

for $|x| > R$, the expression from (a) is correct.

$$\vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$$

on the x-axis, $\hat{r} = \hat{\imath}$ and $r = x$

for $x < 0$, $\vec{E}$ changes direction, $\hat{r} = -\hat{\imath}$

(d) Cont'd

$$\vec{E}_{1x} = \frac{\rho R^3}{3\varepsilon_0 x^2}\hat{\imath} \quad \text{for } x > R$$

$$E_{1x} = \frac{-\rho R^3}{3\varepsilon_0 x^2}\hat{\imath} \quad \text{for } x < -R$$

For $|x| < R$, we have to consider the fact that only part of the charge is enclosed:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \rho\frac{(4/3\pi r^3)}{\varepsilon_0}$$

$$4\pi r^2 E_r = \frac{4\pi\rho r^3}{3\varepsilon_0}$$

$$E_r = \frac{\rho r^3}{3\varepsilon_0 r^2} = \frac{\rho r}{3\varepsilon_0}\hat{r}$$

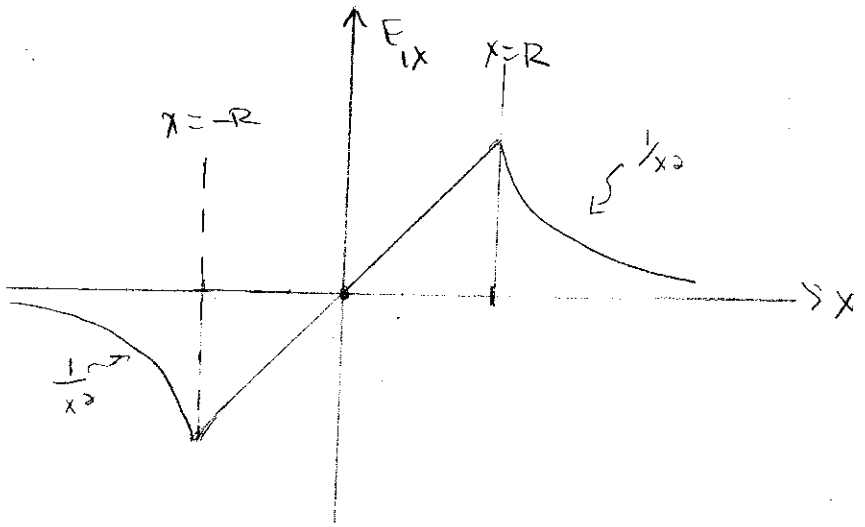
for $x > 0$ $\hat{r} = \hat{\imath}$, $r = x$

for $x < 0$ $\hat{r} = -\hat{\imath}$

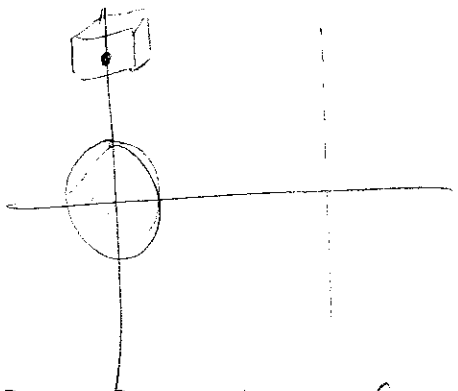
Then we have

$$\vec{E}_{1x} = \frac{\rho x}{3\varepsilon_0}\hat{\imath} \quad \text{for } |x| < R$$

Correct for $x > 0$ and $x < 0$, note that x changes sign



(e)



Φ_E through cube at $x=0, y=2R$

side of cube $= R/3$.

By Gauss's law, $\oint \vec{E} \cdot d\vec{A} = \Phi_E = \frac{Q_{enc}}{\varepsilon_0}$

But $Q_{enc} = 0$, so

$$\boxed{\Phi_E = 0}$$

PHYS102 EXAM #1 February 17, 2005

Last Name_____ First Name_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

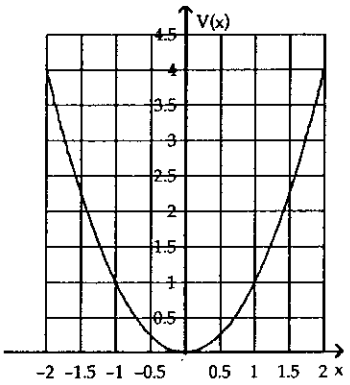
1) A spherical metallic shell carries a charge 2q. A point charge q is placed at the center of the shell. When electrostatic equilibrium is reached, what is the charge carried by the outer surface of the shell?
A) 0 B) q C) 4q D) 2q → (E) 3q

2) A spherical metallic shell carries a charge 2q. A point charge q is placed at the center of the shell. When electrostatic equilibrium is reached, what is the charge carried by the inner surface of the shell?
A) 2q → (B) -q C) q D) 3q E) 0

3) If the electric potential is given by $V(x,y,z) = xy - 3z^{-2}$, then the electric field has a y-component
A) $x + y - 6z^{-3}$. B) x + y. → (C) - x D) x. E) y.

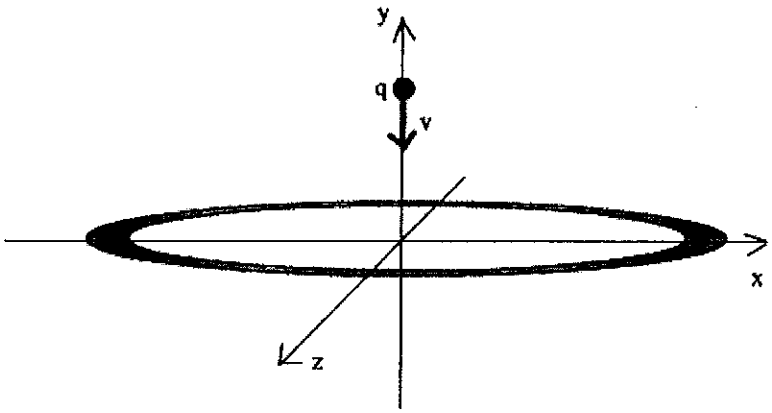
4) An electric dipole of dipole moment $\vec{p} = p_0\hat{i} + p_0\hat{j}$ is placed in a uniform electric field $\vec{E} = E_0\hat{i}$. What is the value of the torque applied on the dipole by the electric field?
A) The torque is equal to zero.
→ (B) $\vec{\tau} = -p_0E_0\hat{k}$
C) $\vec{\tau} = p_0E_0\hat{k}$
D) $\vec{\tau} = -p_0E_0\hat{i}$
E) $\vec{\tau} = -p_0E_0\hat{j}$

5) The figure below shows the variations of the electric potential V (in arbitrary units) as a function of the position x (also in arbitrary units). Which of the choices below correctly describes the orientation of the electric field along the x axis?



A) E is negative from x = -2 to x = 2
B) More information is needed to answer the question
C) E is positive from x = -2 to x = 2
D) E is negative from x = -2 to x = 0, and positive from 0 to x= 2
→ (E) E is positive from x = -2 to x = 0, and negative from 0 to x= 2

6) A ring of negative, uniform charge density is placed on the xz-plane with the center of the ring at the origin. A positive charge moves along the y axis toward the center of the ring as shown in the figure below. At the moment the charge passes through the center of the ring



A) its velocity and its acceleration reach their maximum values.

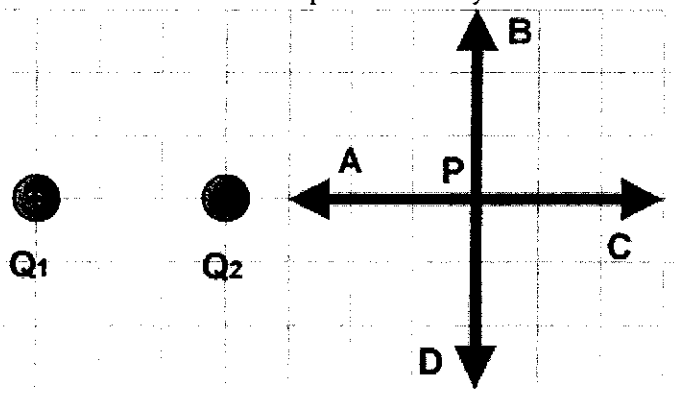
B) its velocity is zero and its acceleration is maximum.

C) its velocity and its acceleration have non-zero values but neither is at its maximum.

D) its velocity and its reach are both equal to zero.

→ (E) its velocity is maximum and its acceleration is zero.

7) Two charges Q_1 and Q_2 of equal magnitudes and opposite signs are positioned as shown in the figure below. Which of the shown arrows represents correctly the electric field at point P?



→ (A) A

B) The field is equal to zero at point P.

C) D

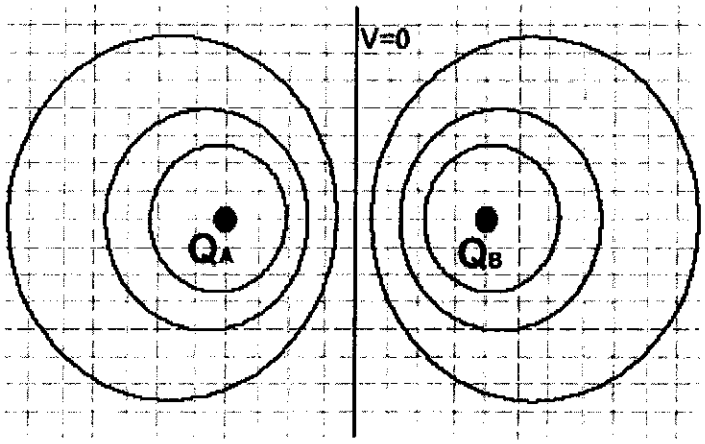
D) B

E) C

8) If a charge is located at the center of a spherical volume and the electric flux through the surface of the sphere is ϕ_O, what is the flux through the surface if the radius of the sphere doubles?

A) 0.125 ϕ_O B) 8 ϕ_O C) 0.500 ϕ_O → (D) ϕ_O E) 5 ϕ_O

9) The figure below shows equipotentials surrounding a pair of charges Q_A and Q_B. The value of the potential half-way between the charges is indicated. Which of the statements below applies to the charges?



A) The two charges have the same sign but different magnitudes

B) The two charges have the same sign and equal magnitudes

→ (C) The two charges have opposite signs and equal magnitudes

D) The two charges have opposite signs and different magnitudes

E) Nothing can be said about the charges

10) When five equal positive charges are uniformly spaced along the x-axis, the force on the next to last charge on the right is

A) zero.

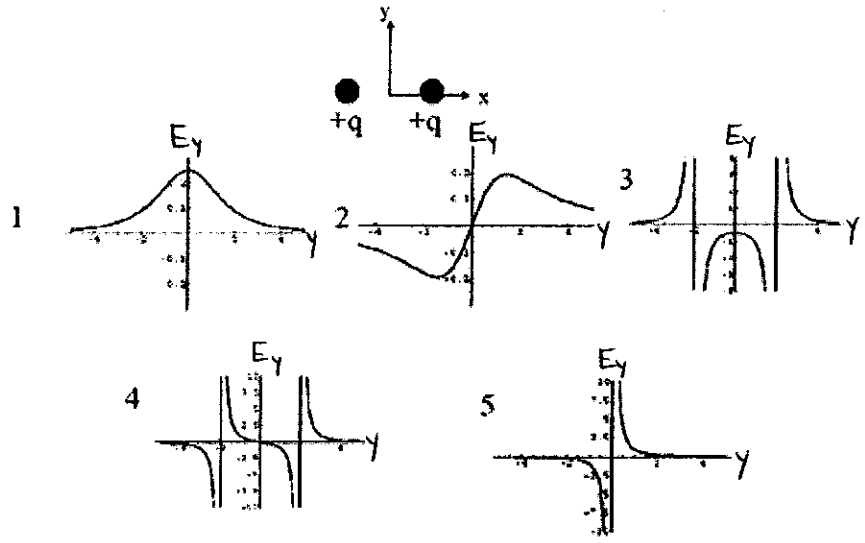
B) possibly in the negative y-direction.

C) possibly in the positive y-direction.

→ (D) to the right.

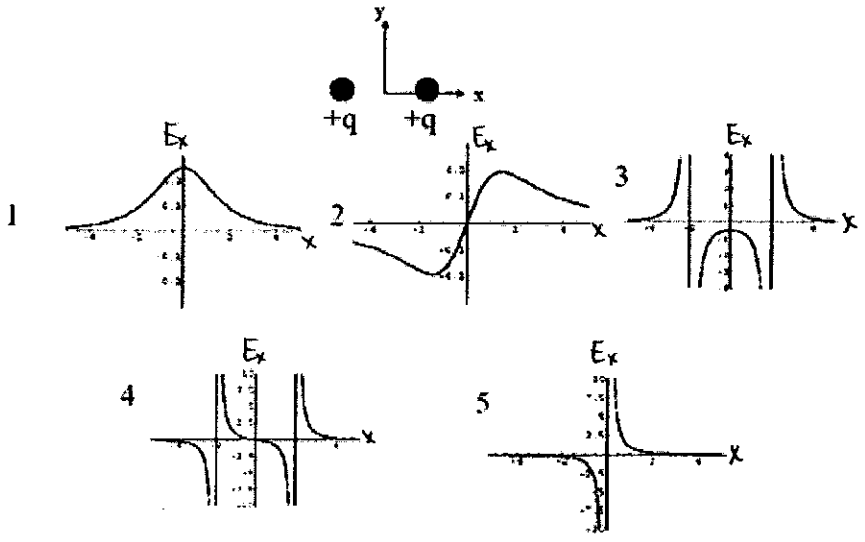
E) to the left.

11) For the assembly of charges shown below, which graph best depicts the y-component of the electric field, E_y, for points along the y-axis?



A) 1 → (B) 2 C) 3 D) 4 E) 5

12) For the assembly of charges shown below, which graph best depicts the x-component of the electric field, E_x, for points along the x-axis?



A) 1 B) 2 C) 3 → (D) 4 E) 5

13) Two charges $Q_A = +q$ and $Q_B = -3q$ are located on the x-axis at x= 0 and x= d respectively. Where is the electric potential equal to zero?

→ (A) x = d/4 B) x= 2d/3 C) x= d/3 D) x= 3d/4 E) x= d/2

14) A particle of positive charge q and mass m moving with a velocity $\vec{v} = v_0 \hat{i}$ enters a region of space where there is an electric field $\vec{E} = E_0 \hat{j}$. At time t after entering the electric field region, the velocity of the particle will be

A) $\vec{v} = v_0 \hat{i} + v_0 \hat{j}$.

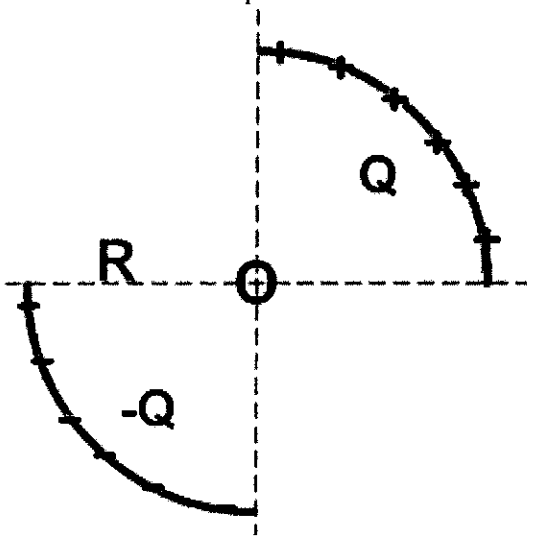
B) $\vec{v} = v_0 \hat{i} - (q E_0 t/m) \hat{j}$.

C) $\vec{v} = (q E_0 t/m) \hat{i} + (q E_0 t/m) \hat{j}$.

→ (D) $\vec{v} = v_0 \hat{i} + (q E_0 t/m) \hat{j}$.

E) $\vec{v} = (q E_0 t/m) \hat{i} + v_0 \hat{j}$.

15) The figure below shows two arcs of a circle on which charges +Q and -Q have been spread uniformly. What is the value of the electric potential at the center of the circle?



→ (A) 0 B) $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{R}$ C) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ E) $\frac{-1}{4\pi\varepsilon_0}\frac{Q}{R}$

Phys 102 - Exam 1
Grading Criteria

I. 35 pts total

(a) 5 pts — -2 if missing one term

(b) 10 pts

+5 if understand $KE = \Delta U$

+5 for correct expression

(c) 10 pts

+3 for understanding $V = \frac{kQ}{r}$

+2 for r_1

+2 for r_2

+3 for final expression

(d) 10 pts

+4 for contribution due to $-6Q$

+6 for contribution due to $-2Q$

II. 35 pts total

(a) 7 pts

+3 Contribution due to sphere
+4 contribution due to line

(b) 7 pts

+3 contribution due to sphere
+4 contribution due to line

(c) 8 pts

+2 Correct general form for line of charge
+4 Correct result for E_x
+2 Sketch

(d) 8 pts

+2 Correct general form for sphere
+2 Correct result for E_x for $|x|>R$
+2 Correct result for E_x for $|x|<R$
+2 Sketch

(e) 5 pts

+2 for definition of Φ_E & Gauss' Law
+3 Correct answer