Last Name: First Name:

Physics 102 Spring 2002: Test 2—March 26, 2002
Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; each free-response question accounts for 35 points, and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

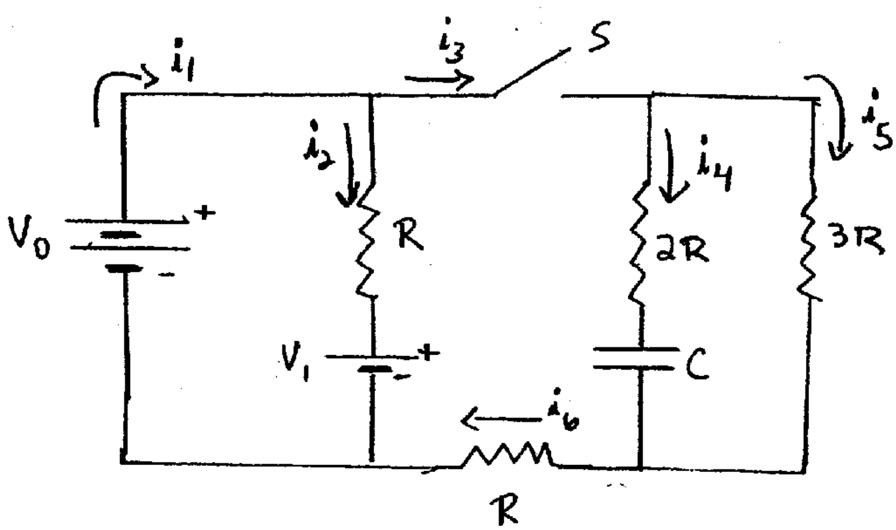
Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

I. In the circuit shown below, the switch S is initially opened. At $t = 0$, it is closed.

(a) Using Kirchoff's Laws, and considering the circuit at a time immediately after the switch is closed, write down a sufficient number of equations to allow you to solve for each of the currents, i_1 through i_6. Use the definitions of the currents shown in the figure.
(b)Solve the above equations and determine all currents immediately after the switch is closed.
(c) Determine all currents a long time after the switch is closed.
(d) Determine the voltage V_C across the capacitor C a long time after the switch is closed.

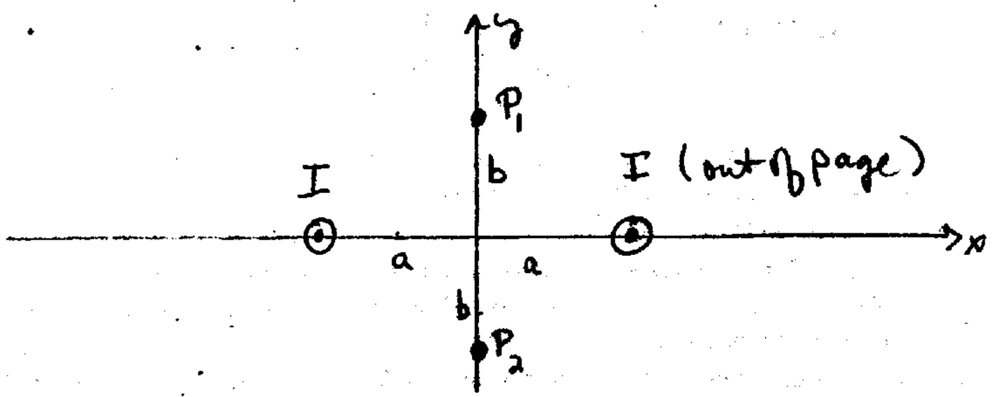
After the switch has been closed for a long time, it is opened again. Redefine $t = 0$ to be the time when the switch is opened.

(e) Determine the current through the resistor with value $2R$ immediately after the switch is opened.
(f) Determine $I(t)$, the current as a function of time.
(g) How long does it take for the current to fall to 0.1 of its initial value?



II. Two very long wires, each carrying a current I out of the page, are located on the x-axis at $x = \pm a$.

(a) Determine the magnetic field $\vec{B}$ at points P_1 and P_2 on the y axis, a distance b above and below the origin.



Now consider the situation of an infinite sheet of current which lies in the $x-z$ plane, with the current flowing toward the positive z direction, as shown below. The current density across the sheet is λ Amp/m.

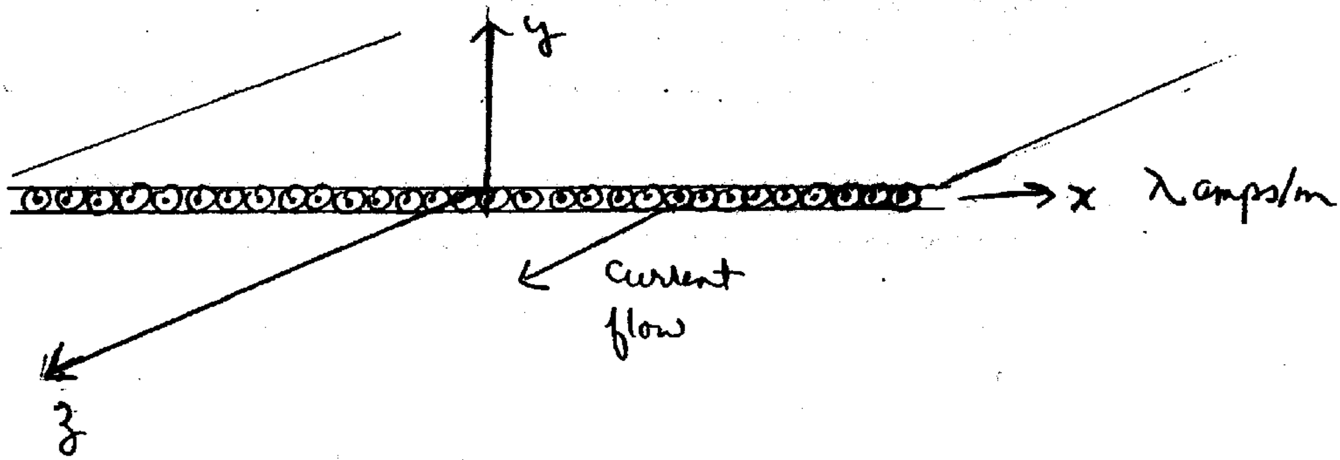
(b) Using symmetry arguments and the result from (a), determine the direction of the resulting magnetic field $\vec{B}$ both above and below the sheet of current. Indicate clearly on a sketch the direction of $\vec{B}$ both above and below the current sheet.

(c) Given the direction of the magnetic field as determined in (a), use Ampere's law to show that the magnetic field both above and below the current sheet is uniform in space. Determine the magnitude of the magnetic field in terms of the current density λ and other constants.

(d) A particle with positive charge $+Q$ and mass m enters the region below the current sheet, moving upward along the negative y axis, with initial velocity in the positive y direction, $\vec{v} = v_0\,\hat{j}$. The particle executes uniform circular motion in the $y-z$ plane. Determine the radius of the circle, the time required to complete one revolution (the period T), and the sense of rotation (clockwise or counterclockwise) as viewed from the positive x axis.

(e) How would the radius of circular motion and the period change if the initial velocity of the particle were doubled in magnitude?

(f) Suppose instead that the particle enters the region of field at 45^0 to the x-axis, $\vec{v} = [v_0\hat{i} + v_0\hat{j}]/\sqrt{2}$. Describe the subsequent motion of the particle qualitatively and determine the radius of circular motion in this case.



I. In the circuit shown below, the switch S is initially opened. At $t = 0$, it is closed.

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4 (c) Determine all currents a long time after the switch is closed.

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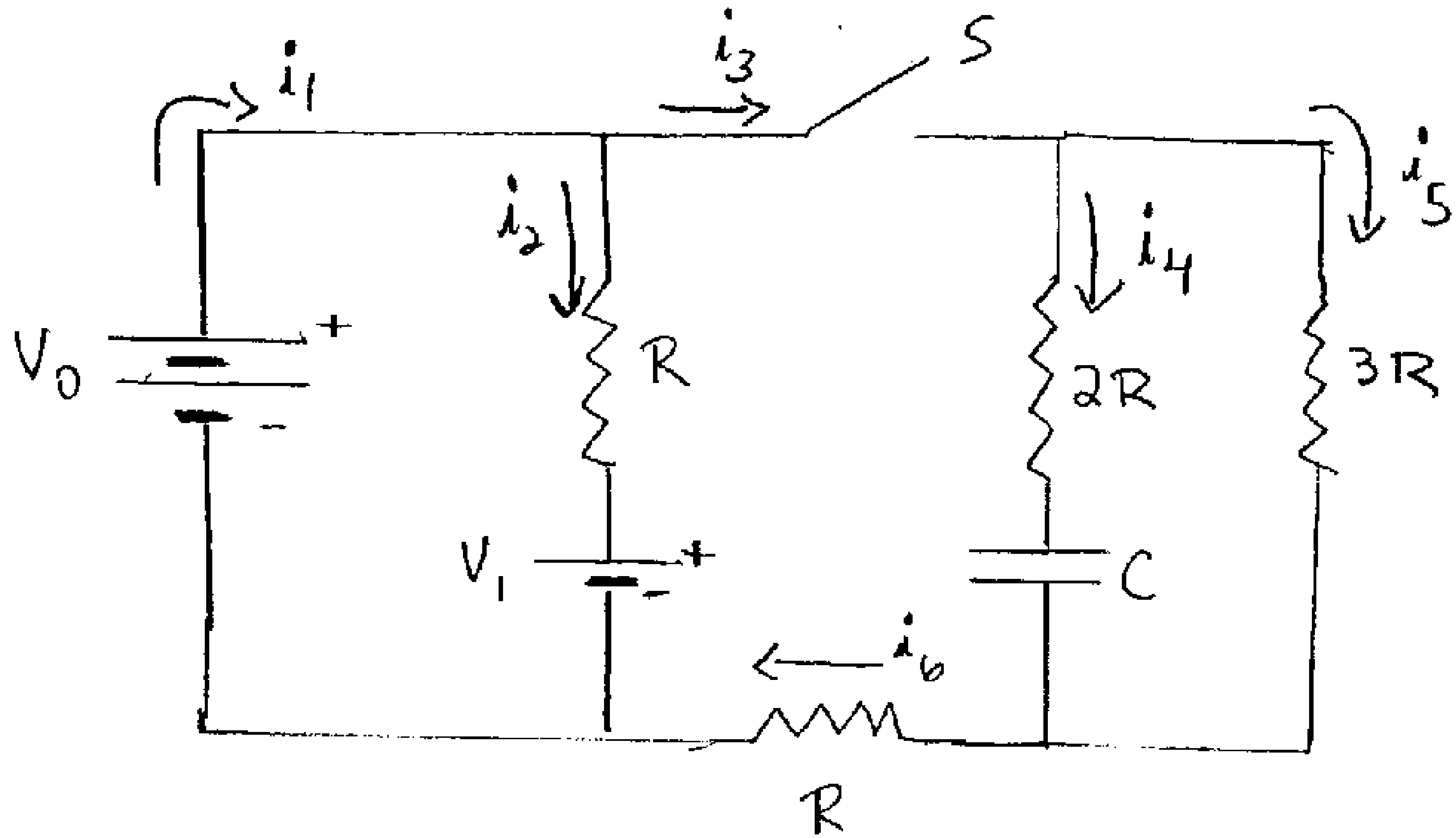
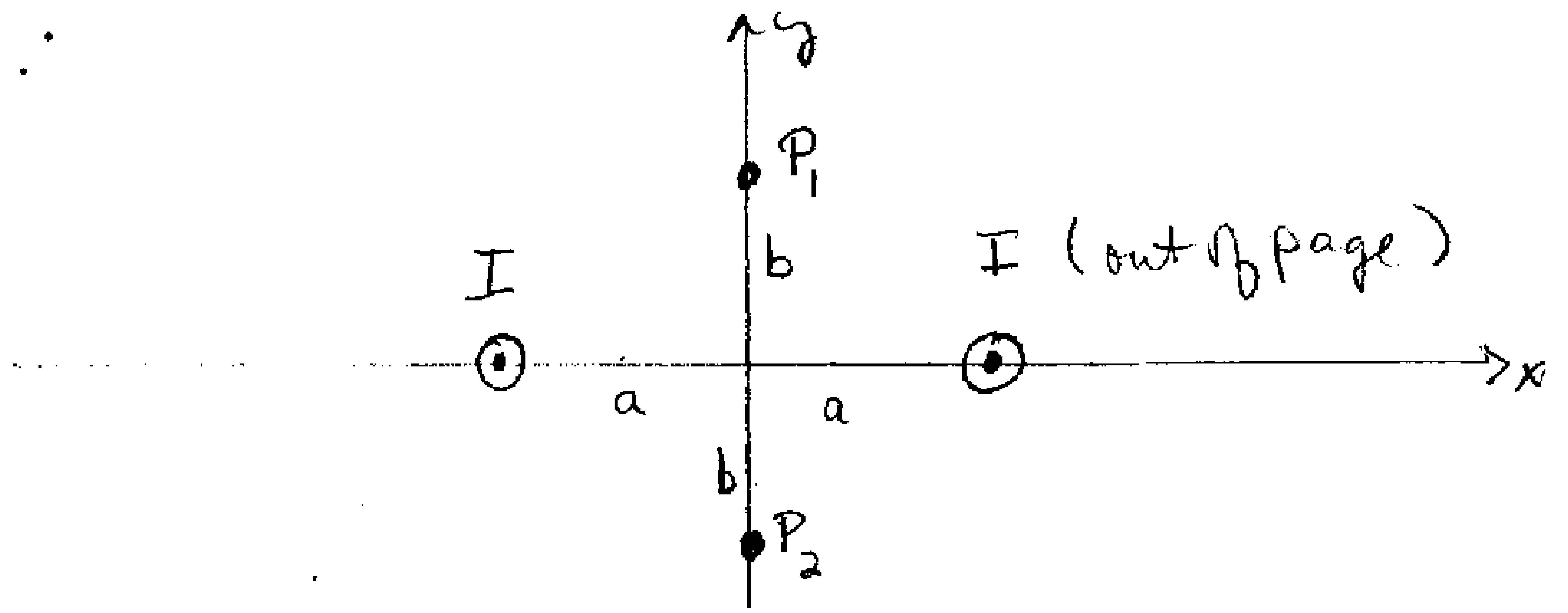


II. Two very long wires, each carrying a current I out of the page, are located on the x-axis at $x = \pm a$.

8 (a) Determine the magnetic field $\vec{B}$ at points P_1 and P_2 on the y axis, a distance b above and below the origin.



Now consider the situation of an infinite sheet of current which lies in the $x - z$ plane, with the current flowing toward the positive z direction, as shown below. The current density across the sheet is λ Amp/m.

4 (b) Using symmetry arguments and the result from (a), determine the direction of the resulting magnetic field $\vec{B}$ both above and below the sheet of current. Indicate clearly on a sketch the direction of $\vec{B}$ both above and below the current sheet.

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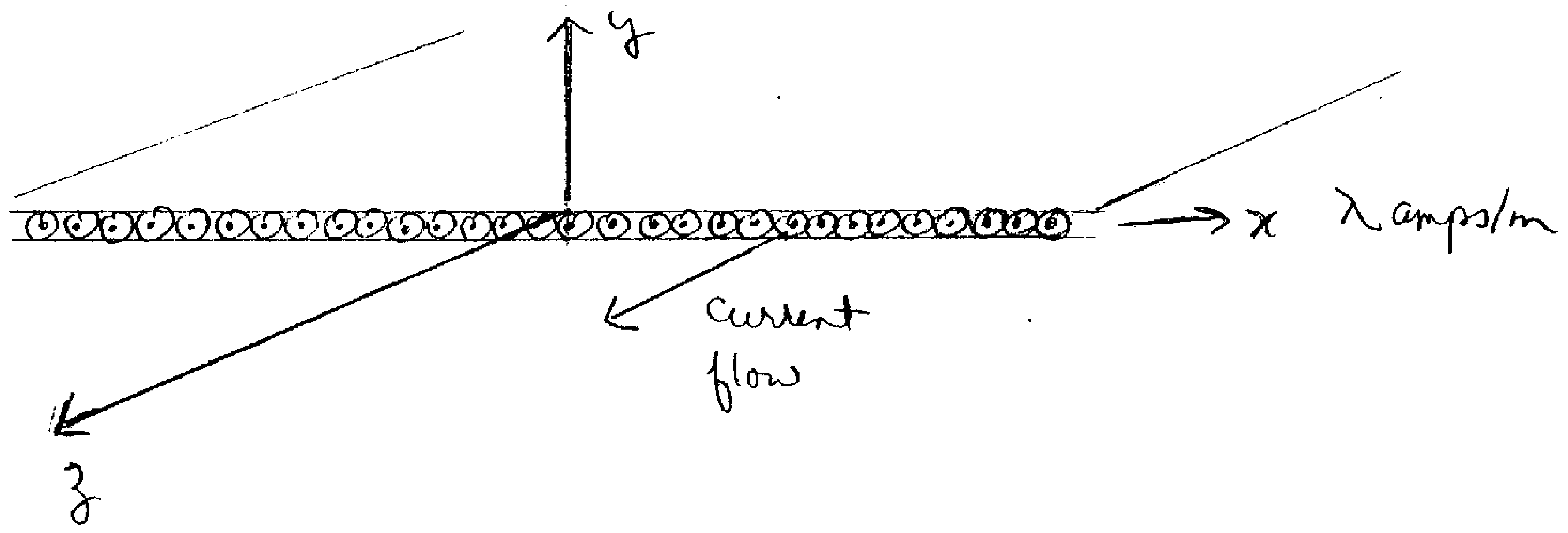
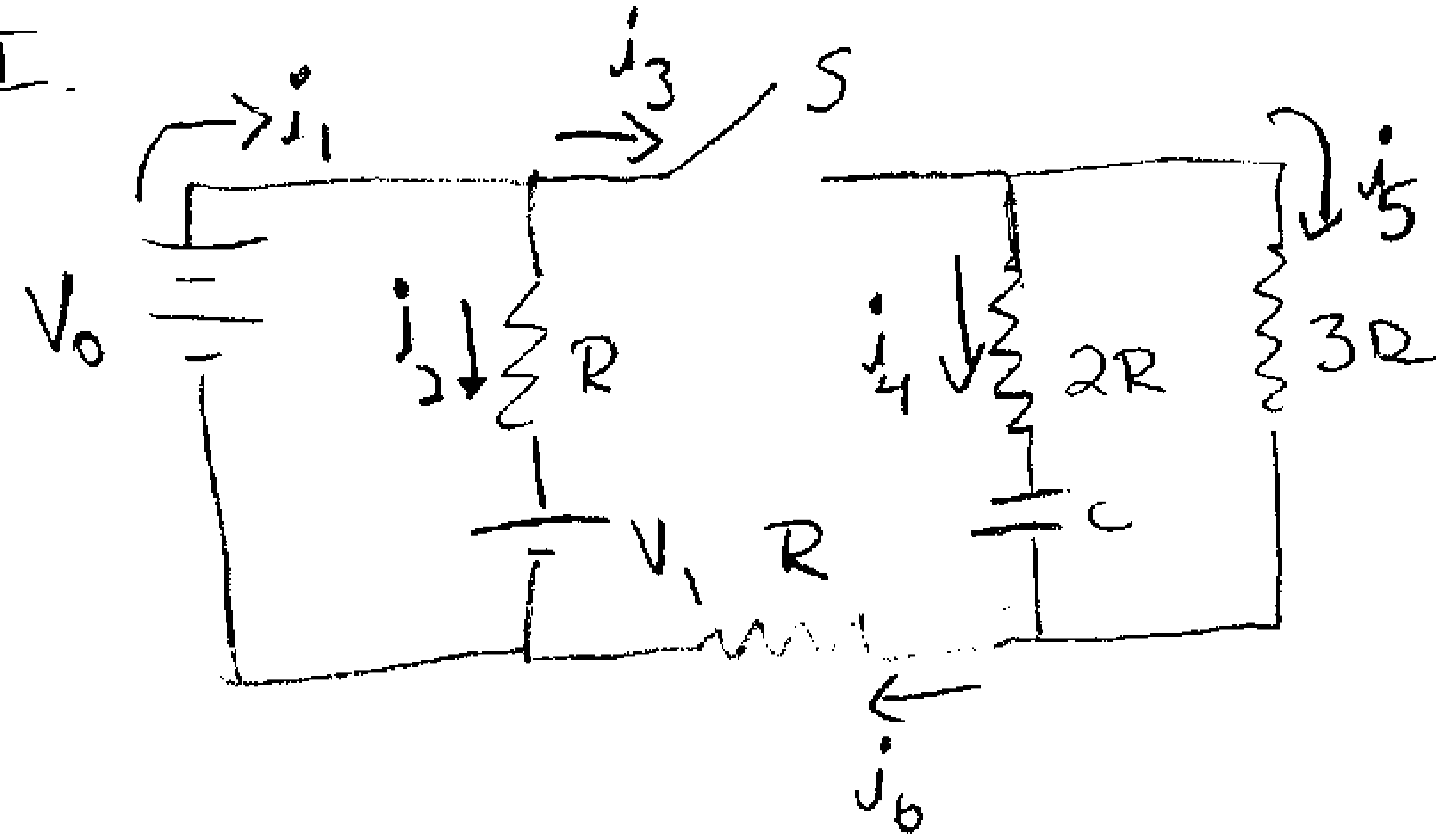


I.



When S is closed, C acts like a short.

(a) Node eqns:

$i_1 = i_2 + i_3$

$i_3 = i_4 + i_5$

$i_6 = i_4 + i_5 = i_3$

Loop eqns:

$* \quad V_0 - Ri_2 - V_1 = 0$

$** \quad V_0 - 2Ri_4 - Ri_6 = 0$

$V_0 - 3Ri_5 - Ri_6 = 0$

$2Ri_4 - 3Ri_5 = 0 \quad ***$

$V_1 + Ri_2 - 2Ri_4 - Ri_6 = 0$

$V_1 + Ri_2 - 3Ri_5 - Ri_6 = 0$

Note that only 6 eqns. are needed to solve for the currents.

(b) Use all node eqns & *'d loop eqns. Note $i_3 = i_6$

* Gives $V_0 - V_1 = Ri_2$

$$\boxed{i_2 = \frac{V_0 - V_1}{R}}$$

*** Gives $2\cancel{R}i_4 = 3\cancel{R}i_5$

$$\boxed{i_4 = \frac{3}{2} i_5}$$

** Gives

$$V_0 - 2Ri_4 - Ri_3 = 0$$

$$V_0 = 2Ri_4 + Ri_4 + Ri_5 = 3Ri_4 + \frac{2}{3}Ri_4 = \frac{11}{3}Ri_4$$

$$V_0 = \frac{11}{3}Ri_4$$

$$\boxed{i_4 = \frac{3V_0}{11R}} \qquad i_5 = \frac{2}{3}\cdot\frac{3V_0}{11R} = \boxed{\frac{2V_0}{11R} = i_5}$$

$$i_3 = i_4 + i_5 = \boxed{\frac{5V_0}{11R} = i_3 = i_6}$$

$$i_1 = i_2 + i_3 = \boxed{\frac{V_0 - V_1}{R} + \frac{5V_0}{11R} = i_1}$$

Check with 3rd loop eqn:

$$V_0 = 3Ri_5 + Ri_3 = 3\cancel{R}\left(\frac{2V_0}{11\cancel{R}}\right) + \frac{5V_0}{11\cancel{R}}\cdot\cancel{R}$$

$$V_0 = \frac{6V_0}{11} + \frac{5V_0}{11} = V_0 \checkmark$$

Summary:

$$\boxed{i_1 = \frac{V_0 - V_1}{R} + \frac{5V_0}{11R}} \qquad \boxed{i_4 = \frac{3V_0}{11R}}$$

$$\boxed{i_2 = \frac{V_0 - V_1}{R}} \qquad \boxed{i_5 = \frac{2V_0}{11R}}$$

$$\boxed{i_3 = i_6 = \frac{5V_0}{11R}}$$

Currents immediately after S is closed.

(c) Find currents a long time after S is closed $\Rightarrow$ C acts like an open circuit. No current flows through resistor 2R

$\boxed{i_4 = 0}$ and $i_3 = i_5 = i_6$. 3R and the lower R are in series.

$i_1 = i_2 + i_3$

$$V_0 - (3R+R)\,i_3 = 0$$

$$\boxed{i_2 = \frac{V_0 - V_1}{R}}$$

as before

$$\boxed{i_3 = \frac{V_0}{4R} = i_5 = i_6}$$

$$\boxed{i_1 = \frac{V_0 - V_1}{R} + \frac{V_0}{4R}}$$

(d) Since $i_4 = 0$, there is no voltage drop across 2R and V_c = voltage across 3R

$$V_c = 3R\,i_5 = \frac{3R \cdot V_0}{4R}$$

$$\boxed{V_c = \frac{3V_0}{4}}$$

(e) Redefine $t=0$ to be when S is opened again.

Opening S isolates C, 2R and 3R from the rest of the circuit, and C discharges through 2R & 3R, which are in series; $R_{eff} = 5R$

$$\boxed{I_0 = \frac{V_c}{5R} = \frac{3V_0}{20R}}$$ Current through 2R & 3R immediately after switch is opened.

(f) $I(t) = I_0 e^{-t/\tau}$ $\quad \tau = R_{eff}C = 5RC$

$$\boxed{I(t) = \frac{V_c}{5R} e^{-t/5RC} = \frac{3V_0}{20R} e^{-t/5RC}}$$

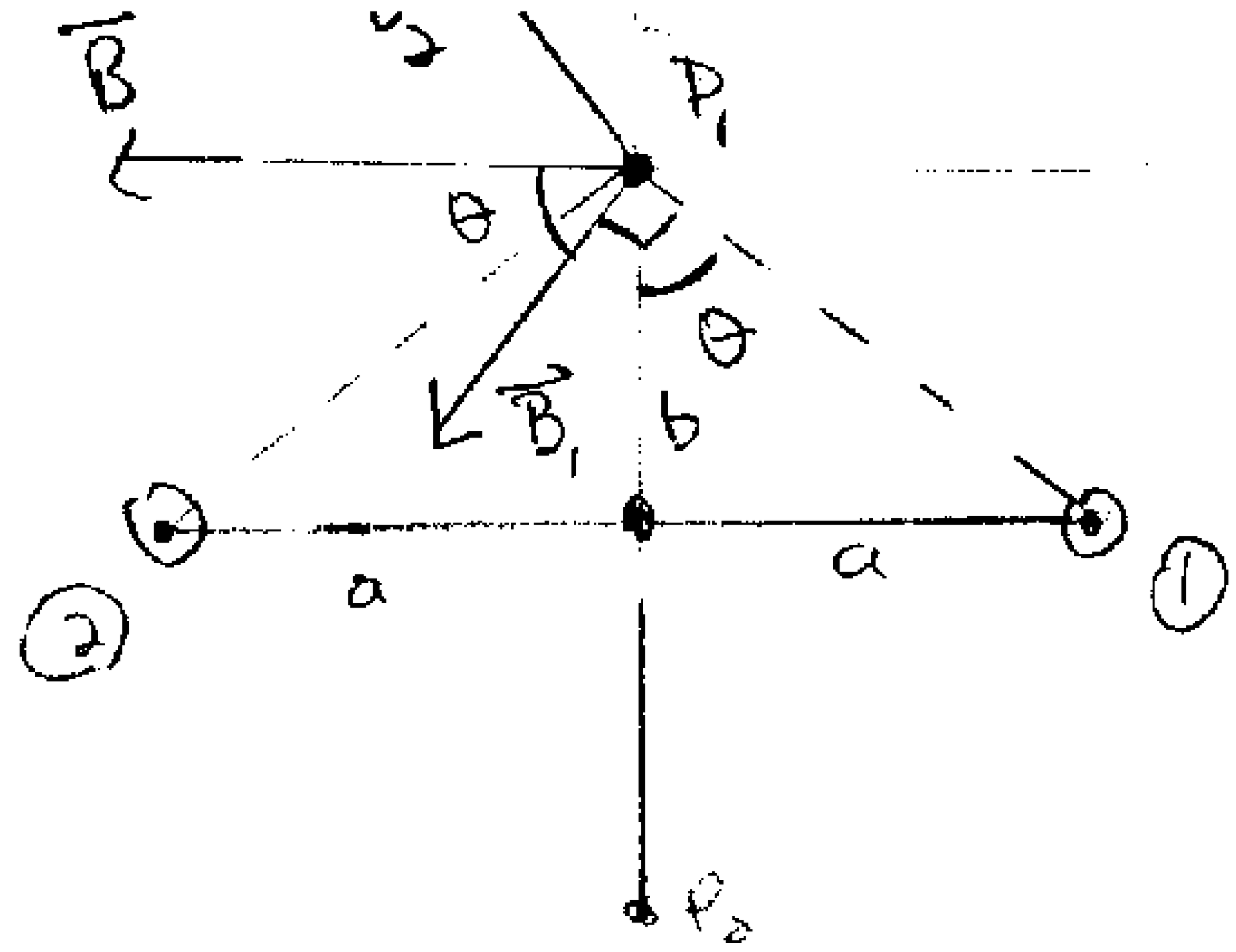
(g) $\frac{I(t)}{I_0} = 0.1 = e^{-t/5RC}$

$$\ln(.1) = -t/5RC$$

$$t = -\ln(.1)(5RC) = (\ln 10)(5RC)$$

$$\boxed{t = 11.5\,RC = \ln 10\,(5RC)}$$

II. (a)



First consider P_1. By symmetry the x-components of $\vec{B}_1$ & $\vec{B}_2$ add, and the y-components cancel.

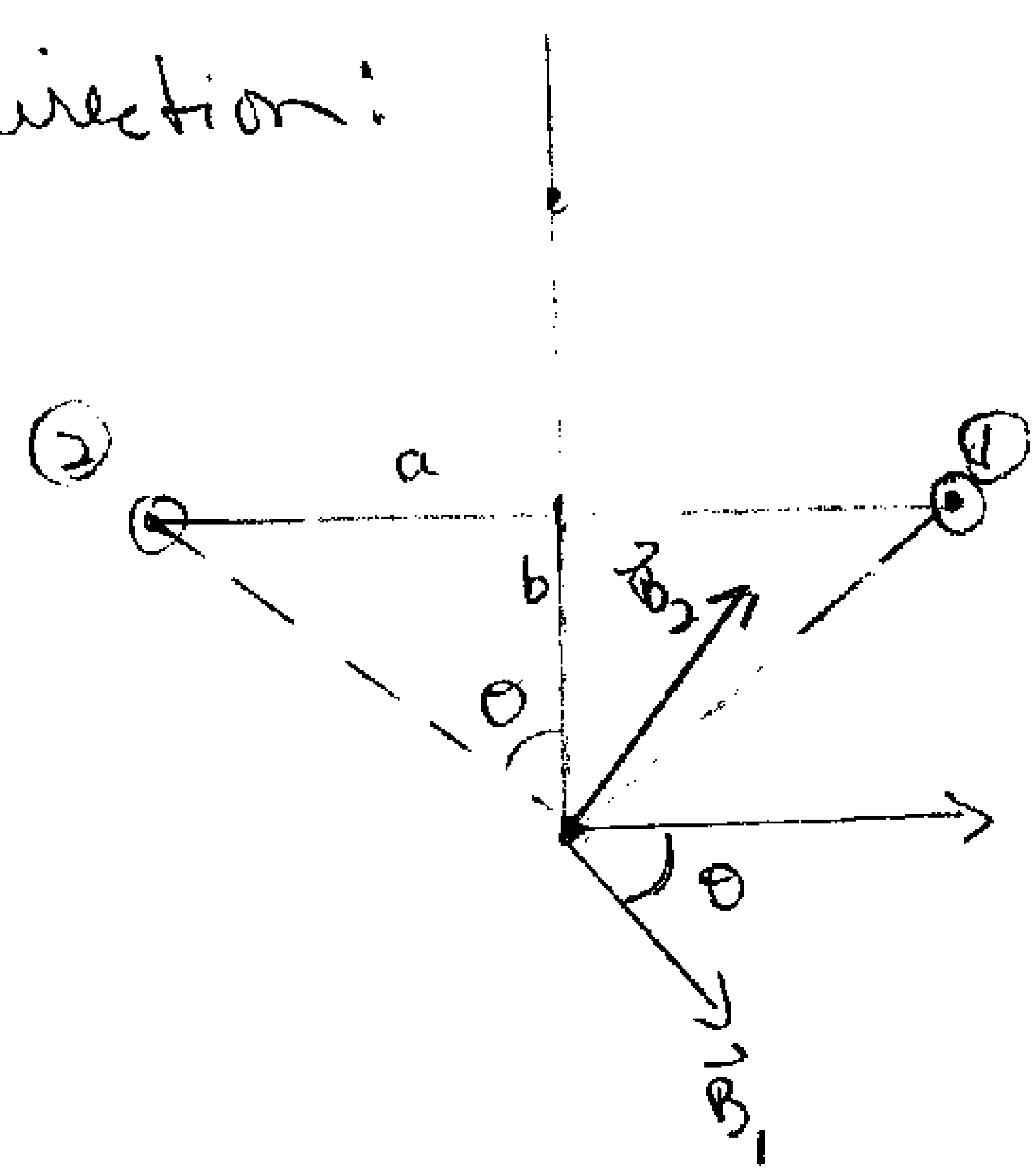
$$|\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi (a^2+b^2)^{1/2}}$$

$$B_{1x} = B_{2x} = \frac{-\mu_0 I \cos\theta}{2\pi (a^2+b^2)^{1/2}} \qquad \cos\theta = \frac{b}{(a^2+b^2)^{1/2}}$$

$$\vec{B}(\text{at } P_1) = -\frac{\cancel{2}\mu_0 I b}{\cancel{2}\pi (a^2+b^2)}\hat{\imath}$$

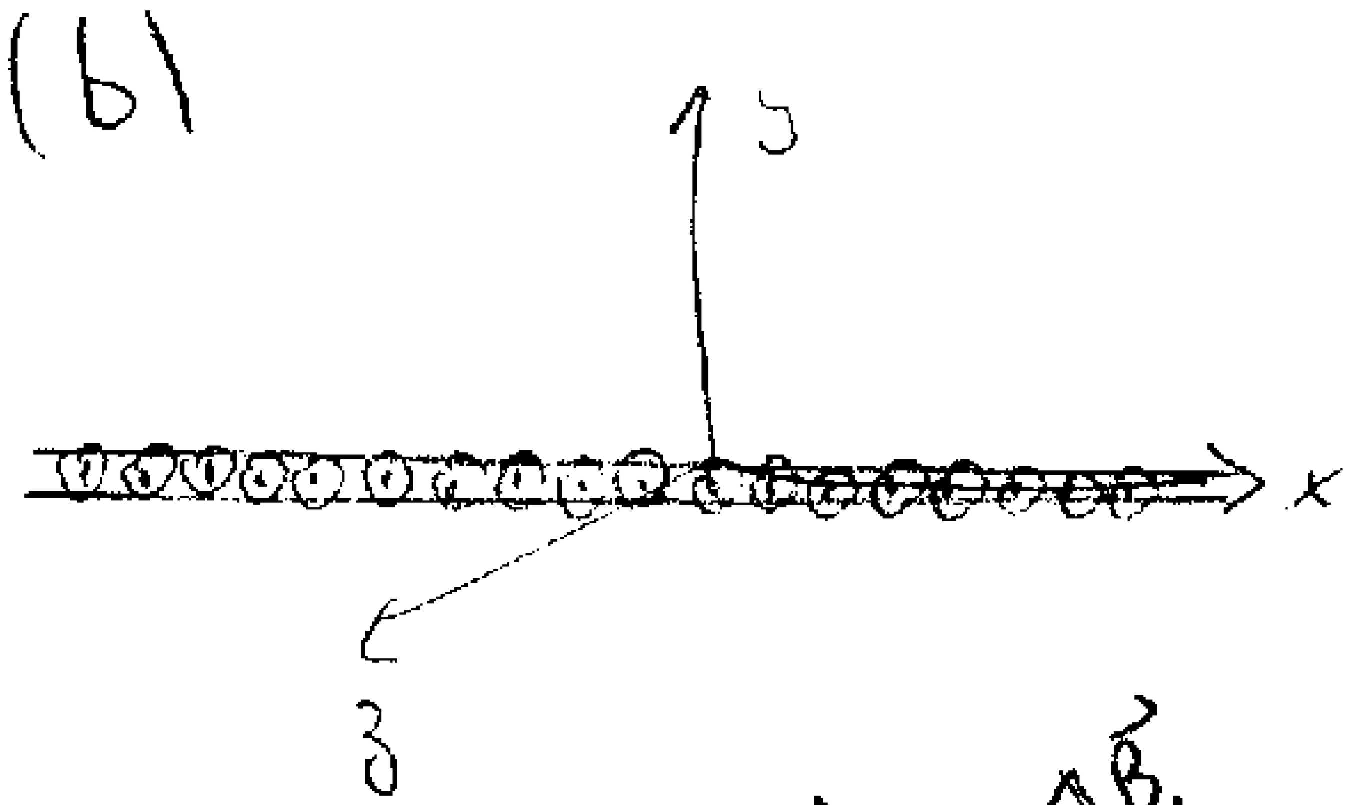
$$\boxed{\vec{B}(\text{at } P_1) = -\frac{\mu_0 I b}{\pi (a^2+b^2)}\hat{\imath}}$$

At P_2, $\vec{B}$ must have the same magnitude but opposite direction:

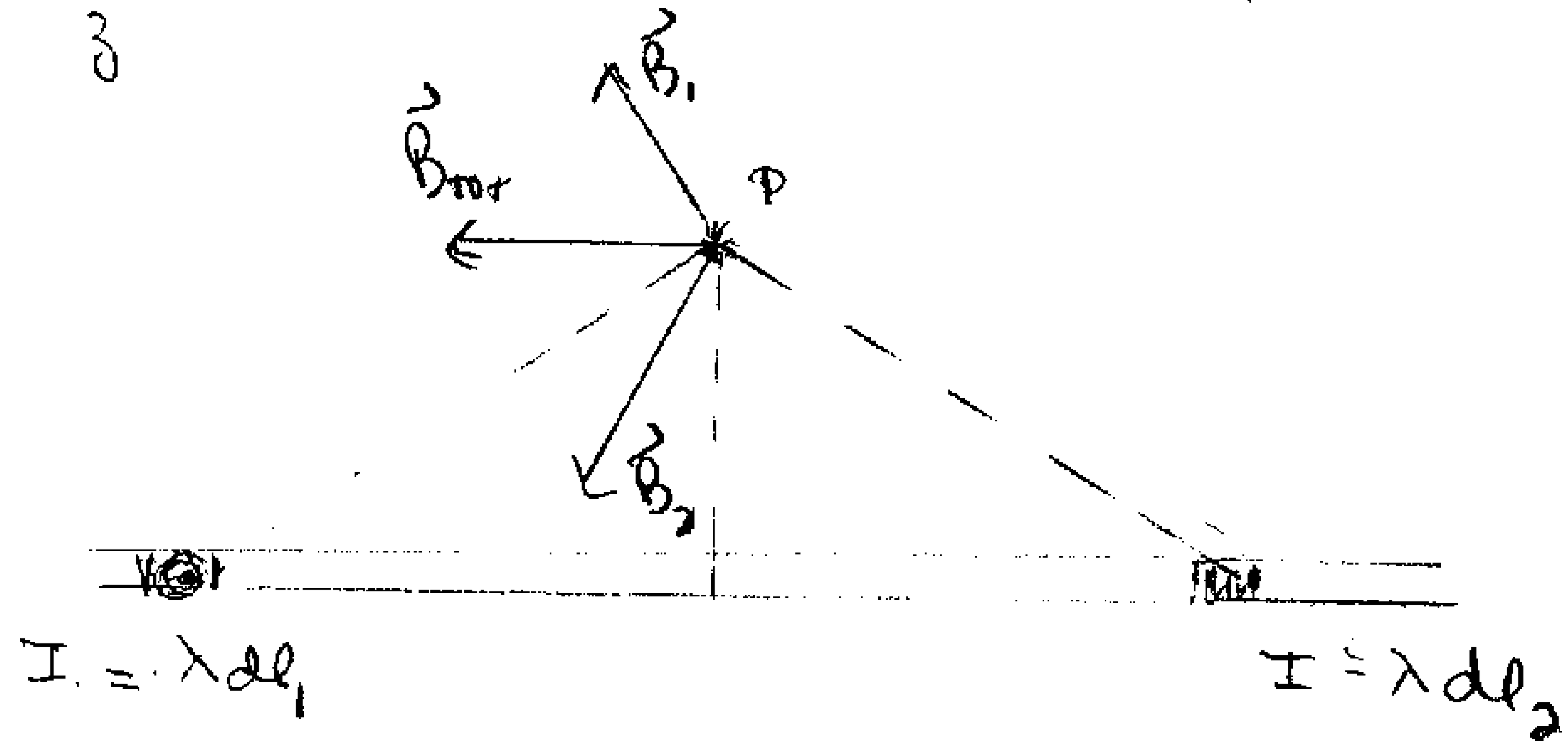


$$\boxed{\vec{B}(\text{at } P_2) = \frac{\mu_0 I b}{\pi (a^2+b^2)}\hat{\imath}}$$

(b)

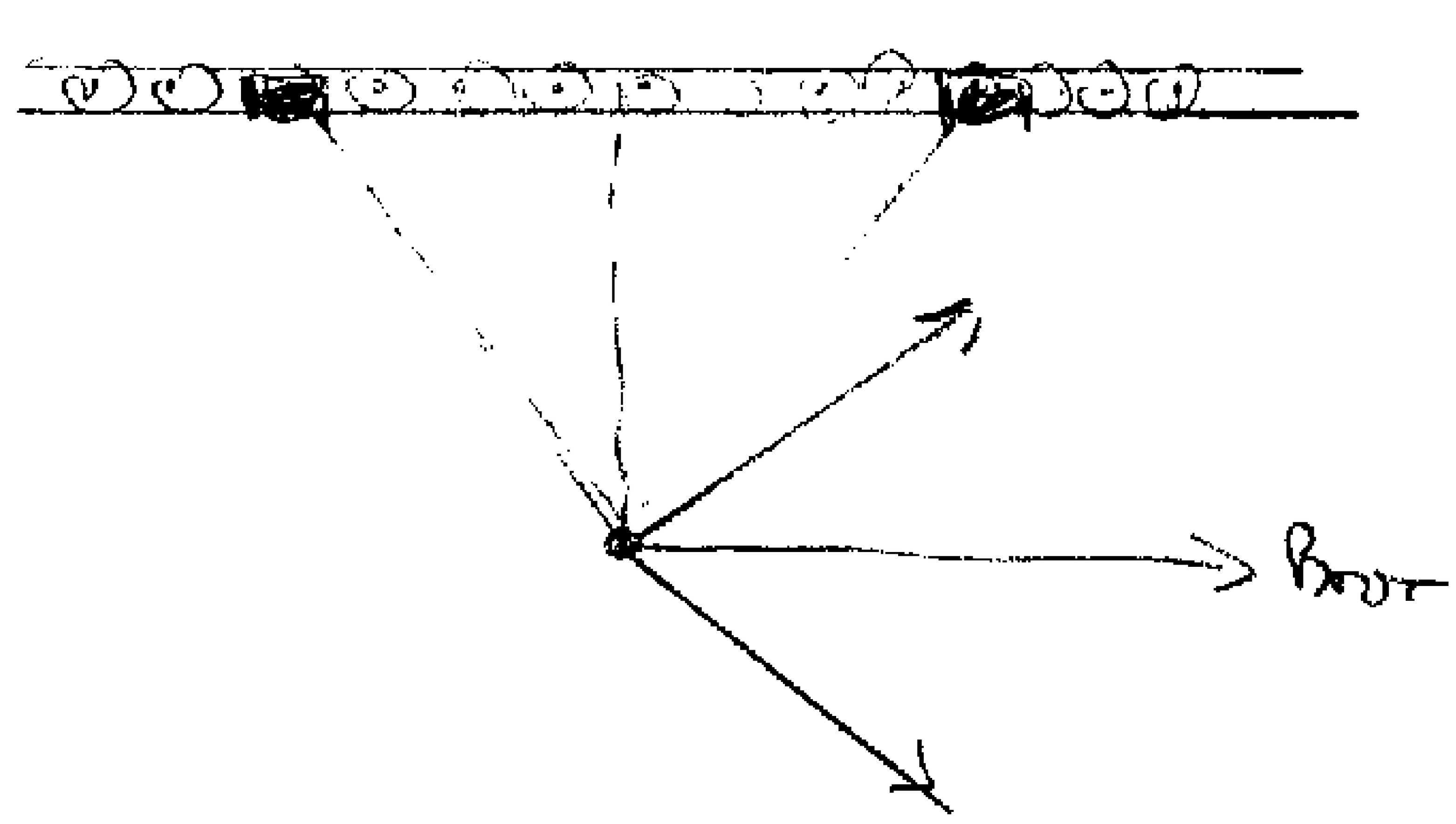


λ A/m in $+z$ direction

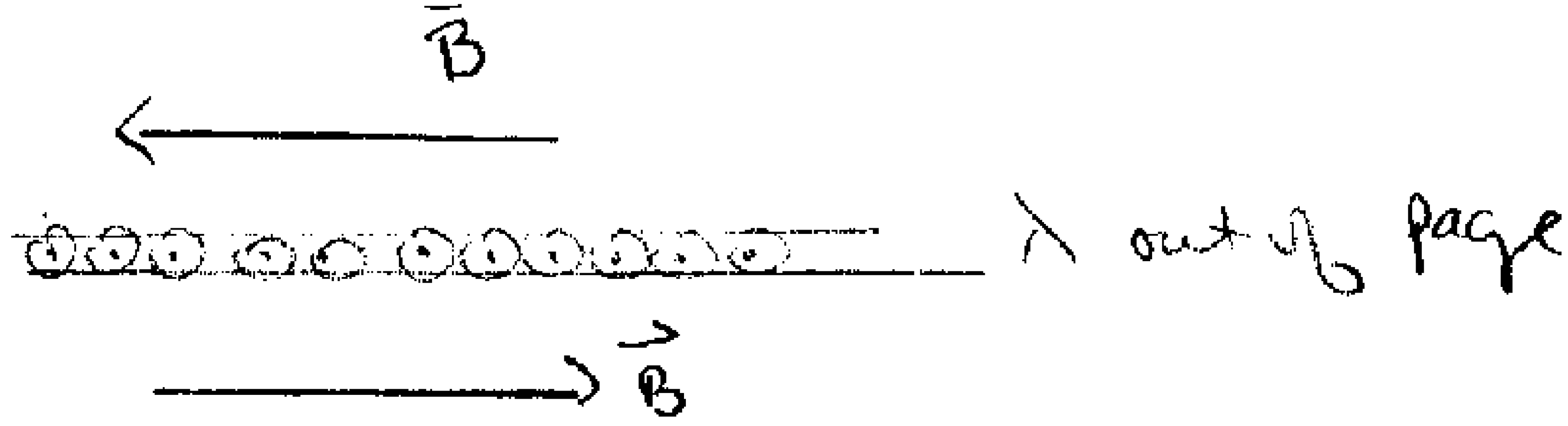


For any point P above the current sheet, we can consider two symmetric segments of the plane, dl_1 and dl_2. As in (a), the y components will cancel, leaving only a component in the $-x$ direction. For every current element dl_1 there is a matching element dl_2, so all y-components cancel, leaving only a horizontal $\vec{B}$ field to the left ($-x$ direction) above the current sheet.

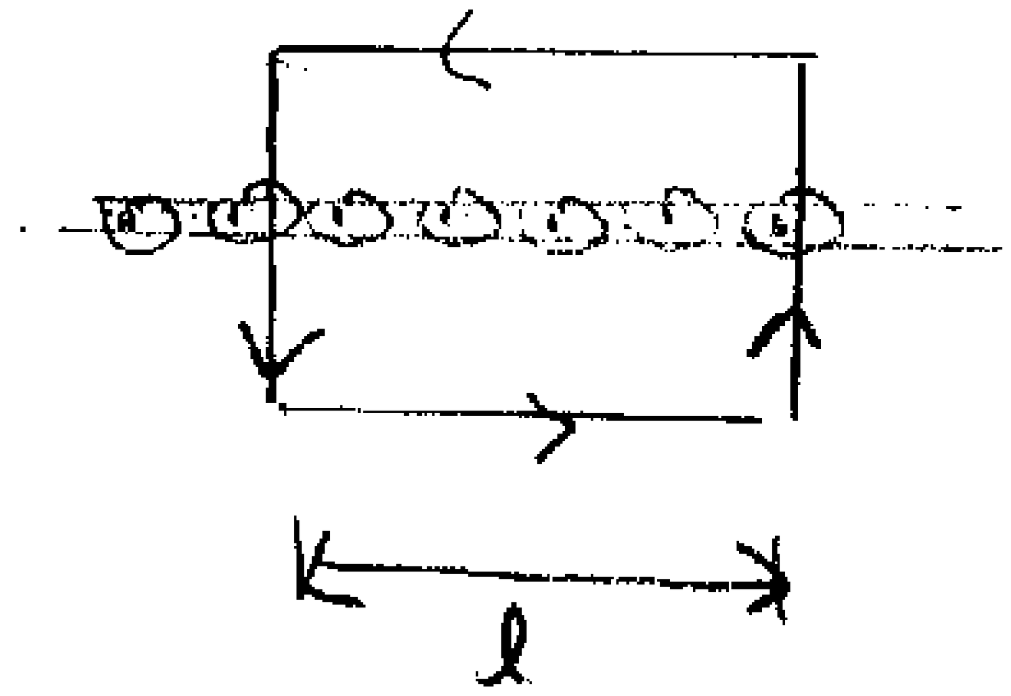
Similarly, below the current sheet (as in (a)), the $\vec{B}$ field will be in the $+x$ direction, to the right.



Moreover, since the current sheet is infinite, $\vec{B}$ must have the same value at every point a given distance above the sheet.



(C)



Take an Amperian loop as shown. The horizontal legs are $\parallel$ to $\vec{B}$ both above & below the sheet, while the vertical legs are $\perp$ to $\vec{B}$. By symmetry, all points on the horizontal legs are equivalent.

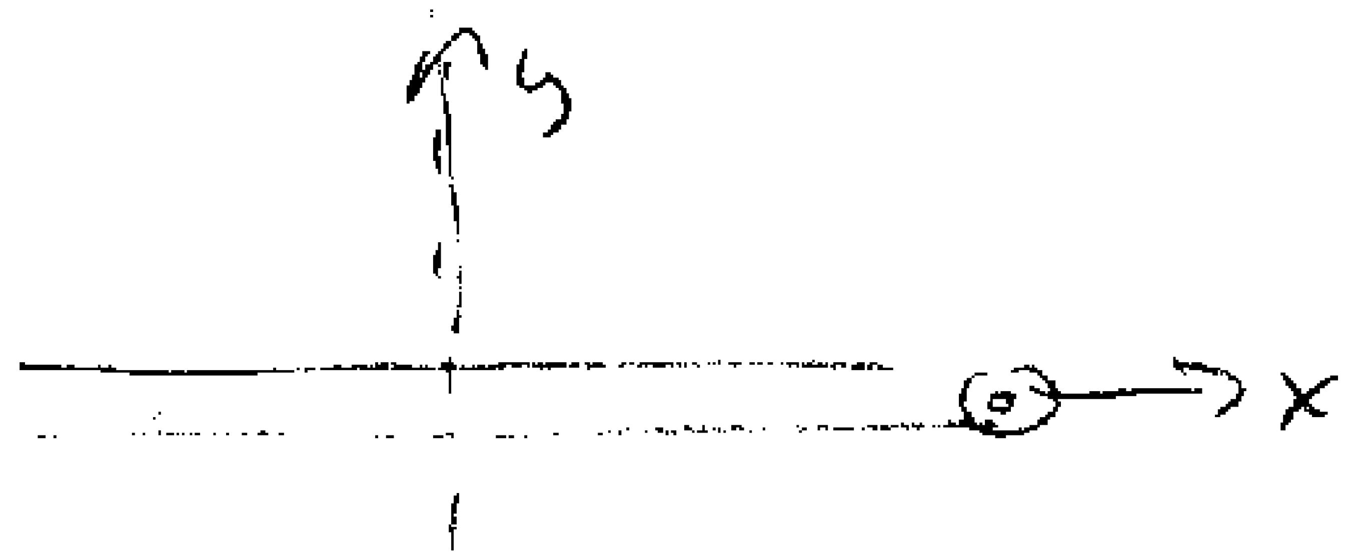
$$\oint \vec{B}\cdot d\vec{l} = \underbrace{Bl + Bl}_{\substack{\vec{B} \parallel d\vec{l} \\ \text{horizontal} \\ \text{legs}}} + \underbrace{0 + 0}_{\substack{\vec{B} \perp d\vec{l} \\ \text{vertical} \\ \text{legs}}} = \mu_0 \underbrace{\lambda l}_{I_{encl} = \lambda \,\mathrm{A/m} \cdot l}$$

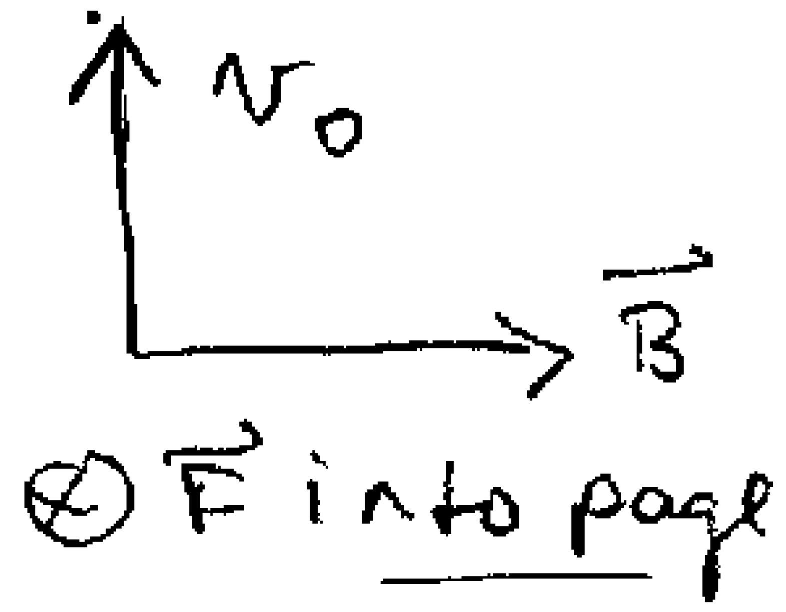
$$2B\cancel{l} = \mu_0 \lambda \cancel{l}$$ l cancels, as it must

$$\boxed{|\vec{B}| = \frac{\mu_0 \lambda}{2}}$$ directions are as given in (b).

Note that the length of the vertical legs of the Amperian loop does not matter, since these segments are $\perp \vec{B}$!
So the field is uniform – ie, independent of the distance from the current sheet!

(d)





$\vec{F}_B = Q\vec{v} \times \vec{B} = Q v_0 B(-\hat{k})$

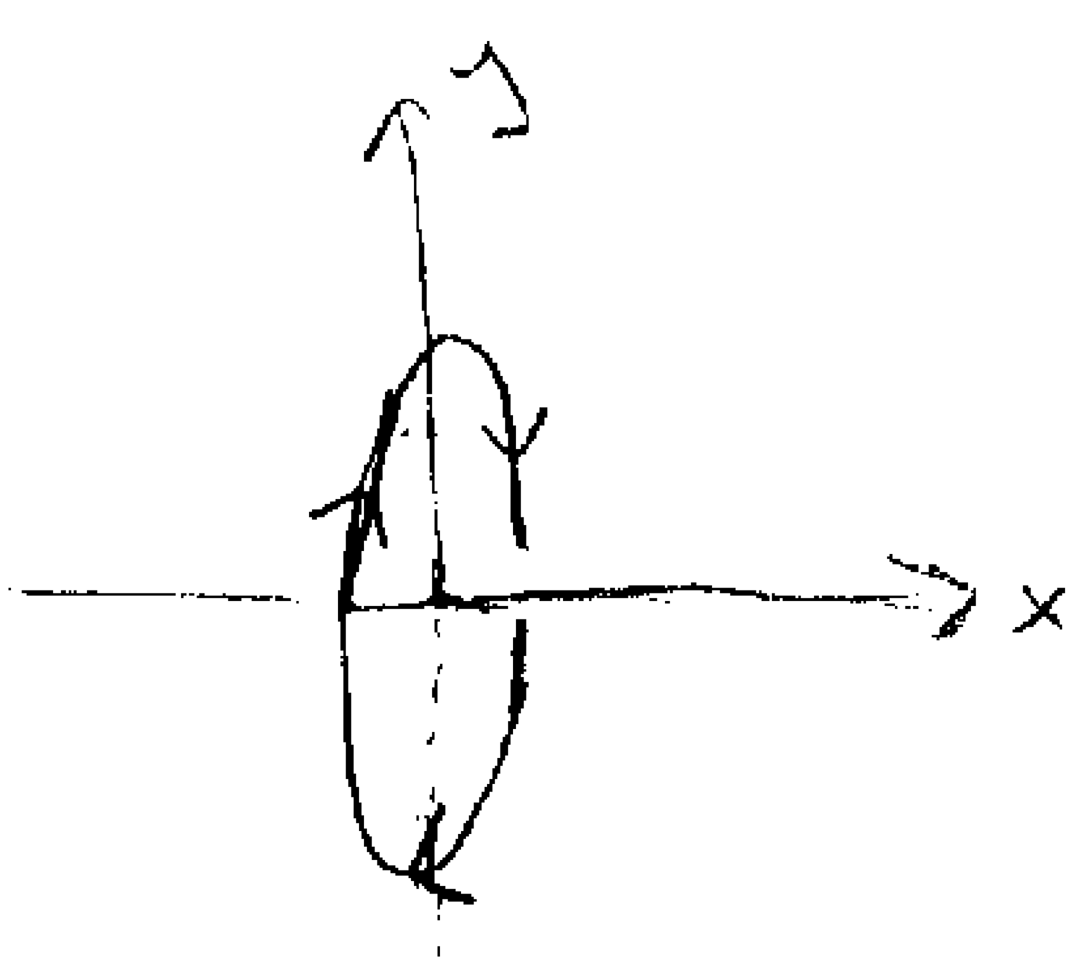
Initially the particle curves in $-z$ direction (into the page)

$$\frac{m v_0^{\cancel{2}}}{R} = Q \cancel{v_0} B$$

$$\boxed{R = \frac{m v_0}{QB} = \frac{2 m v_0}{Q \mu_0 \lambda}}$$

$$T = \frac{2\pi R}{v_0} = \frac{2\pi m \cancel{v_0}}{QB \cancel{v_0}}$$

$$\boxed{T = \frac{2\pi m}{QB} = \frac{4\pi m}{Q\mu_0\lambda}}$$



Motion is clockwise as viewed from $+x$ axis.

(e) If $v_0 \rightarrow 2v_0$ R would double, $R \rightarrow 2R$

T is unchanged.

(f)

If the particle enters at $45°$ to the x-axis,

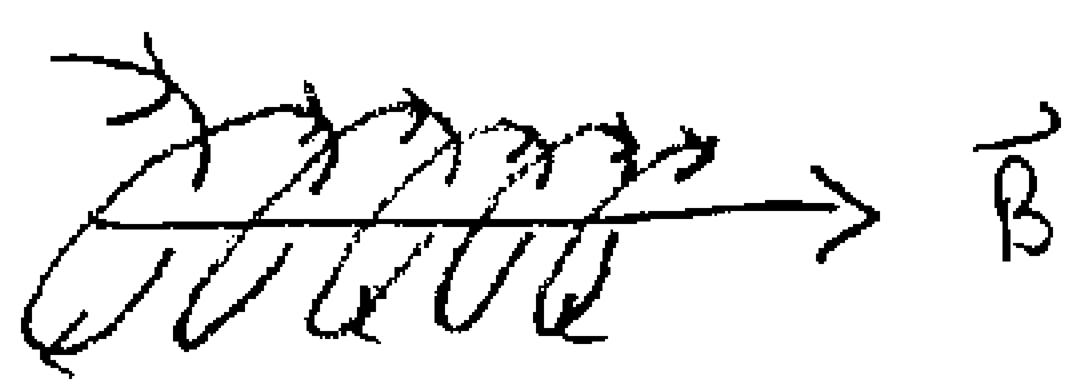
$v_{\parallel}$ (along $\vec{B}$) $= \dfrac{v_0}{\sqrt{2}}$ $\quad v_{\perp}$ ($\perp$ to $\vec{B}$) $= \dfrac{v_0}{\sqrt{2}}$

The particle drifts in the +x direction with $v_{\parallel} = v_0/\sqrt{2}$

In the y-z plane, the particle undergoes circular motion, but the radius of curvature is determined by $v_{\perp}$.

$$R = \frac{m v_{\perp}}{QB} = \frac{m v_0}{\sqrt{2}\, QB}$$

The overall motion is a helix around $\vec{B}$, traveling in +x direction.



Grading Criteria

I. (35 pts total)

(a) 6 pts

one pt. for each correct eqn, up to 6

(b) 6 pts

one point for each current, 6 total

If they don't solve completely, but make an effort (+3)

(C) 4 pts

$i_4 = 0$ (+1) i_1 (+1) i_2 (+1)

i_3, i_5, i_6 (+1)

(d) 4 pts

V_C = voltage across 3R (+2)

Correct answer (+2)

(e) 6 pts

C discharges through 2R & 3R (+2)

2R & 3R in series, $R_{eff} = 5R$ (+2)

$I_0 = \frac{V_C}{R_{eff}} = \frac{V_C}{5R}$ (+2)

(f) 5 pts

$\tau = R_{eff} C = 5RC$ (+2)

$I = I_0 e^{-t/\tau}$ (+3)

(g) 4 pts

$\frac{I}{I_0} = .1 = e^{-t/\tau}$ (+2)

Take ln & solve (+2)

Grading Criteria

II. (35 pts)

(a) 8 pts.

magnitude of $|B_1|$ and $|B_2|$ (+2)

y-components cancel, x-components add (+2)

taking components correctly ($\cos\theta$) (+2)

correct directions above & below (+2)

(b) 4 pts

Matching current segments, canceling y-components (+2)

Correct directions above + below (+2)

(c) 12 pts

Amperian Loop (+2)

$\vec{B} \parallel$ to horizontal segments & constant along horizontal segments (+2)

$\vec{B} \perp$ to vertical segments => zero contribution (+2)

Ampere's law correct (+2)

Length of vertical segments doesn't matter => uniform in space (+2)

Correct answer (+2)

(d) 5 pts

Correct R (+2) Correct T (+2) correct direction (+1)

(e) 2 pts

$R \rightarrow 2R$ (+1)

T unchanged (+1)

(f) 4 pts

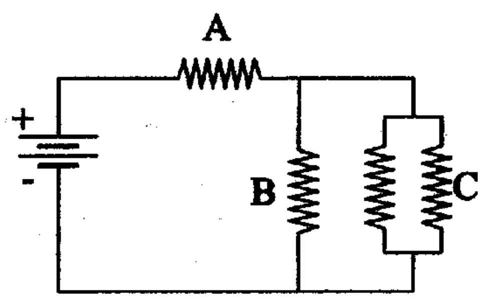
$v_\parallel$, $v_\perp$ and helical motion (+2)

Correct R, using $v_\perp$ (+2)

Last Name: First Name:

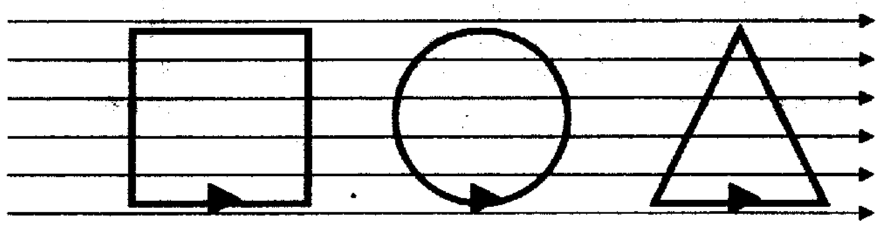
Physics 102 Spring 2002: Test 2—Multiple-Choice Questions

1. **Consider the circuit depicted below in which all resistors are identical. Doubling the resistance of which resistor(s) would diminish the power output by the battery the *least*?**



   (a) A
   (b) B
   (c) C
   (d) A or B
   (e) B or C

2. **Three current loops with *equal perimeters* but different shapes are placed in a uniform magnetic field, as depicted (not to scale) below. The three loops, a square, a circle and an equilateral triangle, have the same magnitude of current flowing in the directions depicted below. The three loops lie in the same plane and the direction of the magnetic field is tangential to that plane. On which loop does the magnetic field exert the greatest torque? (Note: the area of an equilateral triangle with side length L is $A = L^2/2\sqrt{3}$.)**



   (a) The square loop.
   (b) The circular loop.
   (c) The triangular loop.
   (d) The same non-zero torque is exerted on all three.
   (e) No torque is exerted on any of them.

3. **A certain hair dryer converts electrical potential energy into heat and motion at the rate of 1800 W and draws 0.75 A. The energy for the hair dryer is provided by a battery with an internal resistance of 300 Ω. What must the EMF of the battery be to provide enough power?**

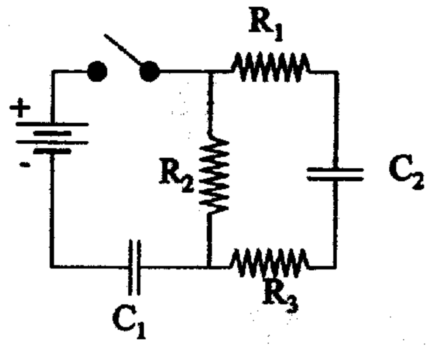
   (a) 0.225 kV
   (b) 1.80 kV
   (c) 2.10 kV
   (d) 2.40 kV
   (e) 2.63 kV

4. In the circuit below, the switch is closed and after a very long time the capacitors are charged by the battery and the circuit reaches a steady state. Then the switch is opened. Assuming all circuit elements work in the idealized way discussed in class, which of the following statements are true a very short time after the switch is opened?

I. Q_1, the charge stored in the capacitor C_1, is less than it was before the switch was opened.

II. $|I_1|$, the magnitude of the current through the resistor R_1, is greater than it was before the switch was opened.

III. $|V_2|$, the magnitude of the potential drop across the resistor R_2, is greater than it was before the switch was opened.



(a) I

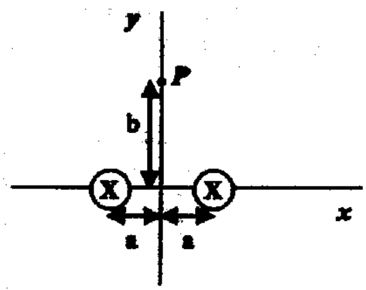
(b) II

(c) I and II

(d) II and III

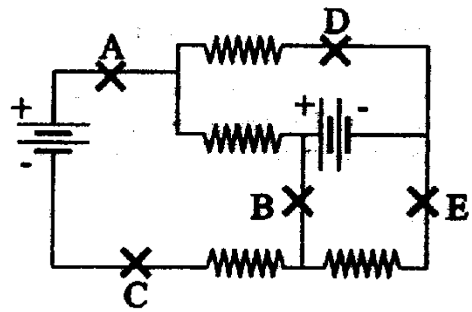
(e) I, II and III

5. Two long, straight wires are located on the x-axis and carry identical currents I in the $-z$-direction (into the page). (The $+z$-direction is out of the page.) The wires are located at ($x = -a$, $y = 0$) and ($x = +a$, $y = 0$). Point P is located on the y-axis at a distance b from the origin. The direction of the magnetic field at point P due to the two current carrying wires is



(a) $+x$-direction.
(b) $+y$-direction.
(c) $+z$-direction.
(d) non-zero x- and y-components.
(e) $-y$-direction.

6. Consider the circuit schematic below. All resistors have the same resistance and all batteries have the same EMF. A voltmeter attached to which two points would register no voltage difference.



(a) A and C

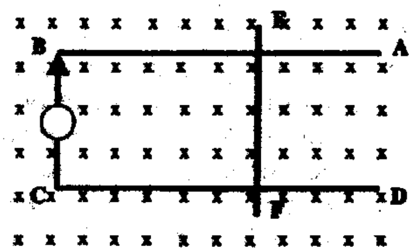
(b) B and C

(c) B and D

(d) B and E

(e) D and E

7. In the figure at right, we look down on a bare wire (ABCD) lying on and fixed to a table top. Lying across ABCD and making good electrical contact with it is another bare wire (EF) that is free to slide without friction. A source of constant current sends current clockwise around the loop, as shown. A uniform external magnetic field blankets the region and is directed into the table top (i.e., into the page). Ignore the Earth's field. The wire EF will



(a) remain in place.

(b) slide to the right.

(c) slide to the left.

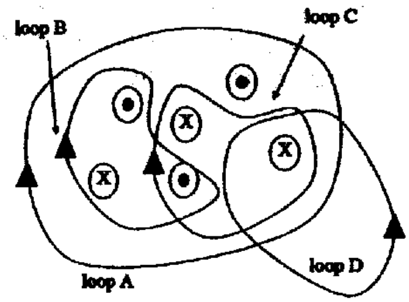
(d) rotate clockwise.

(e) rotate counterclockwise.

8. On the test model of an fancy piece of electronics, certain wires were 1 mm in diameter and made of silver, with resistivity $\rho = 1.6 \times 10^{-8}$ Ωm. In the factory model, these wires will be replaced with copper wires (resistivity $\rho = 2.7 \times 10^{-8}$ Ωm) of the same length. What should the *diameter* of these wires be for the wires to have nearly the same resistance?

(a) 0.59 mm
(b) 0.77 mm
(c) 1.0 mm
(d) 1.3 mm
(e) 1.7 mm

9. Consider the six wires depicted below. Each is directly coming into or going out of the page, and all have the same current. Rank the line integrals of the magnetic field for the paths depicted below from *largest to smallest*, where the direction of the integration around the amperian loop is indicated by the arrow heads.



(a) A>B=C>D

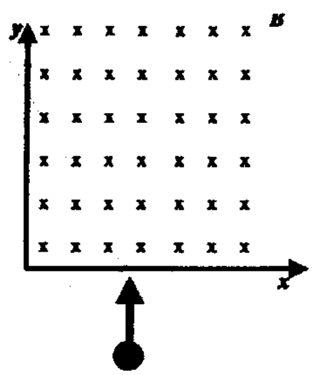
(b) B>C>D>A

(c) C>A>B=D

(d) B>A>C=D

(e) C>B=D>A

10. A positively charged particle is observed to move initially in the $+y$-direction before it enters a region of uniform magnetic field bounded by the x- and y-axes as shown. The magnetic field is directed into the plane of the paper (i.e., the $-z$-direction) as shown. The $+z$-direction is out of the page. The resultant force on the particle can be made to be zero by introducing a uniform electric field of appropriate strength in the



(a) $+x$-direction.
(b) $-x$-direction.
(c) $+y$-direction
(d) $-y$-direction.
(e) $+z$-direction (ie., out of the page).