Last Name: First Name:

Physics 102 Fall 2002: Test 1—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; each free-response question accounts for 35 points, and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

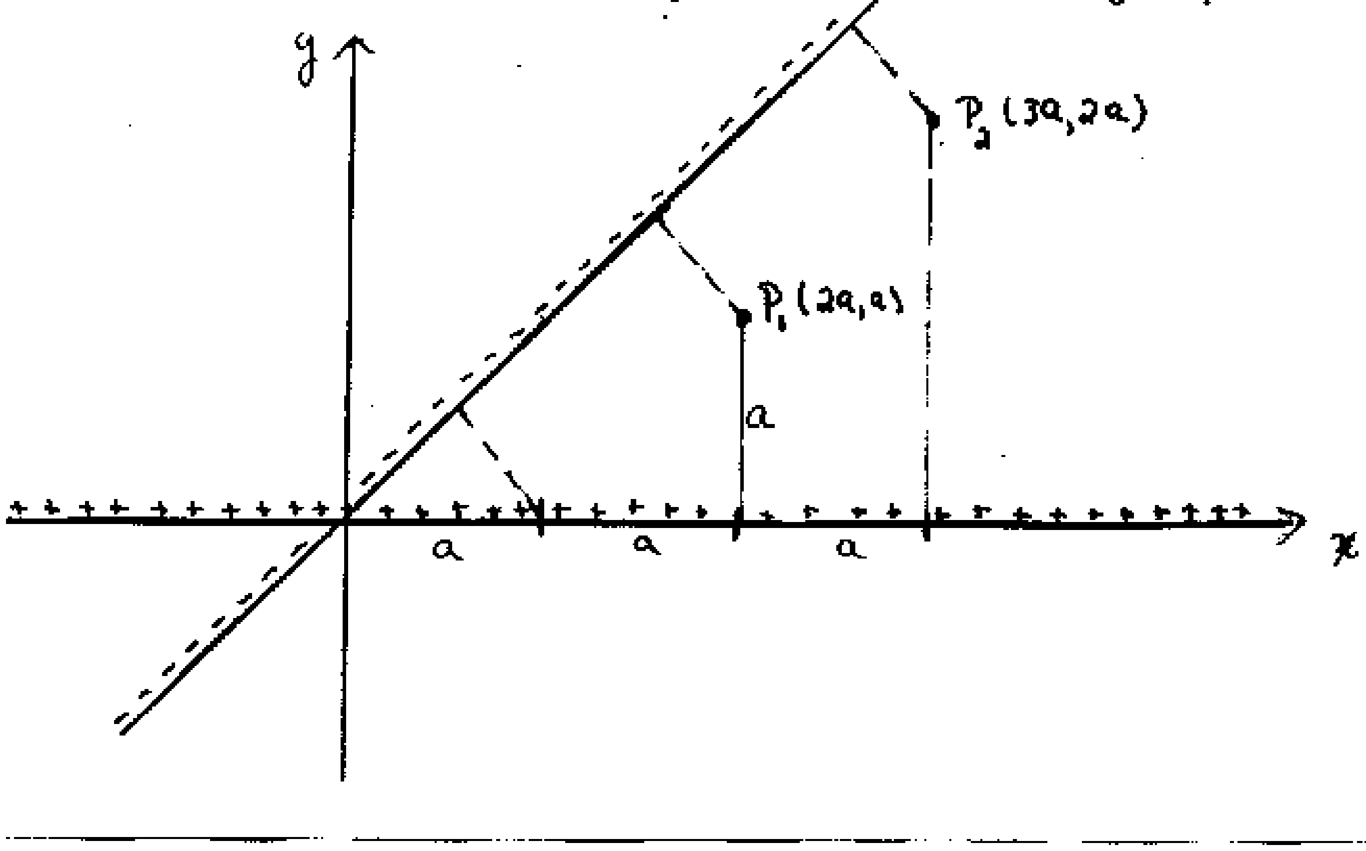
Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

I. An infinite line of charge with positive, uniform linear charge density $+\lambda$ is located on the x-axis.

(a) Using Gauss' Law, determine the electric field $\vec{E}$ at a distance r_1 from the x- axis. Show your work in detail.
(b) Determine the potential difference ΔV between two points which are distances r_1 and r_2 from the x-axis.

Now suppose a second infinite line of charge with uniform, negative linear charge density $-\lambda$ lies in the $x-y$ plane and is oriented at $\theta = 45^0$ to the x-axis as shown in the figure below.

(c) Determine the total electric field $\vec{E}$ at $P_1(2a, a)$. The figure provides a hint as to how to do the geometry.
(d) Determine the potential difference ΔV between P_1 and a second point $P_2(3a, 2a)$.
(e) A Gaussian sphere of radius R is centered at the origin. Determine the total flux Φ through the sphere.

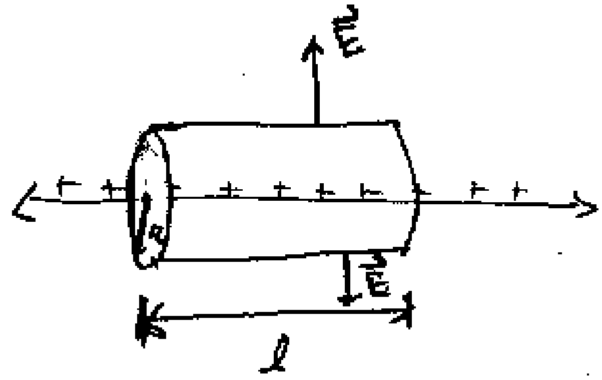


Phys 102

Exam 1 - Spring 2002

35 I.

7 (a)



Consider a Gaussian cylinder of radius r and length l.

By symmetry we know that

1) The electric field $\vec{E}$ is everywhere radially outward

2) $|\vec{E}|$ must be the same at all points on the surface of the cylinder.

Then Gauss' Law becomes

$$\phi = \int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int \vec{E} \cdot d\vec{A} = \underbrace{2\pi r \, \vec{E} \cdot \hat{r} \, l}_{\text{curved outer surface}} + \underbrace{\cancelto{0 \;\; \vec{E} \perp \hat{r}}{\int \vec{E} \cdot d\vec{A}}}_{\text{endcaps}} = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E_r = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda k}{r}}$$

where $E_r \Rightarrow \vec{E}$ is radially outward at every point on the curved cylindrical surface.

(b) $\Delta V = -\int \vec{E} \cdot \vec{dr} =$

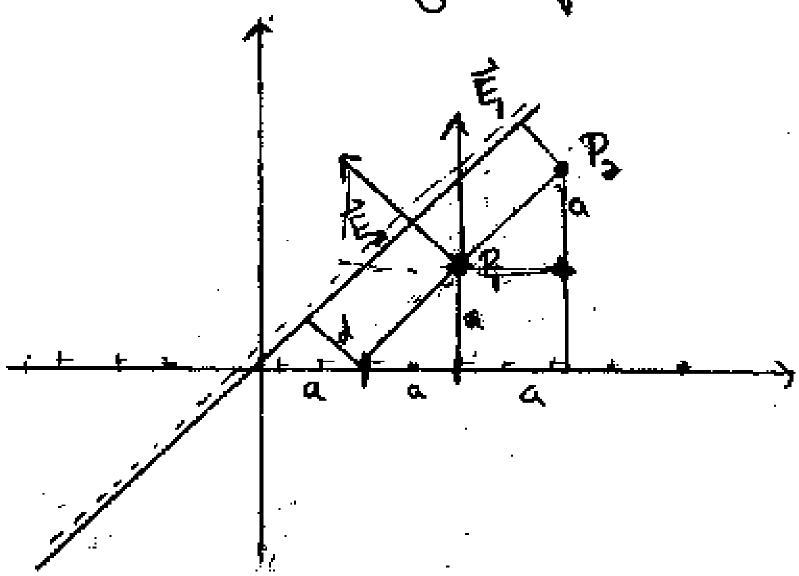
r_2, $\vec{dr}$, r_1

$$\Delta V = -\frac{\lambda}{2\pi\epsilon_0} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(r_2/r_1\right)$$

$$\boxed{\Delta V = V(r_2) - V(r_1) = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(r_2/r_1\right)} = -2\lambda k \ln\left(r_2/r_1\right)$$

Note sign — V is <u>decreasing</u> as you move from $r_1 \to r_2$.

(c) Add second line of charge at 45°



$d = \frac{\sqrt{a^2}}{2}$

There are two contributions to the field — one from each line of charge.

$\vec{E}_1$ (due to $+\lambda$) $= \dfrac{\lambda}{2\pi\varepsilon_0 a}\hat{j}$ (from (a))

$\vec{E}$ $\vec{L}$

$|\vec{E}_2|$ (due to $-\lambda$) $= \dfrac{\lambda}{2\pi\varepsilon_0 d} = \dfrac{\lambda\sqrt{2}}{2\pi\varepsilon_0 a} = \dfrac{\lambda}{\sqrt{2}\pi\varepsilon_0 d}$

$\vec{E}$ $\vec{L}$

$\vec{E}$ $\vec{L}$

$\vec{E}_2$ has both x & y components

$$E_{2x} = \frac{-\lambda}{\sqrt{2}\pi\varepsilon_0 a}\cdot\cos 45^\circ\,\hat{i} = \frac{-\lambda}{2\pi\varepsilon_0 a}\hat{i}$$

$$E_{2y} = \frac{\lambda}{\sqrt{2}\pi\varepsilon_0 a}\cdot\sin 45^\circ\,\hat{j} = \frac{+\lambda}{2\pi\varepsilon_0 a}\hat{j}$$

$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{TOT} = \frac{-\lambda}{2\pi\varepsilon_0 a}\hat{i} + \frac{2\lambda}{2\pi\varepsilon_0 a}\hat{j}$$

$$\boxed{\vec{E}_{TOT} = \frac{-\lambda}{2\pi\varepsilon_0 a}\hat{i} + \frac{\lambda}{\pi\varepsilon_0 a}\hat{j}}$$

(d) $\Delta V(P_2 - P_1) = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l} = -\int(\vec{E}_1+\vec{E}_2)\cdot d\vec{l} = -\int\vec{E}_1\cdot d\vec{l} - \int\vec{E}_2\cdot d\vec{l}$

By superposition, we can consider $\vec{E}_1$ and $\vec{E}_2$ separately. From (b) we know that $\Delta V = \frac{-\lambda}{2\pi\varepsilon_0}\ln(r_2/r_1)$ for a single line of charge. Note that P_1 & P_2 are the <u>same distance</u> from the negative line of charge, so

ΔV (due to $\vec{E}_2$) = 0 since $\ln(1) = 0$.

So we only need to consider the contribution to $+\lambda$

$$\Delta V = -\int \vec{E}_1 \cdot d\vec{l} = \frac{-\lambda}{2\pi\varepsilon_0}\int_a^{2a} \frac{dy}{y} = \frac{-\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2a}{a}\right)$$

$$\boxed{\Delta V(P_2 - P_1) = \frac{-\lambda}{2\pi\varepsilon_0}\ln 2}$$

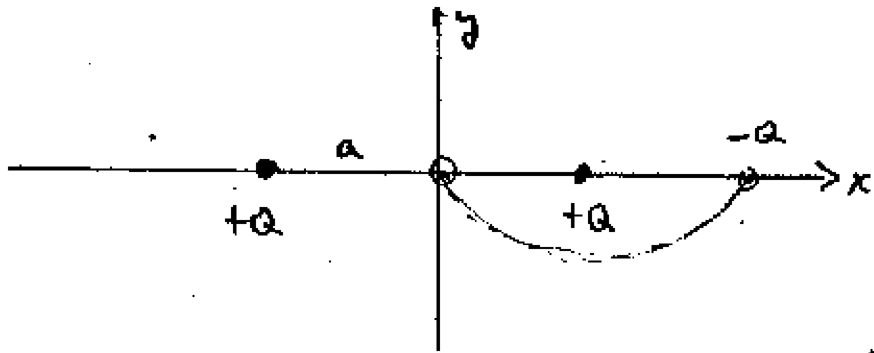
Note that V decreases as we move away from $+\lambda$.

(d) $\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

$Q_{encl} = \lambda(2R) - \lambda(2R) = 0$ Equal amounts of + and − charge are enclosed

$$\boxed{\phi = 0}$$

II.



(a) When just +Q is present, $V(x=a) = \dfrac{kQ}{2a}$

Then the work to bring a second +Q to $x=a$ is

$$W = qV = \frac{kQ^2}{2a}$$

$$\boxed{W = \frac{kQ^2}{2a}} = \boxed{U = \frac{kQ^2}{2a} = \text{Potential energy in charge configuration}}$$

The work done by an external agent to bring +Q to $x=a$ is stored as potential energy in the electric field

(b) With both charges present

$$V(x=0) = \frac{kQ}{a} + \frac{kQ}{a} = \frac{2kQ}{a}$$

Potentials add as scalars

$$\boxed{V(x=0) = \frac{2kQ}{a}}$$

Work to bring -Q to $x=0$

$$\boxed{W = qV = \frac{-2kQ^2}{a}}$$

Note that $W<0$. An external agent would do negative work, which means the external agent must keep the charge from accelerating. The electric field does positive work.

There is **less** PE in the charge configuration with $-Q$ at $x=0$ than with $-Q$ at ∞.

(c) Work to move $-Q$ from $x=0$ to $x=2a$ is <u>independent</u> of the path

$$W = q\Delta V \qquad V(x=2a) = \frac{kQ}{a} + \frac{kQ}{3a} = \frac{4kQ}{3a}$$

$$V(x=0) = \frac{2kQ}{a}$$

$$\Delta V = V_f - V_i = \frac{4kQ}{3a} - \frac{2kQ}{a} = \frac{4kQ}{3a} - \frac{6kQ}{3a} = -\frac{2kQ}{3a}$$

$$\boxed{W = q\Delta V = \frac{+2kQ^2}{3a}}$$

(d) Work is path <u>independent</u>, $W = +\frac{2kQ^2}{3a}$

(e) $U = \frac{kQ^2}{2a} - \frac{kQ^2}{a} - \frac{kQ^2}{3a} = \frac{3kQ^2 - 6kQ^2 - 2kQ^2}{6a}$

$$\boxed{U = \frac{-5kQ^2}{6a}}$$

The (−) sign means that the PE of this charge configuration is **less** than the configuration w/ $-Q$ at ∞.

II. A positive point charge $+Q$ is located at $x = -a$ on the x axis. Other charges are brought from infinity to regions near the first charge, as described below. Assume in all cases that the additional charges begin and end their motions at rest. Take the zero of the electrostatic potential and potential energy to be at infinity.

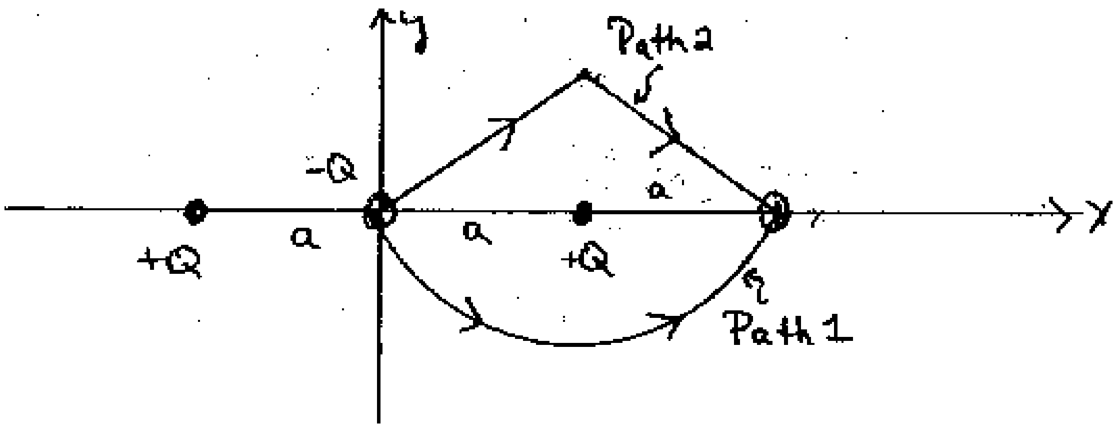
(a) How much work is required to bring a second positive point charge $+Q$ from infinity to $x = +a$ on the x-axis? What is the total energy of this charge configuration?

(b) With the two charges at $\pm a$, determine the electrostatic potential V at the origin. How much work is required to bring a third charge of $-Q$ from infinity to the origin?

(c) How much work is required to move the $-Q$ charge from the origin to $x = 2a$ along the semicircular path (Path 1) shown?

(d) How much work is required to move the $-Q$ charge from the origin to $x = 2a$ along Path 2 as shown?

(e) What is the total potential energy stored in the final charge configuration? (That is, $+Q$ at $x = \pm a$, and $-Q$ at $x = 2a$).



Last Name:______________ First Name:______________

Physics 102 Spring 2002: Test 1—Multiple-Choice Questions

1. Three capacitors, each of identical capacitance C, are combined in a variety of ways using copper wires. What is the minimum effective capacitance that can be found in a combination that uses all three capacitors?

   (a) $C/6$

   (b) $C/3$

   (c) $C/2$

   (d) C

   (e) $3C/2$

2. Three pithballs are suspended from threads. Up to 3 of the pithballs are charged by contact of other charged objects. It is found that pithballs 1 and 2 attract each other and pithballs 2 and 3 repel each other. From this we can conclude with certainty that

   (a) pithballs 1 and 3 carry charge of opposite sign.

   (b) pithballs 1 and 3 carry charge of same sign.

   (c) all three pithballs carry charge of same sign.

   (d) one of the pithballs carries no charge.

   (e) we need more data to reach a conclusion with certainty.

3. Call U_1 the work required to bring charges from infinitely far away and give a surface charge density σ to a conducting sphere of radius R. Call U_2 the work required to build up the same surface charge density σ on a conducting sphere with twice the radius. Which of the following is a true statement?

   (a) $U_2 = U_1/2$

   (b) $U_2 = U_1$

   (c) $U_2 = 2U_1$

   (d) $U_2 = 8U_1$

   (e) $U_2 = 16U_1$

4. A parallel plate capacitor is first charged by connecting it, by means of wires and a switch, to a battery with a potential difference between its terminals of 120 V. After the capacitor is fully charged, the switch is opened. Another metal wire is now inserted between the charged plates to connect the surfaces of the two plates. What happens?

   (a) The positive and negative plates are reversed.

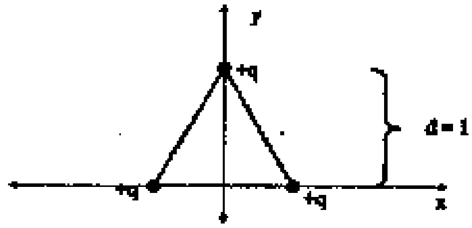
   (b) The electric field between the plates is reversed.

   (c) The electrostatic potential energy of the system is increased.

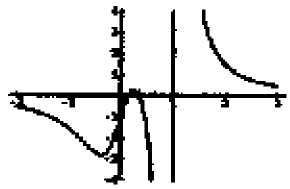
   (d) The electrostatic potential difference between the plates becomes zero.

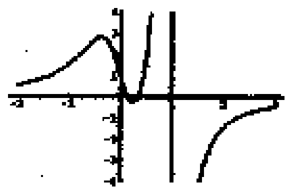
   (e) None of the above.

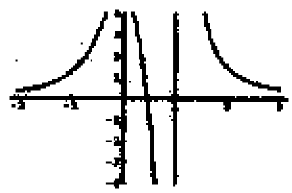
(Questions 5–6) The following two questions are concerned with the configuration of three equal positive point charges $+q$ placed at the vertices of an equilateral triangle lying in the x-y plane, as depicted to the right. The scale is set so that the altitude of the triangle is 1, in arbitrary units.

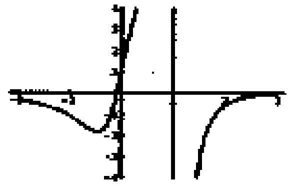


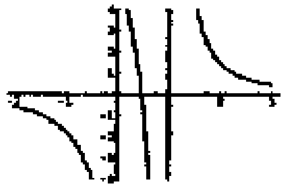
5. Which of the graphs below most accurately depicts the y-component of the electric field E_y on the y-axis due to this charge configuration?


(a)

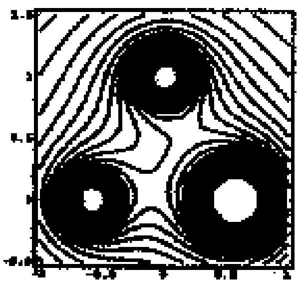

(b)

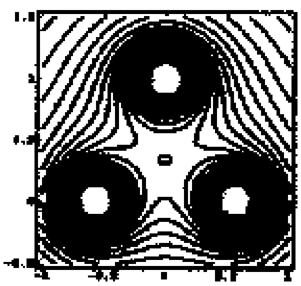

(c)

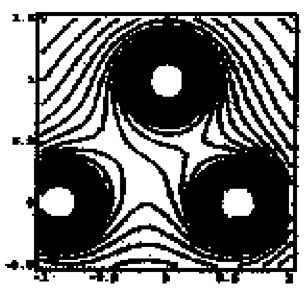

(d)

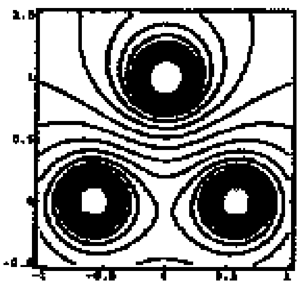

(e)

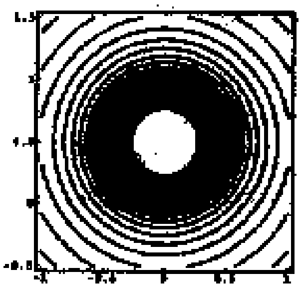
6. Which of the graphs below most accurately depicts the equally-spaced equipotential surfaces in the plane of the triangle due to this charge configuration?


(a)


(b)


(c)


(d)


(e)

7. Consider the picture below which depicts four possible paths traced by an electron in a constant electric field directed to the right. Each path begins at the end without the arrow head and ends at the point of the arrow, and the drawing is to scale. Rank the change in electrostatic potential energy of the system for each of the paths the electron follows in order from greatest negative change to greatest positive change.

(a) 1, 2, 3, 4
(b) 1, 4, 2, 3
(c) 4, 3, 2, 1
(d) 3, 1, 2, 4
(e) 3, 2, 4, 1

8. A charge of $q = +6.0$ nC sits at the origin ($x = 0, y = 0$). What is the ratio of the y-component of the electric field to the x-component of the electric field (that is, E_y/E_x) at the point ($x = 0.4$ m, $y = 0.3$ m)?

(a) 0.56
(b) 0.60
(c) 0.75
(d) 1.33
(e) 1.78

9. A parallel plate capacitor with capacitance C_0 is charged to a voltage V_0 by a battery. While still connected to the battery, a dielectric slab with dielectric constant κ is inserted and fills the space between the plates. Consider the following statements.

I. The charge density on the plates increases by a factor κ.

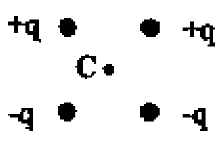
II. The electric field between the plates increases by a factor κ.

III. The energy stored in the capacitor increases by a factor κ.

Which of the above statements are true?

(a) I
(b) I and II
(c) I and III
(d) II and III
(e) I, II and III

10. The figure below depicts four charges, two with positive charge $+q$ and two with negative charge $-q$, placed at the corners of a square. Which arrow below most accurately depicts the force on a positively charged particle at the center point C?



A. ↑ B. → C. ↓ D. ↗ E. ←