Open Book, Open Notes, Open Web, Time Limited Test1 for Mech417

JEA11417

Available beginning Feb. 17, 2011 with hardcopy solution due 5pm Feb 24
at the ME "In-Box" wall slot between rooms ME 109 and 108.

Cover Page

Instructions

You must work alone on this test. If you are not familiar with the Rice Honor system, then review it before proceeding. Once you go beyond this cover page, you have a maximum of two hours to work on this test, excluding an optional 30-minute break that prohibits consulting reference materials.

This is an open book, open notes and open course web page test.

Name __SOLUTION_________________ Begin Time ______ End Time ______

1. Briefly explain the operations of scatter and gather and the type of data upon which they operate. **A *gather* brings nodal data or solution values from the system to an element. A *scatter* ADDS element matrix terms to the corresponding system matrices. Both use the mapping from element equation numbers to system equation numbers, which in-turn are obtained for the element connection list. Mathematically, a gather is $Data^e = \beta^e Data^{system}$ while a scatter is $Data^{system} = Data^{system} + \beta^{e^T} Data^e$, and thus always requires an addition.**

2. What are the essential and non-essential boundary conditions for an ordinary differential equation that is: a) second order, b) fourth order? **For an ODE of order 2m, the EBC apply to the function and derivatives through (*m-1*). The NBC apply to derivatives of order *m* through (*2m-1*). A) *m=1* so EBC is on the function and NBC is on its slope (first derivative), B) *m=2* so EBC are the function and slope and NBC are on the second and third derivatives.**

3. What are the major advantages and disadvantages of a finite element analysis? **It treats arbitrary curved geometry of mixed materials and directionally dependent (anisotropic) materials. It is applicable to many fields of engineering and physics. However, it requires a large amount of input data and usually involves a numerical solution.**

4. An axial bar has a distributed axial load per unit length, *w*, from its fixed left support to half way to its free right end. Create and show a mesh of three two noded bar elements, form the three element load vectors. Do not form the stiffness matrices. **There are at least three solutions. A) two loaded elements of length L/4 and one unloaded element of length L/2. 1*---2*---3*------4*. The first two have constant load vectors of $C_w^e = \frac{wL}{4}\begin{Bmatrix}1\\1\end{Bmatrix}$ while the third element has zero load. Assembling (scattering) the system load is $C_w = \frac{wL}{4}\begin{Bmatrix}1\\1+1\\1+0\\0\end{Bmatrix} = \frac{wL}{4}\begin{Bmatrix}1\\2\\1\\0\end{Bmatrix}$. B) Reverse the mesh (not good) then $C_w = \frac{wL}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$. C) Use three equal length elements: one loaded, one half-loaded, one unloaded. The middle one requires a new integral definition, $C_w^e = \int_0^1 H(x)^T w(x) L^e d\blacksquare = wL^e \int_0^{0.5} H(x)^T d\blacksquare = \frac{wL^e}{8}\begin{Bmatrix}3\\1\end{Bmatrix}$ and assembly gives $C_w = \frac{wL}{24}\begin{Bmatrix}4\\4+3\\1+0\\0\end{Bmatrix} = \frac{wL}{24}\begin{Bmatrix}4\\7\\1\\0\end{Bmatrix}$. In all cases, the total resultant is wL/2.**

5. Four line elements, with two nodes each, are joined as listed in the connection table and sketch shown below.

```
e  j  k      1              2                3  nodes
1  2  1      *------(1)-------*-------(2)-------*                 typical element
2  2  3      |                |                 | (element)       *----(e)----*
3  1  2      *------(3)-------*                 |                 1           2
4  3  1      *-------------------(4)-------------*
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Let the source vector for a typical element be written as $\boldsymbol{C}^e = \begin{Bmatrix}C_1\\C_2\end{Bmatrix}^e$, write out the four resultant source terms that result from scattering (assembling) all elements to the system level. (That is, write each entry as the sum of scalar contributions C_k^e for *e = 1, 2, 3, 4 and k = 1, 2.*) $\boldsymbol{C} = \begin{Bmatrix}C_2^1 + C_1^3 + C_2^4\\ C_1^1 + C_1^2 + C_2^3\\ C_2^2 + C_1^4\end{Bmatrix}$

6. What are the primary uses of the geometric Jacobian in a typical element? **The determinant of the Jacobian relates the parametric differential volume to the physical differential volume, $d\Omega = |J| d\blacksquare$. The inverse of the Jacobian converts the parametric gradient to the physical gradient, $\frac{\partial}{\partial x} = J^{-1} \frac{\partial}{\partial \blacksquare}$.**

7. List the major phases involved in the numerical solution of a finite element problem.

8. We have presented two ways for modifying a singular matrix system to include an essential boundary condition, necessary to render it non-singular. Briefly describe one of the methods. What is a disadvantage of that method? **A) Delete the rows and column of the known dof, and then insert and identity for that dof. It destroys the system reaction data, unless it is saved before the modification.**

9. A three node unit triangle has nodal property values of $\boldsymbol{P}^{e^T} = [p_1 \quad p_2 \quad p_3]$. What is the corresponding property value at the interior point (0.3, 0.2), in unit coordinates? $p(r,s) = \boldsymbol{H}(r,s)\boldsymbol{P}^e$ **here** $p(r,s) = (1 - r - s)p_1^e + r\, p_2^e + s\, p_3^e = 0.5p_1^e + 0.3\, p_2^e + 0.2p_3^e$

10. The above triangle has corner x-coordinates of $\boldsymbol{x}^{e^T} = [x_1 \quad x_2 \quad x_3]$. Evaluate $\partial x / \partial r, \partial x / \partial s$ for the triangle. $x(r,s) = \boldsymbol{H}(r,s)\boldsymbol{x}^e$ **so** $\partial x / \partial r = -1x_1^e + 1x_2^e + 0$ **and** $\partial x / \partial s = -1x_1^e + 0 + 1x_3^e$

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11. Explain why the sum of the Gaussian quadrature weights tabulated over 0 <= r <= 1, must equal unity, while those tabulated over -1<= a <= 1 must sum to 2.
$\boldsymbol{Measure} = \int_{\blacksquare} d\blacksquare = \int_{\blacksquare} 1\, d\blacksquare = \sum_q^{n\_q} 1\, w_q = \sum_q^{n\_q} w_q$ **and the parametric measures are 1 and 2, respectively.**

12. The concentration C(x) of a species in 1D mass transport is:

$$-\frac{d}{dx}\left(D(x)\frac{dC(x)}{dx}\right) + u(x)\frac{dC(x)}{dx} + K(x)C(x) = m$$

where D, u, K and m are supplied data. Write the integrated by parts Galerkin equivalent integral form for this system over 0 ≤ x ≤ L.

$$- C(x)\left(D(x)\frac{dC(x)}{dx}\right)\Big|_0^L + \int_0^L \frac{dC(x)}{dx} D(x)\frac{dC(x)}{dx}dx + \int_0^L C(x)\, u(x)\frac{dC(x)}{dx}dx + \int_0^L C(x)\, K(x)C(x)dx - \int_0^L C(x)m\, dx = 0$$

13. The consistent mass matrix for any element is $\mathbf{m}^e = \int_L \mathbf{N}^{e^T} \rho \mathbf{N}^e dx$. For a four-node line element (L4) in the unit space 0 <= r <= 1 explain how many quadrature points would be required to integrate this matrix exactly if the density ρ is: a) constant, b) $\rho = \rho_0\, x^2$. **N is cubic, degree three, so the integrand is A) degree 6, so 6 <= 2 n_q – 1 and n_q = 4. B) is degree 8 so n_q = 5.**