EXAMPLE 4-10

The A-36 steel bar shown in Fig. 4–18 is constrained to just fit between two fixed supports when $T_1 = 60°F$. If the temperature is raised to $T_2 = 120°F$, determine the average normal thermal stress developed in the bar.

SOLUTION

Equilibrium. The free-body diagram of the bar is shown in Fig. 4–18*b*. Since there is no external load, the force at A is equal but opposite to the force acting at B; that is,

$$+\uparrow \Sigma F_y = 0; \qquad F_A = F_B = F$$

The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at A that would occur, Fig. 4–18*c*, is counteracted by the force **F** that would be required to push the bar δ_F back to its original position. The compatibility condition at A becomes

$$(+\uparrow) \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

Applying the thermal and load–displacement relationships, we have

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

Thus, from the data on the inside back cover,

$$\begin{aligned} F &= \alpha \Delta T A E \\ &= [6.60(10^{-6})/°F](120°F - 60°F)(0.5\text{ in.})^2[29(10^3)\text{ kip/in}^2] \\ &= 2.87\text{ kip} \end{aligned}$$

From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members.

Since **F** also represents the internal axial force within the bar, the average normal compressive (thermal) stress is thus

$$\sigma = \frac{F}{A} = \frac{2.87\text{ kip}}{(0.5\text{ in.})^2} = 11.5\text{ ksi} \qquad \textit{Ans.}$$

Fig. 4–18