Rice University, Mechanical Engineering
Prof. J. E. Akin, November 2015

Review of Planar Dynamics and Equilibrium (Statics)

Consider a rigid body accelerating in the x-y plane and rotating about the z-axis. Newton's Laws of translational motion for the force effects at the center of mass C, at $(\bar{x}, \bar{y})$, are

$$\sum F_x = m\, \bar{a}_x\, ,\ \ \sum F_y = m\ \bar{a}_y$$

where m denotes the mass $\vec{a}$ is the acceleration vector and $\sum \vec{F}$ is the resultant external force vector. Let M_0 be the resultant moment, about the z-axis, calculated at an arbitrary point 0 in the x-y plane and I_o be the polar mass moment of inertia calculated at the same point. Then Newton's Law of angular motion with respect to the same arbitrary point is

$$\sum M_0 = \ I_0\alpha + m(\overline{x}\, a_{y_0} - \overline{y}\, a_{x_0})\ .$$

where α is the angular acceleration and the linear acceleration components of point 0 are (a_{x_0}, a_{y_0}). There are common special cases of these Laws:

1. If point 0 is a fixed point, say FP, then its linear acceleration is zero, $\vec{a}_{FP} = \vec{0}$, so the angular motion law is

$$\sum M_{FP} = \ I_{FP}\alpha\ .$$

2. If point 0 is the center of mass, C, then $\overline{\mathrm{x}} = 0 = \overline{\mathrm{y}}$, so the angular motion law is

$$\sum \mathrm{M_C} = \ \mathrm{I_C}\alpha$$

3. If point C is on a body with pure translation then $\alpha = 0$, so the moment equation reduces to

$$\sum \mathrm{M_C} = 0$$

4. If the translation has a constant velocity (including zero) then $\vec{a}\ = \vec{0}$ and we say the system is in equilibrium and Newton's Laws reduce to

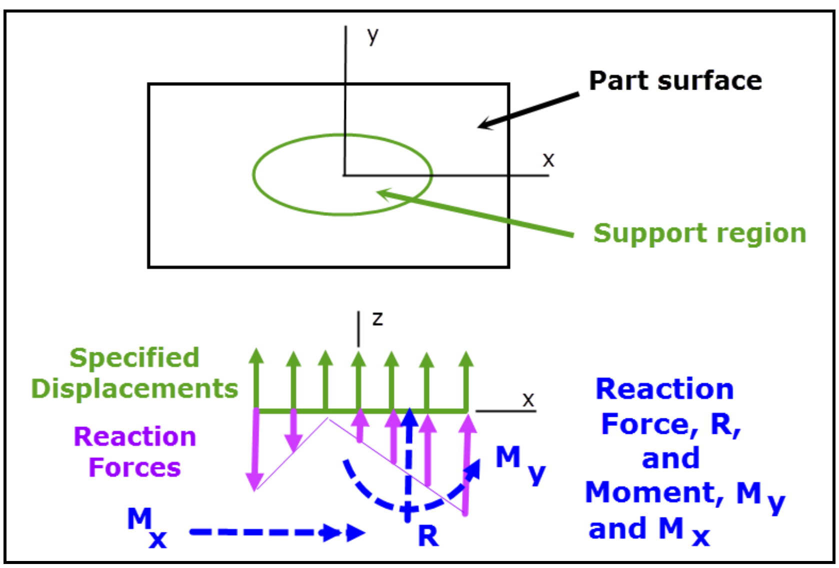
$$\sum \mathrm{M_0} = 0,\ \ \sum F_x = 0\ \ ,\ \ \sum F_y = 0$$

Reaction Forces and Moments from FEA Simulation Studies

Different displacement boundary conditions (or fixtures, or supports) cause the development of various reaction forces and moments needed to maintain such conditions. In the classic displacement based finite element analysis (FEA) any point on a solid has only three translations and therefore only three force components can act there. In displacement based FEA of beams and shell surfaces (including flat plates) at a point on the object one can prescribe either displacements or infinitesimal rotations. If any displacement component is specified at a point on a solid, shell or beam then (only) a reaction force, in the direction of that component, develops to maintain that condition. If an infinitesimal rotation (usually zero) is specified at a point on a shell surface or a beam then a reaction moment, in the direction of the rotation, develops to maintain that condition.

SolidWorks defaults to a solid body formulation for FEA simulations (even though a well-trained engineer can often get more accurate and more efficient results with 2-D models or beams). While only reaction forces develop at a supported point on a solid they cause moments at any other location. If a curve or surface area on a solid has specified constant displacement values (fixtures) then the reaction forces at that curve or surface also cause a static moment to develop as well.

In other words, a cluster of adjacent nodes with an imposed set of constant displacements (usually zero) prevent that region from rotating also. In mechanics rotations are related to moments (because their product yields mechanical work). When drawing a free body diagram (FBD) of the part one must include both a force and moment at the support region.



Distributed displacement restraints cause force and moment reactions

(Note: the support region NEVER should be touching just air!)