Forecasting Using Moving Averages

1. The moving average method is quite simple.

$$\hat{Y}_t = Y_{t-1} + Y_{t-2} + ... + Y_{t-q}$$

2. The length of the MA is also referred to as the span.

3. Note that if the span is 1 then we have the random walk with $m = 0$.

4. Exponential smoothing (ES). Criticism of the simple moving average is that it puts equal weight on each value. ES deals with this problem using a smoothing constant. Simple ES has a single smoothing constant. Forecasts are constructed using two equations. The first is the level of the series at time t. The value is not observable but can only be estimated. It is where we think the series would be at time t were there NO random noise.

   $$L_t = a Y_t + (1 - a) L_{t-1}$$

   The forecast is then

   $$F_{t+k} = L_t$$

   The smoothing constant is similar to the span in the moving average. Note that the forecast error, E_t, made in forecasting Y_t at time t-1 is $Y_t - F_t = Y_t - L_t$ then we can write

   $$L_t = L_{t-1} + a E_t .$$

5. Exponential smoothing with trend (Holt's method).

Simple ES works ok if there is no obvious trend. However, if there is a trend in the series then this method consistently lags behind it. If the series is increasing then simple ES will be consistently low. Holt's method introduces a time trend explictly (T_t), and a corresponding smoothing constnat for the trend effect (b). The interpretation of L_t is the same as before whereas the interpretation of T_t is that it represents an estimate of the change in the series from one period to the next. The equations are:

$$L_t = a Y_t + (1-a)(L_{t-1} + T_{t-1})$$
$$T_t = b(L_t - L_{t-1}) + (1-b) T_{t-1})$$
$$F_{t+k} = L_t + kT_t.$$

6. Winter's Model for Seasonality-either deseasonalize the series or use this (or others) method on the seasonal data. Add a new parameter (g) which controls how quickly the mthod reacts to percieved changes in the pattern of seasonality. If gamma is small then the method reacts slowly to it. The equations are:

$$L_t = a \frac{Y_t}{S_{t-m}} + (1-a)(L_{t-1} + T_{t-1})$$
$$T_t = b(L_t - L_{t-1}) + (1-b)T_{t-1}$$
$$S_t = g \frac{Y_t}{L_t} + (1-g)S_{t-m}$$
$$F_{t+k} = (L_t + kT_t)S_{t+k-m}$$