COMP 210, Spring 2002
Lecture 3: Conditional Logic

Things to do

- Explain homework hand-in procedure…
- Reading sections 1-5

Review

Last class, we:

1. Wrote some trivial Scheme programs (called functions, procedures, or methods in other language). I got the syntax slightly wrong.
2. Discussed *rewriting* as a way of evaluating Scheme programs.
3. Saw that complex programs can be built on the shoulders of smaller programs. *Always reuse an existing function if one exists.*
4. Introduced the notions of a contract and a purpose

Introducing the methodology

Did not define the header yet

To develop a program, we follow a simple design procedure:

1. Write a contract, purpose, and header
2. Develop several examples, chosen to elucidate the program's behavior
3. Write the program's body
4. Test the program to ensure that it behaves as specified

This class, we'll apply the methodology and extend it with more detail.

Another Example

Let's go back to paying for your pizza. Of course, even work-study jobs require you to pay taxes. Since the actual tax system is too complex for a second program, lets assume that Steve Forbes stages a massive political comeback and becomes President in 2004. To stretch things even further, let's assume that Congress adopts his flat tax, at a 17% rate.

Can we write a program that calculates our tax burden? We already wrote one that calculates wages

```
; Wage: num -> num
; Purpose:  calculate gross wage from # hours worked
;           assume pay rate is $6 per hour
(define  (Wage Hours) … )
```

Step 1

Test case	*Expected Output*	*Actual Output*
(Wage 1)	6	6
(Wage 2)	12	12
(Wage 15)	90	90

Step 2 *Step 4*

```
; Wage: num -> num
; Purpose:  calculate gross wage from # hours worked
;           assume pay rate is $6 per hour
(define  (Wage Hours)
   (*  6  Hours))
```

Step 3

To compute the taxes on our pizza:

The tax on wages of W is just (* W (/ 17 100)), or

```
; Taxes:  num -> num
; Purpose:  compute taxes on a gross wage
;           assume a 17% flat tax a la Steve Forbes
(define (Taxes W) … )
```

Test Case	*Expected Output*	*Actual Output*
(Taxes 1)	17/100	17/100
(Taxes 2)	34/100	34/100
(Taxes 15)	255/100	255/100
(Taxes 0)	0	0

```
; Taxes:  num -> num
; Purpose:  compute taxes on a gross wage
;           assume a 17% flat tax a la Steve Forbes
(define (Taxes W)
    (*  W  (/  17  100)) )
```

In Class Exercise (3 to 5 minutes):

Develop a program that computes your take home pay, under the flat tax system.

Finally, we might want to know our take home pay, so that we can figure out how much pizza to order.

```
; Take-home-pay: num -> num
; Purpose: compute take home pay, under Forbes' flat tax, from hours worked
(define (Take-home-pay Hours-worked)
  (- (Wage Hours-worked) (Taxes (Wage Hours-worked)) ) )
```

Together, these functions compute gross wages, net taxes, and net income. If we need to change the minimum wage, or the tax rate, each occurs in exactly one place.

Writing good programs requires good organization. Re-use existing programs, or break a program's task into smaller ones.

The Real World & Conditional Computation

One of the hardest aspects of working for pizza is computing your net pay, as opposed to your gross pay. Recall that your gross pay could be computed as

```
; Wage: num -> num
; Purpose:  compute gross pay from hours worked
;           assume a $6 per hour pay rate
(define (Wage H)
  (*  6   H))
```

Assuming that your pay rate is $6 per hour, and that H is the number of hours worked during a week.

Under the current tax code, wages up to $500 per week are taxed at 15%, above $500 to $1,500 per week are taxed at 28%, and wages above $1,500 per week are taxed at 33%. (To keep this example simple, we assume that the 36 and 39% brackets do not exist.)

How do we write our tax program to handle this more complex tax scenario? In other words, if I give you the amount of weekly earnings, how do we figure out the taxes? This is the program "Taxes" in the previous example.

If WW stands for the weekly wage, then we first determine which tax bracket includes WW, then we multiply it by the proper tax rate.

In math, we know that we can test equality and various inequalities.

```
3 < 10        => true
10 <= 3       => false
```

3 = 4 => false

In Scheme, we can write expressions that test equalities and inequalities and return either **true** or **false**

(< 3 10)

(<= 10 3)

(= 3 4)

We can combine **true**s and **false**s using **and**, **or**, and **not**. We write **true** and **false**, in Scheme as **#t** and **#f**.

To use this insight in a program, we need to lay out the intervals on the number line, and then construct an appropriate set of tests.

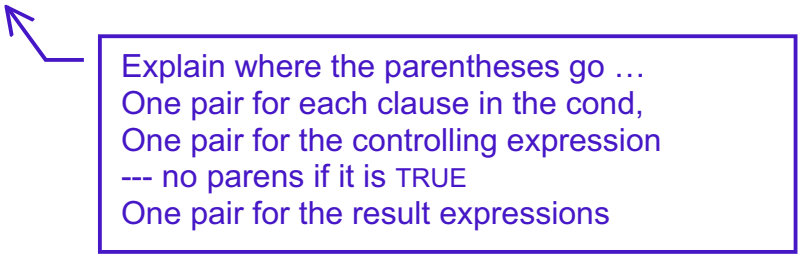
```
0------------------500--------------------------------1500-------------------------
      15%                  28%                            33%
```

If WW stands for our weekly wage, then the appropriate conditions are

```
(<= WW 500)
(and (< 500 WW) (<= WW 1500))
(< 1500 WW)
```

To put this together in Scheme, we use a construct called the **cond** construct (for conditional).

```
;  Real-Tax:  num -> num
; Purpose: compute the graduated income tax on a given weekly wage
(define (Real-Tax  WW)
    (cond
       ((<= WW 500)                          (* (/ 15 100) WW))
       ((and (< 500 WW) (<= WW 1500))  (* (/  28 100) WW))
       ((< 1500 WW)                          (*  (/ 33 100) WW)) ))
```



When a problem specifies different behaviors for different sets of numeric data, draw an interval picture, translate the interval into conditions, and use a **cond** expression to formulate the program.

Work an example, like (Real-Tax 300).

```
(Real-Tax 300)
=  (cond
      ((<=  300  500)                          (*  (/  15  100)  300))
      ((and (<  500  300) (<=  300  1500))     (*  (/   28 100)  300))
      ((<  1500 300)                           (*   (/  33  100)  300))  ))
=  (cond
      (#t                                      (*  (/  15  100)  300))
      (#f                                      (*  (/   28 100)  300))
      (#f                                      (*   (/  33  100)  300))  ))
= (*  (/  15  100)  300)
=  45
```

Of course, this isn't how DrScheme evaluates the cond expression. It evaluates each case, in the order that they appear.

```
(Real-Tax 300)
=  (cond
      ((<=  300  500)                          (*  (/  15  100)  300))
      ((and (<  500  300) (<=  300  1500))     (*  (/   28 100)  300))
      ((<  1500 300)                           (*   (/  33  100)  300))  ))

=  (cond
      (true                                    (*  (/  15  100)  300))
      ((and (<  500  300) (<=  300  1500))     (*  (/   28 100)  300))
      ((<  1500 300)                           (*   (/  33  100)  300))  ))

=   (*  (/  15  100)  300)
=   (*  15/100       300)
=   45
```

(RealTax 2000)

```
=  (cond
      (false                                    (*  (/  15  100)  2000))
      ((and (<  500  2000) (<=  2000  1500))    (*  (/   28 100)  2000))
      ((<  1500 200)                            (*   (/  33  100)  2000)) ))

=  (cond
      ((and (<  500  2000) (<=  2000  1500))    (*  (/   28 100)  2000))
      ((<  1500 200)                            (*   (/  33  100)  2000)) ))

=  (cond
      ((and false false)                        (*  (/   28 100)  2000))
      ((<  1500 200)                            (*   (/  33  100)  2000)) ))

=  (cond
      (false                                    (*  (/   28 100)  2000))
      ((<  1500 200)                            (*   (/  33  100)  2000)) ))

=  (cond
      ((<  1500 200)                            (*   (/  33  100)  2000)) ))

=  (cond
      (true                                     (*   (/  33  100)  2000)) ))
=  (*   (/  33  100)  2000))
```