COMP 210, Spring 2001
Lecture 2: Conditional Logic

Things to do

- Explain homework hand-in procedure…
- Reading sections 1-4 (for homework), 5-6 (for class)

Review

Last class, we:

1. Discussed the *rewriting* approach to evaluating Scheme programs.
2. Saw that complex programs can be built on the shoulders of smaller programs. This led to the 1st principle of COMP 210: *Always reuse an existing function if one exists.*
3. *Introduced a design recipe for writing programs*
   a) Write a contract, purpose, and header
   b) Develop several examples (mapping inputs to outputs), chosen to elucidate the program's behavior
   c) Write the program's body
   d) Test the program to ensure that it behaves as specified

This class, we'll apply the recipe and extend it with more detail.

In-class Exercise (3—5 minutes)

Recall our function Area

```
(define (area radius)
    (* pi (* radius radius)))
```

In your New Media Design Class, you are looking at some of the tradeoffs that arise in designing a new physical format for reusable optical disks. A critical parameter is the useful area of the optical disk. We can model its area as a disk with a hole in the middle.

Design a program, named **`area-of-disk-with-a-hole`**, that calculates the area of an optical disk. It should take two ARGUMENTs, the radius of the disk and the radius of the hole in its center. Recall the area program that we wrote earlier.

Work with the person seated next to you. One of you will get to write your solution on the board.

Skipped this section in Fall 2000.

Another Example

Let's go back to paying for your pizza. Of course, even work-study jobs require you to pay taxes. Since the actual tax system is too complex for a second program, lets assume that President Bush and Congress get carried away with the idea of tax reform and adopt Steve Forbes's flat tax proposal which stipulates a 17% tax rate.

Can we write a program that calculates our tax burden? We already wrote one that calculates wages

```
; wage: Num -> Num
; Purpose:  calculate gross wage from # hours worked
;           assuming pay rate is $6 per hour
(define (wage hours)
    (* 6 hours))
```

The tax on wages of `w` is just `(* w (/ 17 100))`, or

```
; taxes:  Num -> Num
; Purpose:  compute taxes on a gross wage
;           assuming a 17% flat tax with no exemptions
(define (taxes w)
    (* w (/ 17 100)))
```

Finally, we might want to know our take home pay, so that we can figure out how much pizza to order.

```
; take-home-pay: Num -> Num
; Purpose: compute take home pay from hours worked, given pay rate of $6
           per hour and a 17% flat tax with no exemptions
(define (take-home-pay hours-worked)
    (- (wage hours-worked) (taxes (wage hours-worked))))
```

Together, these functions compute gross wages, net taxes, and net income. If we need to change the minimum wage, or the tax rate, each occurs in exactly one place.

Writing good programs requires good organization. Re-use existing programs, or break a program's task into smaller ones.

The Real World

One of the hardest aspects of working for pizza is computing your net pay, as opposed to your gross pay. Recall that your gross pay could be computed as

```
; wage: Num -> Num
; Purpose:  calculate gross wage from # hours worked
;           assuming pay rate is $6 per hour
(define (wage hours)
    (* 6 hours))
```

Under the current tax code, wages up to $500 per week are taxed at 15%,

above $500 to $1,500 per week are taxed at 28%, and wages above $1,500 per week are taxed at 33%. (To keep this example simple, we assume that the 36 and 39% brackets do not exist.)

How do we write our tax program to handle this more complex tax scenario? In other words, if I give you the amount of weekly earnings, how do we figure out the taxes? This is the program "Taxes" in the previous example.

If `ww` *stands for the weekly wage, then we first determine which tax bracket includes* `ww`*, then we multiply it by the proper tax rate.*

In math, we know that we can test equality and various inequalities.

3 < 10 => *true*
10 <= 3 => *false*
3 = 4 => *false*

In Scheme, we can write expressions that test equalities and inequalities and return either *true* or *false*

```
(< 3 10)
(<= 10 3)
(= 3 4)
```

We can combine the values *true* and *false* using the operations *and*, *or*, and *not*. We write *true* and *false*, in Scheme as **`true`** and **`false`**.

To use this insight in a program, we need to lay out the intervals on the number line, and then construct an appropriate set of tests.

0----------500----------1500----------
15% 28% 33%

If WW stands for our weekly wage, then the appropriate conditions are

```
(<= ww 500)
(and (< 500 ww) (<= ww 1500))
(< 1500 ww)
```

To put this together in Scheme, we use a construct called the `cond` construct (for conditional).

```
; real-Tax:  Num -> Num
; Purpose: compute the graduated income tax on a given weekly wage
(define (real-tax ww)
    (cond
        ((<= ww 500)  (* (/ 15 100) ww))
        ((and (< 500 ww) (<= ww 1500))  (* (/ 28 100) ww))
        ((<  1500 ww)  (* (/ 33 100) ww))))
```

Let us work an example, like **`(real-tax 300)`**.

```
  (real-tax 300)
```

```
= (cond
      ((<= 300 500)  (* (/ 15 100) 300))
      ((and (< 500 300) (<= 300 1500))  (* (/ 28 100) 300))
      ((< 1500 300)  (* (/ 33 100) 300))))
= (cond
      (true  (* (/ 15 100) 300))
      ((and (< 500 300) (<= 300 1500))  (* (/ 28 100) 300))
      ((< 1500 300)  (* (/ 33 100) 300))))

= (* (/ 15 100) 300)

=  45     [two steps]
```

When a problem specifies different behaviors for different sets of numeric data, draw an interval picture, translate the interval into conditions, and use a **cond** expression to formulate the program.

The preceding program contains a gross logical error. What is it? Hint: Compute:

```
(real-tax 500)
(real-tax 500.01)
```

How much extra tax is paid on the extra penny of income?

How can we fix this problem? Use some simple algebra.

```
realtax(x) = .15*x                                    if 0 < x <= 500
           = .15*500 + .28*(x-500)                    if 500 < x <= 1500
           = .15*500 + .28*(1500-500) + .33*(x-1500)  if 1500 < x
```

which reduces to:

```
realtax(x) = .15*x                      if 0 < x <= 500
           = 75 + .28*(x-500)           if 500 < x <= 1500
           = 75 + 280 + .33*(x-1500)    if 1500 < x
```

Let us rewrite this function definition in Scheme:

```
; real-Tax:  Num -> Num
; Purpose: compute the graduated income tax on a given weekly wage
(define (real-tax ww)
   (cond
      [(<= ww 500)  (* (/ 15 100) ww)]
      [(and (< 500 ww) (<= ww 1500))  (+ 75 (* (/ 28 100) (- ww 500)))]
      [(< 1500 ww)  (+ 75 280 (* (/ 33 100) (- ww 1500)))]))
```

Comment: Scheme accepts square brackets in lieu of parentheses provided that they match.